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DISPLACEMENT INTERFEROMETRY BY THE AID OF THE ACHROMATIC FRINGES

PART III

BY CARL BARUS
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PREFACE.

The present report is chiefly devoted to the investigation of methods of research in which displacement interferometry, conducted by the aid of the achromatics discussed in the preceding report, gives promise of fruitful applications. Thus, in Chapter I the method of measuring small angles hitherto suggested is given a practical test. The general theory of the subject in its bearing on the two possible methods is developed at some length and a variety of interferometer devices, with mirror, ocular, and collimator micrometers, are instanced. Unfortunately, it was not till after the end of these experiments that I detected the method of reducing the fringes to the smallest number possible, practically to a single fringe; otherwise the work would have been more satisfactory throughout.

As the achromatic fringes can not (in general) be found without first finding the corresponding spectrum fringes and, conversely, since for each type of spectrum fringes (direct or reversed) a corresponding group of achromatic fringes may be associated, I have devoted Chapter II to spectrum fringes differing in their manner of production. The endeavor here has been to obtain interferences from distant slender luminous objects, without the aid of a slit. Partially at least the work has succeeded, but not as far as I hoped. The experiments are very difficult.

The work in the third chapter was undertaken at the request of Prof. W. G. Cady, of Wesleyan University, in the endeavor to obtain the elastic constants of small bodies. The application of the displacement method proved at once to be astonishingly easy in a case where a degree of rough handling is inevitable; but there lurked in the elastic apparatus some discrepancy, both of viscosity and hysteresis, the nature of which escaped detection even after many attempts to locate its origin.

Chapter IV contains applications of the rectangular interferometer using achromatic fringes to geophysical problems. A method for the determination of the Newtonian constant is worked out. Again, the same interferometer is associated with the horizontal pendulum for the detection of small changes in the inclination of the earth's surface. Series of observations extending between January and August are recorded.

Finally, in the last chapter, I have investigated corresponding methods for the interferometry of vibrating systems. The luminosity of the achromatic fringes lends itself easily to this purpose and it was merely necessary to design an appropriate vibrating telescope. To test the method, a study is made of the vibration of telephonic apparatus. For the first time I have obtained clear-cut interference vibration curves for two identical telephonic systems joined directly in series, while these forms subsided completely when the telephones were joined differentially. Such a system, from another point of view, is an electric dynamometer capable of appreciating an average alternating current well within a microampere. It could, moreover, be synchronized with an external impulse by aid of the Lissajous curves with the same accuracy as two tuning-forks.

CARL BARUS.

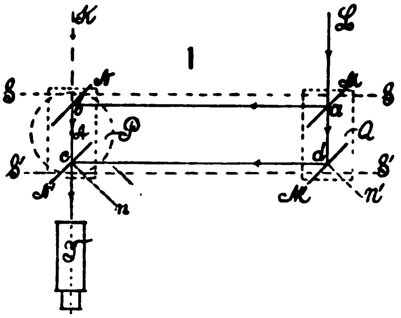
Providence, August 1918.

CHAPTER I.

THE DISPLACEMENT INTERFEROMETRY OF LONG DISTANCES.

1. Introduction.—Methods for the measurement of small angles and of long distances were broached at the end of the last report. It is the purpose of the present chapter to continue the work experimentally, with a view to further development. It will therefore be desirable to collect the useful equations in this place in relation to the form of apparatus to be adopted, as well as to deduce the consequences of these equations in relation to their bearing on displacement interferometry in general. Throughout the chapter the work is done chiefly with the aid of the achromatic fringe groups, as I have called them; but these, as a rule, can be found only by means of the spectrum fringes, wherefore the latter become of coördinate importance.

2. Apparatus.—This is an interferometer of the Jamin-Mach type (fig. 1) with four vertical plate mirrors, M,M', N,N', in parallel and at 45° to the horizontal beam of impinging white daylight L, from the country beyond. Three of these mirrors are half-silvered, viz, M,N,N', while M' is or may be opaque. The equal distances $ab=cd=b$ constitute the base-line (b) of the apparatus and the rectangle $abcd$ will be called the ray parallelogram. Its area is $2Rb$, where $ad=bc=2R$. Each of the mirrors must be provided with three adjustment screws for fine motion, so that the mirrors are each slightly revolvable about a vertical and a horizontal axis. In case of the mirrors N, M', these screws must be convenient to manipulate (thumb-screws), for adjustment here will frequently be necessary. The opaque mirror M' is on a Fraunhofer micrometer slide with its screw in the direction of the normal n' to M' and adapted to readings of at least 10^{-4} cm.


1

In the figure S, S' may be regarded as slides of a lathe-bed on which the carriages P and Q carrying the mirrors may move longitudinally. Any distance $ab=cd$ is thus available for a base-line.

The telescope is at T and receives both the light from L after two reflections from the paired mirrors and the direct light from K through the half-silvers. It is desirable that the two beams from K and L be of about equal intensity.

The pair of mirrors N and N' is on a vertical axis A, so that it may be rotated as a whole. The amount of rotation may be read off either directly on a divided circle corresponding to the axis A or indirectly by the displacement of M' on the screw at n' in relation to the observed sweep of interference fringes.

Thus there are three objects seen in the telescope: the direct landscape from K, the reflected landscape from L, and the achromatic interference fringes due to the partial beams *abc* and *adc* when the apparatus is in adjustment. To find the latter the method pursued in the preceding report suffices. It is first necessary to find the spectrum interferences when L is a beam of intense white light from a collimator and fine slit and the telescope is provided with a direct-vision prism. These are to be centered by moving the micrometer at M' and adjusting the pair of mirrors $M'N$. The spectroscope is now removed (swiveled out) and the slit broadened, whereupon the intense achromatic fringes will appear covering the position of the image of the originally fine slit. If not vertical, the fringes may be made so by further slightly rotating M' and N on a horizontal axis, in the absence of compensators.

The collimator at L is then removed and the fringes will be found superposed on the foreground. If they are not bright enough from the light of the landscape, they may be given any intensity by reflecting (by way of *adc* and *abc*) a narrow horizontal strip of skylight from white paper into the telescope from L. Intense fringes will be seen transverse to the strip.

3. Rigorous equations.—In addition to the sides of the ray parallelogram b (base) and $2R$ (R radius of rotation) we shall have to consider the following angles or angular increments: $\Delta\alpha$ the angular rotation of the paired mirrors, $\Delta\theta$ the corresponding angular displacement of the fringes, ΔN the linear displacement of the micrometer mirror in a direction normal to its face, and $\Delta\varphi$ the angle subtended by two consecutive fringes. If n is the order of the fringe, we may write

(1) $$\Delta\varphi = \frac{\Delta\theta}{\Delta n}$$

Moreover, if i is the angle of incidence (45°) of L at the mirrors and λ the wave-length in question,

(2) $$\frac{2\cos i\,\Delta N}{\Delta n} = \lambda$$

or on substitution,

(3) $$\frac{\Delta\theta}{\Delta N} = \frac{\Delta\varphi}{\Delta N/\Delta n} = \frac{2\Delta\varphi\cos i}{\lambda}$$

Moreover, if s is the angle at the apex of the distance triangle on the base b,

(4) $$\Delta s = 2\Delta\alpha$$

(5) $$\Delta\alpha = \frac{\Delta N\cos i}{R}$$

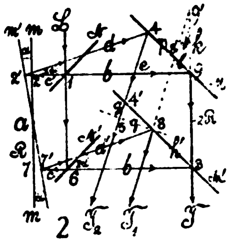
And since the distance $d = b/2s = b/2\,\Delta\alpha$, from (5),

(6) $$d = \frac{bR}{2\Delta N\cos i} = \frac{F}{\Delta N}$$

so that the sensitiveness is from (6),

$$\delta d = \frac{2d^2 \cos i}{bR}\,\delta(\Delta N) = \frac{\delta \Delta N}{Fd^2} \tag{7}$$

(*a*) It will next be desirable to deduce the above fundamental equations more rigorously than has thus far been done. Figure 2 is supplied for this purpose, and represents the more sensitive case where, in addition to the mirrors $MM,'$ NN' (all but M being necessarily half-silvers), there is an auxiliary mirror, *mm*, capable of rotation (angle α) about a vertical axis a. The mirrors, M----N', in their original position, are conveniently at 45° to the rays of light, while *mm* is normal to them. Light arriving at L is thus separated by the half-silver N at 1, into the two components 1, 2, 1, 9, 3, T and 1, 6, 7, 6, 3, T, interfering in the telescope at T.



2

When *mm* is rotated over a small angle α, these paths are modified to 1, 2, 2', 4, 4', 5, T_2 and 1, 6, 7, 8', T_1 and T_2 enter the telescope in parallel and produce interferences visible in the principal focal plane, provided the rays T_1 and T_2 are not too far apart, in practice not more than 1 or 2 mm. Interference fringes therefore will always disappear if the angle α is excessive, but the limits are adequately wide for all purposes. The essential constants of the apparatus are to be:

$(9, 1) = (6, 3) = b \qquad (1, 2) = (6, 7) = C \qquad (9, 3) = (1, 6) = (2, 7) = 2\,R$

R being the radius of rotation.

Where the mirror *mm* is rotated to *m'm'* over the angle α the new upper path will be:

$$C + R \tan a + d + e + g$$

where $(2'\ 4) = d$, $(4, 4') = e$, $(4,'5) = g$, the plane $(8, 5) = q$ normal to T_1, and T_2 being the final wave-front. The lower path is similarly

$$2\,R + (C\text{-}R \tan a) + d'$$

to the same wave-front (8, 5), where $(7', 8) = d'$. Hence (apart from glass paths, which are preferably treated separately) the path-difference $n\lambda$ (n being the order of interference) should be

$$n\lambda = 2\,R\,(\tan \alpha - 1) + d - d' + e + g$$

The figure, in view of the laws of reflection, then gives us in succession

$$\begin{aligned}
d &= (b + c + R \tan \alpha) \,/ (\cos 2\,\alpha + \sin 2\,\alpha) \\
d' &= (b + c - R \tan \alpha) \,/ (\cos 2\,\alpha + \sin 2\,\alpha) \\
e &= 2\,R \,/ (\cos 2\,\alpha + \sin 2\,\alpha) \\
g &= 2\,R \sin 2\alpha\,(1 + \tan \alpha)\,(\cos 2\,\alpha - \sin 2\,\alpha)/(\cos 2\,\alpha + \sin 2\,\alpha) \\
q &= 2\,R \sin 2\,\alpha\,(1 + \tan \alpha)
\end{aligned}$$

To obtain g it is sufficient to treat the similar triangles (3, 8, 9′) and (9, 8′, 9′), where $h=(9,4)$, $h'=(3,8)$, $k=(9,9')$, $l=(9,8')$ may be found in succession, as the normal distance between the mirrors M and M' is $R\sqrt{2}$, so that finally

$$g=(h\text{-}l)\sin(45^\circ-2\alpha) \qquad q=(h\text{-}l)\cos(45^\circ-2\alpha)$$

If these quantities are introduced into the above equation for $n\lambda$, we may obtain, after some reduction,

$$n\lambda=4R\sin\alpha(\cos\alpha-\sin\alpha)$$

Since $n\lambda=2\,\Delta N\cos i$, ΔN being the normal displacement of the mirror M' and $i=45^\circ$, the corresponding equation to the second order of small quantities, α, is

$$\frac{\Delta N}{\Delta\alpha}=\frac{2R}{\cos i}(\cos\alpha-\sin\alpha)=2\sqrt{2}R(1-\alpha-\alpha^2/2)$$

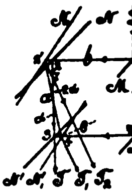
If α is sufficiently small, the coefficient is simply $2R/\cos i$ as used above.

There remain the glass paths which for the rays d and d' are compensated. Additionally the upper ray has a glass path (3) displaced to (4′), The lower ray has the fixed path at (1), and this is equal to the other at (1), since the angles are 45°. Thus the variable part of the glass paths at (3) to (4′) is uncompensated and the angle of incidence changes from 45° to $45^\circ-2\alpha$. The reflecting sides of the plates are silvered. Hence

$$e(\sin i-\cos i\tan r)2\Delta\alpha=\sqrt{2}(1-\tan r)e$$

must be added to the equation.

(*b*) The second case, figure 3, in which the auxiliary mirror of the preceding apparatus is omitted, is curiously enough inherently simpler. MM', NN' are mirrors (half-silvered at (1) and (3), and the two latter on a vertical axis a, and rigidly joined by the rail (2, 3). The mirrors being preferably at 45°, the component rays are 1, 2, 3, T and 1, 5, 3, T, the mirror M' being on a micrometer with the screw normal to the face. The ray parallelogram is made up as before of $(1,2)=b=(3,5)$ and $(1,5)=2R=(2,3)$. When the rail (2, 3) is rotated over an angle α, the mirrors take the position N_1 and N'_1, at an angle α to their prior position, and the angle of incidence is now $45^\circ-\alpha$. The new paths, if (4, 6) is the final wave-front, are thus (1, 2, 2′, 6, T_2) and (1, 5, 4, T_1). The rays T_1 and T_2 are parallel and interfere in the telescope. Hence the path-difference introduced by rotation is (n being the order of interference)



3

$$n\lambda=b+R\tan\alpha+\frac{2R}{\cos\alpha}\cos\alpha-(2R+b-R\tan\alpha)$$

or

$$n\lambda=2R\tan\alpha$$

for the triangle (a, 7, 2′) is isosceles and its acute angles each α.

The rays T_1 and T_2 have now separated and the amount (4, 6) is also $2R\tan\alpha$. When this exceeds a few millimeters the interferences vanish.

A correction must, however, be applied, since in the practical apparatus the mirrors rotate at a fixed distance apart, $2R$. Hence the mirror N_1 must be displaced toward the right (shortening the path) by the normal distance.

$$e=(R/\cos\alpha-R)\cos 45^\circ$$

and the mirror N_1' toward the left by the same amount. The path-difference introduced is thus a decrement and is twice the $2e\cos(45^\circ-\alpha)$ of each mirror. Thus the total correction to be subtracted from the equation is

$$\frac{4R}{\cos\alpha}(1-\cos\alpha)\frac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)\frac{\sqrt{2}}{2}=2R(1-\cos\alpha)(1+\tan\alpha)$$

Hence the equation becomes after reduction

$$n\lambda=2R\ (\tan\alpha-1-\tan\alpha+\sin\alpha+\cos\alpha)$$

or

$$n\lambda=2R\ (\sin\alpha+\cos\alpha-1)$$

To the second order of small quantities, if $i=45^\circ$ is the angle of incidence and ΔN the normal displacement of M',

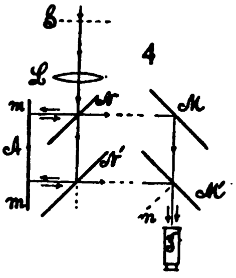
$$\frac{\Delta N}{\Delta\alpha}=R\frac{1+\Delta\alpha/2}{\cos i}$$

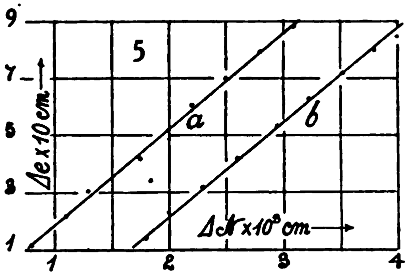
As all the mirrors receive the light on their silvered sides, M originally compensates N if the mirrors are identical in thickness and glass. But the transmission at (3) varies as the angle of incidence changes from $i=45^\circ$ to $45^\circ-\alpha$. The glass path here decreases by

$$e\ (\sin i-\cos i\tan r)\ \Delta\alpha$$

where e is the plate thickness, r the angle of refraction. The path-difference as above reckoned has thus been increased by this amount, and this quantity is to be added to the right-hand member. The effect will not usually exceed a small percentage of the air-path-difference and the ratio is the same as above.

4. Ocular micrometer.—It has been stated that the motion of the fringes across the field of the telescope (T, fig. 4) is astonishingly swift; hence it is often desirable to insert a micrometer here, as the displacement of fringes can thus be much more accurately and easily measured than at the micrometer along the normal n of the opaque mirror M of the interferometer. If the latter is of the type using an auxiliary mirror (mm, fig. 4) the fringes may even be established of a size to correspond with the ocular micrometer by rotating the auxiliary mirror; but this is not usually necessary. A good ocular plate micrometer was at hand, dividing the width of field (about 1 cm.) into 100 parts, the divisions being 0.1 mm. One-tenth of this is easily estimated by the eye in view of the eye lens. The light from the collimator SL should completely fill the field, a condition which may be fulfilled by suitably placing the former, or modifying its objective L. After completing such preliminary adjustments with the fringes, made very sharp, and the ocular scale equally so, this is to be placed at right angles to the fringes. Let Δe denote their displacement measured in centimeters on the ocular scale and ΔN (cm.) the displacement of the opaque mirror M of the interferometer. The question is whether Δe and ΔN are nearly enough proportional quantities for practical purposes. A number of such standardizations were carried out throughout 1 cm. of Δe, two of which are shown in detail in figure 5. The fluctuation of data is due to air-currents across the interferometer. It was not easy to obviate these, and it was not thought necessary for the present purposes. Otherwise the data would have been smooth. There is no doubt that a linear relation may be assumed. In curve *a* the readings of the interferometer micrometer increase, in curve *b* they decrease. If the means be taken from doublets far apart the ratios are





$$(a)\quad \Delta N/\Delta e = 0.00310 \qquad\qquad (b)\quad \Delta N/\Delta e = 0.00310$$

and they happen to coincide. Thus Δe is 323 times as large as ΔN and correspondingly easy to measure. The impossibility of setting the micrometer for ΔN accurately enough, since it is graduated to only 5×10^{-5} cm., is completely obviated in Δe. Moreover, as $2\Delta N \cos i = \lambda$ (i being the angle of incidence, 45°, and λ the mean wave-length), we now have

$$0.0061\ \Delta e \cos i = \lambda$$

so that the fringe displacement

$$\Delta e = \frac{6\times 10^{-5}}{6.1\times 10^{-3}\times 0.71} = 0.014 \text{ cm.}$$

measured on the ocular micrometer, corresponds to the wave-length of light in the interferometer measurements. This is more than one scale part. There is, however, no difficulty in making the fringes larger and obtaining a much more sensitive apparatus in proportion. The achromatic fringes, moreover, when properly produced, contain a distinctive central black line compatible with the measurement of 0.1 scale part, as here given; i. e., measurement to a few millionths of a centimeter are thus easily feasible under proper surroundings. The apparatus will be used below.

If $\Delta\varphi$, the angular fringe breadth, is given, $\Delta\theta/\Delta N$ may be computed from the above equation, Chapter I, No. 1, since

$$\frac{\Delta\theta}{\Delta N}=\frac{2\Delta\varphi\cos i}{\lambda}$$

or

$$\frac{\Delta e}{\Delta N}=\frac{2L\Delta\varphi\cos i}{\lambda}\text{ and }Ld\varphi=\frac{\lambda}{2\cos i\,\Delta N/Ne}$$

as both $\Delta\theta$ and Δe have the same radius; i. e., the length $L=19.5$ cm. of the telescope. Hence the fringe breadth in centimeters is, if $\lambda=6\text{x}10^{-5}$ cm., $i=45°$ and $10^3\Delta N/\Delta e=3.1$

$$L\Delta\varphi=6\times10^{-5}/(2\times0.071\times3.1\times10^{-3})=0.014\text{ cm.}$$

the value actually observed. Thus, if $\Delta\varphi$ is given or measured, $\Delta e/\Delta N$ may be deduced.

The question finally to be determined is thus the value and the meaning of the fringe breadth $\Delta\varphi$. Since $2\Delta N\cos i=n\lambda$, if $\Delta N=\Delta N_0$ is constant and also λ, we may then write

(8)
$$\Delta\varphi=\frac{di}{dn}=-\frac{\lambda}{2\Delta N_0\sin i}$$

Furthermore,

(9)
$$\Delta\theta=n\Delta\varphi=-\frac{\Delta N}{\Delta N_0}\cot i$$

if $n\lambda$ is replaced by its value and ΔN is small compared with ΔN_0. If it is not, since ΔN_0 involves ΔN, we must state the case thus:

$$-\Delta\theta=\cot i\int_{\Delta N_0}^{\Delta N_0+\Delta N}dN/\Delta N_0=\cot i\log\frac{\Delta N_0+\Delta N}{\Delta N_0}$$

or, on expanding the natural logarithm,

(10)
$$-\Delta\theta=\cot i\left(\frac{\Delta N}{\Delta N_0}-\frac{1}{2}\left(\frac{\Delta N}{\Delta N_0}\right)^2+\dots\right)$$

and

$$-\Delta e=L\Delta\theta$$

In the above measurements

$$\Delta\varphi=\frac{L\Delta\varphi}{L}=\frac{0.014}{19.5}=7.2\times10^{-4}$$

whence (apart from signs)

$$\Delta N_0=\frac{\lambda}{2\Delta\varphi\sin i}=0.06\text{ cm., nearly}$$

whereas the maximun displacement ΔN throughout the whole series (or field width in fig. 5) does not exceed $\Delta N=5\times10^{-3}$ cm. Hence $(\Delta N/\Delta N_0)^2/2$ may here be neglected to about 1/300 and (again apart from signs) since $i=45°$,

$$\Delta e=L\Delta N/\Delta N_0=19.5\frac{\Delta N}{0.06}=325\Delta N$$

as it should be; i. e., the relation of Δe and ΔN is practically linear, if the displacement ΔN is not excessive or goes beyond the field width.

As the determination of ΔN_0 is inconvenient, we thus come back to the practical equation (3) already used, or

$$\Delta N/\Delta\theta=\lambda/2\Delta\varphi\cos i$$

or if $Ld\varphi=\delta e$ and Δe and δe are the displacement and the fringe breadth measured on the same ocular micrometer,

$$\frac{\Delta N}{\Delta e}=\frac{\lambda}{2\delta e\cos i} \tag{11}$$

With this deduction, equations (5), (6), (7) now take on the new form in terms of the fringe breadth δe and the fringe displacement Δe, which it is well to record here;

$$\Delta\alpha=\frac{\lambda}{2R\delta e}\Delta e \tag{12}$$

$$d=\frac{bR\delta e}{\lambda}\frac{1}{\Delta e} \tag{13}$$

$$\delta d=\frac{d^2\lambda}{bR\delta e}\delta(\Delta e) \tag{14}$$

If into the last equation we insert such values as are easily obtained and will be used in the sequel, viz, $d=$ kilometer $=10^5$ cm.; $b=200$ cm.; $R=10$ cm.; $\delta e=0.015$ cm.; $\delta(\Delta e)=0.001$ cm. (1/10 scale part of ocular micrometer); then

$$\delta d=\frac{10^{10}\times 6\times 10^{-5}\times 10^{-3}}{200\times 10\times 15\times 10^{-3}}=20 \text{ cm.}$$

that is, an object should be located at a distance of a kilometer to 20 cm. If there is sufficient light, however, fringes may be easily made larger than 0.015 cm. in breadth, certainly 3 to 5 times, in which case δd would be correspondingly smaller. With ordinary daylight, however, $10^3\delta e=10$ to 20 cm. should not be exceeded.

Results similar to the preceding may be obtained if the glass paths of the interferometer are alone considered.

If μ is the index of refraction and e is the effective thickness of the plates, i. e., the difference of effective thickness of M and N and of compensators (fig. 1) through which the beams pass, we may write as in the colors of thin plates (since the beam passes the plate but once)

$$n\lambda=e\mu\cos r-2N\cos i \tag{8}$$

if r is the angle of refraction corresponding to the angle of incidence i. If n, i, r alone vary, while e, λ, N are fixed (i. e., if the eye travels through the field of the telescope from left to right), $\Delta\varphi=di/dn$, and since $\sin i=\mu\sin r$,

$$\Delta\varphi=\frac{di}{dn}=\frac{\lambda}{2N\sin i-e\tan r\cos i} \tag{9}$$

so that $\Delta\varphi$ depends inversely on e and N. When the spectrum ellipses are centered, $N=N_0$, a condition necessary for the occurrence of achromatic fringes, where

(10′) $$2\,N=2\,N_0=e\,(\mu\cos r+2\,B/\lambda^2\cos r)/\cos i$$

if $\lambda d\mu/d\lambda=2\,B/\lambda^2$ is adequate. Equation (10) may now be inserted in equation (9) and the coefficient of e, viz, the long parenthesis containing circular functions evaluated for $i=45°$, $\lambda=60\times10^{-6}$, $\mu=1.55$, $2\,B/\lambda^2=0.026$. Its value is slightly greater than 1. Hence we may write approximately but with much greater convenience,

(11′) $$\Delta\varphi=\lambda/e$$

We thus obtain the breadth of fringes for different values of the micrometer e, roughly. However, it would be possible to compute the accurate value of i, fulfilling the condition (10′).

These equations placed in the above (3), (5), (6) give in succession

(12′) $$\Delta\theta=\frac{2\Delta\varphi\,\Delta N\cos i}{\lambda}=\frac{2\cos i}{e}\Delta N$$

(13′) $$\Delta\theta=\frac{2R}{\lambda}\Delta\varphi\Delta\alpha\quad=\frac{2R}{e}\Delta\alpha$$

(14′) $$d=\frac{bR}{2\,\Delta N\cos i}\quad=\frac{bR}{e\Delta\theta}$$

so that the measurement of the long distance d depends ultimately on the area $2bR$ of the ray parallelogram, the differential thickness of paired glass plates e, and the displacement $\Delta\theta$ of achromatic fringes.

From equation (14) we obtain the sensitiveness by differentiation, or

(15′) $$-\delta d=\frac{d^2e}{bR}\,\delta(\Delta\theta)$$

Let the angle $\Delta\theta$ or its variation be measured in a telescope of length L and provided with an ocular micrometer, so that the angle $\Delta\theta=x/L$, x being the linear magnitude measured on this micrometer. Hence

(16′) $$\delta d=\frac{d^2e}{bRL}\,\delta x$$

It will be noticed that equations (12′) to (14′) are the same as equations (12) to (14), if e in the latter is replaced by $\lambda/\Delta\varphi$ (equation 11′), since $\Delta e=L\Delta\theta$ and $\delta e=L\Delta\varphi$.

5. Collimator micrometer.—It is obvious that the ocular micrometer may be an image of a scale, S (fig 4), placed in the wide slit of the collimator SL. The method is even more sensitive than the last and has the additional advantage that the telescope, containing no fiducial lines, may be shifted at pleasure. This is a great convenience when promiscuous experiments are contemplated. It was found best, in case of the preceding apparatus without ocular micrometer, to remove the slit altogether (fig. 4) and install a glass scale, S, about 2.5 cm. long and divided into 100 parts, in its place. A millimeter scale is usually too coarse. The scale S must be so inclined to the horizontal that its image is normal to the fringes in the telescope. No difficulty was found either as to clearness or precision with this adjustment. Measurements were made, from which (ΔN is the normal displacement at the mirror M, fig. 4) and $\Delta\epsilon$ the displacement of fringes along the imaged micrometer in the ocular)

$$\frac{\Delta N}{\Delta\epsilon}=0.00101$$

was found for the tenth-millimeter fringes considered in the preceding paragraph. Here $\Delta\epsilon$ is therefore nearly 1,000 times larger than ΔN. We may thus write $0.00202\ \Delta\epsilon \cos i=\lambda$, so that

$$\Delta\epsilon=\frac{6\times 10^{-5}}{2\times 10^{-3}\times 0.71}=0.042 \text{ cm.}$$

or nearly half a millimeter, corresponds to the wave-length of light in the interferometer measurements. With larger fringes the precision may be enhanced and sharp achromatic lines are available as before.

If we denote by $\delta\epsilon$ and $\Delta\epsilon$ the fringe breadth and the displacement of fringes in case of the collimator micrometer (imaged in the ocular) and by δe and Δe the corresponding quantities in case of the plate ocular micrometer, and if F and f be the principal focal distances of the objectives of the collimator and of the telescope (fig. 4), the relations are obviously

$$\frac{\delta e}{\delta\epsilon}=\frac{\Delta e}{\Delta\epsilon}=\frac{f}{F}$$

so that equations (10) to (14) are equally true when e is replaced by ϵ; but

$$\Delta N/\Delta e=\frac{F}{f}\frac{\Delta N}{\Delta\epsilon}$$

Hence $\Delta N/\Delta e$ is larger than $\Delta N/\Delta\epsilon$ in the ratio of F/f as actually found; but the data of the collimator micrometer at the slit are at once admissible in computing.

6. Half-silvered films.—The case of regular repetition of the achromatic phenomenon in case of white light, with the ghosts successively fainter, have frequently been mentioned. To possibly interpret this phenomenon a pair of half-silver plates were pressed together on the half-silver sides, so as to inclose a thin film of air between them. This double plate was then put into the beam from the collimater normally, whereupon a succession of ghosts was seen in the telescope, whereas none appeared without the air film. Hence they must be produced by reflection. In one case three repetitions were seen on the left and two (much enlarged) on the right of the main interference fringes, usually at an angle to them and more or less curved. The inclination of fringes was usually opposite on the two sides. In spite of differences in size the successive repetitions of fringes were nearly equidistant. Thus four at a distance of $\Delta e = 0.075$ cm. were seen. At another part of the double half-silver, $\Delta e = 0.085$ cm. was measured. Putting a steel clip on the plates to force them more closely together, Δe was reduced to 0.040 cm. Taking the clip off increased Δc. No ghosts occurred with a single half-silver plate (no air film). Again, the distance Δe varied with the angle of incidence of the plate pair with the collimated beam of light. Thus, in case of three vivid interference grids (two repetitions), if i is the angle of incidence, the measurements gave, when the plate was placed at different angles i (estimated)

$i = 0°$	$i = 30°–40°$	$i = 50°–60°$	$i = 70°–80°$
$e = 0.15, 0.48, 0.87$	$0.22, 0.48, 0.79$	$0.30, 0.49, 0.70$	$0.29, 0.39, 0.49, 0.65, 0.80$

In the last case the number of ghosts increased, but they also grew more irregular and confused.

Hence the cause of this originally puzzling phenomenon must be some reflection on the two sides of an air film, or a glass film. If x is the normal thickness of the film, and i the angle of incidence, the direct rays and those twice reflected interfere with a path-difference of $2x \cos i$. To restore the fringes to the center the main micrometer would have to move over ΔN or annul a path-difference of

$$2\frac{\Delta N}{\Delta e}\Delta e \cos i$$

so that

$$x \cos i = (\Delta N/\Delta e)\Delta e \cos i = 0.0033 \times 0.7\Delta e = 0.0023\Delta e$$

Thus in the above data the reproductions would be at

$\delta e =$	0.36	0.28	0.20	0.80
$10^5 x =$	83	83	82	82 cm.
$i =$	0°	36°	54°	77°

provided the angles were such as here given, which was sufficiently near the case.

It is nevertheless difficult to surmise what reflections can occur in the earlier work and in the absence of a specially half-silvered biplate; for the effect of an air film between clear glass plates is only visible with difficulty.

A similar problem is the measurement of the index of refraction, μ, of a normal film of glass in terms of δe. The relation is here

$$((\mu - 1) + 2\ B/\lambda^2)\ d = 2\ (\Delta N/\Delta e)\ \Delta e \cos i = 0.0047\ \Delta e$$

A film of mica $d = 0.0050$ cm. thick was inserted between the mirrors of the interferometer and the displacement $\Delta e = 0.65$ cm. read off. Hence, if $2B/\lambda^2 = 0.026$ roughly,

$$\mu = 1 + 0.0047 \times 0.65/0.0050 - 0.026 = 1.58$$

This is slightly low, for the thickness d and the dispersion constant B are not adequately guaranteed; but the result is interesting, as it may be obtained instantly, either in terms of Δe or ΔN.

7. Direct observations.—The first experiments were made upon objects lying across the campus of Brown University, since these distances, though relatively small, were measurable. Unfortunately there were few available long clear stretches, and distances of two objects at $d=9{,}990$ cm. and 3,060 cm. were used. They were not, of course, in the same straight line, so that the constant small angle between them must be borne in mind. The constants of the apparatus were also correspondingly small (as it was necessary to look out of a window), being $b=51.8$ cm., $2R=9.4$ cm., obtained by passing a beam of collimated sunlight through the system of mirrors and measuring the length and breadth of the ray parallelogram. The angle of incidence was $i=45°$. Hence, since $d=bR/2 \cos i\ \Delta N=F/\Delta N$, the factor is

$$F=bR/2 \cos i=172.2.$$

It is first necessary to get ΔN_0, the micrometer position for parallel rays, as the plate mirrors were common plate glass. This is done by inverting the equation, since $\Delta N=F/d$ and $\Delta N'=F/d'$ for the two objects. The results were $\Delta N=0.01723$ cm., $\Delta N'=0.05626$ cm. Hence the computed equivalent of the angle between the two objects is $\Delta N'-\Delta N=0.03903$ cm. (computed). The value directly observed in the interferometer was $\Delta N'-\Delta N=0.0389$ cm. (observed), a very satisfactory agreement. Here it is to be carefully noted that the fringes in the second measurement $\Delta N'$, must be placed in coincidence with the displaced image of the first object, not with the coincident images of the second object, in which case the micrometer reading ($\Delta N'=0.0411$ cm.) would be too large, or with the actual direction of the first object (non-reflected beam K), in which case the reading ($\Delta N'=0.0368$ cm.) would be too small; for the two objects are not in the same straight line. This is of course very important, even if the angles are small.

We now have

$$\Delta N-\Delta N_0=0.01723 \text{ cm.} \qquad \Delta N'-\Delta N_0=0.05626 \text{ cm.}$$

where $\Delta N=0.0459$ and $\Delta N=0.0839$ are the observed values. Thus $\Delta N_0=0.0277$ cm. and the practical equation now reads

$$d=bR/2 \cos i\ (\Delta N-\Delta N_0).$$

Inverting the operation, we thus find

First object, observed: $\Delta N=0.0450$ cm., $d=9{,}992$ cm. (computed), $d=9{,}990$ cm. (observed).

Second object, observed 0.0339 cm., 3,063 cm. (computed), 3,060 cm. (observed).

The constant ΔN_0 is independent of the fatcor F. Hence the base b and R may be changed at pleasure, from the exceptionally small value 51.8 cm. here used. Finally, for this small base b of but about half a meter the micrometer play between $d=10$ meters and infinity would be ($\Delta N-\Delta N_0=F/d$)

$d=10$ meters $\quad \Delta N=0.172$ cm.
$d=$kilometer, $\quad \Delta N=0.0017$ cm.

and the sensitiveness δd if $\delta\ (\Delta N) = 10^{-4}$ cm. is the least micrometer reading at

$d = 10$ meters $\quad \delta d = 0.58$ cm.
$d = 100$ meters $\quad 58$ cm.
$d =$ kilometer $\quad 5,814$ cm.

in view of the small base 52 cm. and small $R = 4.7$ cm. The observed data were well within this. Of course on using the fringes individually these results could be immensely improved.

The endeavor was now made to work at larger distances d and a larger base b. But the laboratory being surrounded almost on all sides by trees, it was found that only at one upper window was a distance prospect visible, and hence, since it was again necessary to look through a window obliquely, the base was restricted to but $b = 36.6$ cm., not much above a foot. Still, as the values of ΔN are proportional to b, the purpose of the experiments could be adequately carried out within a range of about a mile, using the distant hill or horizon for ΔN_0 corresponding to $d = \infty$. Three objects were selected, a school-house gable, a church spire, and a house on the hill. The results were as follows:

TABLE 1.—Measurement of larger distances; base $b = 36.6$ cm.; $2R = 9.4$ cm.; $i = 45°$.

	$\Delta N \times 10^3$	$(\Delta N - \Delta N_0) \times 10^3$	d(cm.)	d (miles)	$\Delta N \times 10^3$	$(\Delta N - \Delta N_0) \times 10^3$	d cm.	d miles
Hill..	30.1 cm.	0.0 cm.	∞	∞	26.6 cm.	0.0 cm.	∞	∞
Spire.	31.8 cm.	1.7	72,000	0.44	28.4	1.8	68,000	0.42
Gable	....	...	....	....	31.9	5.3	23,000	0.14

The results for the spire obtained on different days and with different adjustments are as close as the limit of the micrometer (10^{-4} cm.) admit. All were in agreement with the data taken from a city map. With a larger b and a larger R, the accuracy would increase as the product of these quantities, so that the results are entirely satisfactory, notwithstanding the limited range. Measurements from the roof of the laboratory, which would have been very onerous, in view of the improvised apparatus, were therefore abandoned. I may add that by screening off the direct ray (K), the apparatus may be adjusted (if out of order) by putting the two images of the reflected (L) rays in coincidence. The fringes with daylight are perhaps not as intense as one would wish and too many are visible, differing in this respect from the case where an intense light and a collimator are used. In these directions further work is necessary. With ordinary plate and vertical fringes, the two images will usually be one above the other. Vertical coincidence in such a case is secured by aid of a fine vertical cross-hair in the ocular. Since all coincidences are subject to this, its precise trend is not of importance. Complete coincidence may, however, always be obtained with the use of compensators, as indicated in § 10.

8. Indirect observations.—These refer particularly to the actual use of the large angular displacement $\Delta\theta$ of the group of fringes in the ocular as a whole, where $\Delta\theta = 2\Delta N \cos i/e$. If Δx corresponds to $\Delta\theta$ on an ocular micrometer and if the length of the telescope is L,

$$\Delta\theta = x/L \text{ or } \Delta x = 2L\Delta N \cos i/e$$

so that Δx may be very large as compared with ΔN. If, therefore, the distance of one object in the field is given, the distance of others might be advantageously found from $\Delta\theta$ or Δx. There is a difficulty, however, since $\Delta\theta$ and e vary together; i e., $d = bR/e\Delta\theta$. From this it follows that

$$d - d' = (dd'/bR)\ (e'\Delta\theta' - e\Delta\theta)$$

and only for such small differences of d and d' as may leave e appreciably constant could $d - d'$ be regarded as proportional to $\Delta\theta' - \Delta\theta$.

To get rid of the e it is necessary to introduce the fringe-breadth $\Delta\varphi$, which is measurable, while e is not. Since $e = \lambda/\Delta\varphi$ nearly, the equation becomes

$$d - d' = \frac{dd'\lambda}{bR}\left(\frac{\Delta\theta'}{\Delta\varphi'} - \frac{\Delta\theta}{\Delta\varphi}\right)$$

But this is virtually counting the number of fringes which pass between d and d'. If there are n fringes the equation may be written

$$d - d' = \frac{d^2}{bR} n\lambda$$

If $d' = \infty$, this becomes $d = bR/n\lambda = bR/(\Delta\theta/\Delta\varphi)\lambda$, where n is zero if $d = \infty$. There are but few fringes visible, however, and hence such an equation has but the specified limited application for distances close together, if fringes only are to be used. An example of the use of the latter equation follows, $\Delta\theta$ and $\Delta\varphi$ being estimated by an ocular micrometer. As before, if $b = 36.6$ cm.; $2R = 9.4$ cm.; $\lambda = 6 \times 10^{-5}$ cm., it was found that

$$\Delta\varphi = 0.010 \text{ cm.} \qquad \Delta\theta = 0.6 \text{ cm.}$$

Hence

$$d = \frac{36.6 \times 4.7}{(0.61/0.010) \times 6 \times 10^{-5}} = 5 \times 10^4 \text{ cm.}$$

This is too large as compared with $d = 4.3 \times 10^4$ cm. above, but no more so than the difficulty of measuring $\Delta\theta$ for a group of fringes $\Delta\varphi$ and the estimated λ imply. These 60 odd fringes as they passed by definite points of the ocular could have been counted here. In conclusion, measurement in terms of $\Delta\theta$ require a brightening of fringes and a diminution of the visible number. The number here was about 20 or more and they were visible during a displacement of $\Delta\theta = 100\Delta\varphi$, about. They were not quite equidistant, moreover, decreasing about 10 per cent in distance apart toward the ends of the group. A narrow strip of white paper illuminated by sunshine and visible in the field was the best background for their illumination. A few experiments were made on the relation of $\Delta\theta$, ΔN, and $\Delta\varphi = 0.01$ cm. on the ocular micrometer. The results were

$\Delta N\times 10^3$	$\Delta\theta\times 10^3$	$(\Delta N/\Delta\theta)\times 10^3$
2.5	620	4.0
2.2	500	4.4
2.4	570	4.2
3.6	850	4.2
3.9	870	4.5

The mean value $\Delta N/\Delta\theta = 0.00425$ agrees with the theoretical value $\lambda/2\cos i = 6\times 10^{-5}/1.41 = 0.0043$ quite as well as the observations warrant.

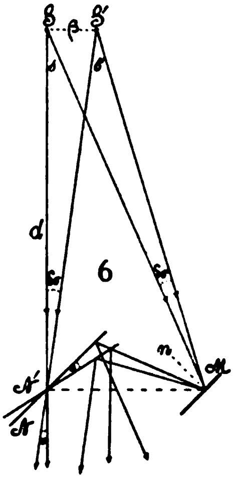
Another interesting application of measurement by fringes is shown in figure 6. This is concerned with the distance apart, β, of two objects S and S' (β parallel to b), both at a distance d from the observer. In passing from S to S' the angles at the base b are incremented by $\delta\sigma$ and decremented by $\delta\sigma'$, where

$$s - s' = \delta s + \delta s' = 2\delta s$$

if S is very distant relatively to b. But $2\ \delta\alpha = \delta s$; whence $\delta\alpha = \delta\sigma$ and $\beta = d\ .\ \delta s = d\ .\ \delta\alpha$. Since $2\ \Delta\alpha = 2\Delta N\cos i/R = n\lambda/R$, we obtain finally

$$\beta = d\frac{n\lambda}{2R}$$

if n fringes correspond to $\delta\alpha$, or the passage from S to S'.



Thus in case of the spire above, where $d = 7\times 10^4$ cm. or 0.43 mile, about 120 fringes were counted in passing from one side to the other of a conspicuous ledge of rock. Hence, since $2R = 9.4.$ cm.,

$$\beta = 7\times 10^4\frac{120\times 6\times 10^{-5}}{9.4} = 54\text{ cm.}$$

9. Ellipses and hyperbolas.—The occurrences of spectrum ellipses and of hyperbolas, the rotation of fringes, centering of ellipses, etc., may be sufficiently explained as follows. Let the equation

$$(1)\qquad n\,\lambda = 2e\mu\cos r$$

refer to the horizontal axis, x, passing through the center of ellipses in the field of the ocular of the telescope. Here n is the order of a given dark fringe in wave-length λ, e the thickness, μ the index of refraction, and i and r the angles of incidence and refraction, all referring to the half-silver plate. There is no compensator. In the vertical direction in the spectrum the light is homogeneous in λ and the equation would be at the center upward or downward $n\lambda = 2e\mu\cos\beta$ . . . . (2), if α and β are here the angles of incidence and of refraction (vertical plane). Since the angle α at least is very small, we may consider as an approximation that these equations hold throughout the field of the spectrum. Hence in general the e of equation (1) is to be replaced by $e\cos\beta$ in accordance with equation (1) and an equation applying throughout the spectrum may therefore, under these simplifying conditions, be written

$$n\lambda = 2e\mu \cos r \cos \beta \qquad (3)$$

This changes to (1) or (2) according as β or r is zero. Moreover, if x and y are the linear coördinates of points of the spectrum and the length of the telescope tube is L, horizontal and vertical angles will be $\theta = x/L$, $\beta = y/L$, with the center of ellipses as an origin.

By the law of refraction (3) may then be changed to

$$\mu\, n^2\lambda^2 = 4e^2\,(\mu^2 - \sin^2 i)\,(\mu^2 - \sin^2 \alpha) \qquad (4)$$

Since α is small,

$$\lambda = D \sin \theta' = D \sin (\theta_0 + \theta) = D\,(\sin \theta_0 + \theta \cos \theta_0)$$

where θ is small in comparison with θ_0 (D being the grating space and $\theta' = \theta + \theta_0$ the angle of diffraction), we may write further

$$\mu\, n^2D^2\,(\sin \theta_0 + \theta \cos \theta_0)^2 + e^2\alpha^2K = e^2K\mu^2$$

where $K = 4\,(\mu^2 - \sin^2 i)$.

This is an ellipse if n at the center corresponds to a maximum value, in terms of the variables θ and α, so long as K and μ are considered constant. But as μ and therefore K vary with λ, though slowly, it is true the equation is more complicated.

When the center of ellipses is not in the field, but passes through the vertical plane corresponding to the center of the field of view, the ellipses may soon become appreciably straight lines in their *visible* contours, and the fringes must rotate in one direction or the other, according as the center is above or below the field. Rotation will be rapid when the vertical axis of the ellipses is relatively long. To bring the center into the field (for a proper value of N), the angle α must be zero, i. e., the two corresponding opaque mirrors which reflect the interfering beams must be rotated on a horizontal axis towards each other, or from each other, until $\alpha = 0$, or the horizontal plane through the field is a plane of symmetry.

Furthermore, since the fringes necessarily move toward or from the center of ellipses with change of N, the motion of fringes will necessarily be oblique if the center of ellipses is obliquely outside the field of view. In the limit, if the center is in the vertical plane specified, horizontal fringes will rise or fall.

Finally, if n passes through a minimum instead of a maximum, the fringes will be roughly of the hyperbolic type.

At the center of ellipses in case of spectrum fringes, n is therefore a maximum relatively to points of the spectrum in the same vertical or transverse line of homogeneous color. This maximum is due to obliquity, the horizontal one to change of λ. In the case of fringes produced with white light (without dispersion), like the colors of thin plates generally or the achromatic fringes discussed elsewhere, the center of ellipses (which are now circles) is an absolute maximum, horizontally or vertically, i. e., relative to points in all directions from the center and for each color. The center of spectrum ellipses, therefore, has no direct relation to the center of white light fringes; for the latter occur only when the rays pass the plate normally. On the other hand, when the white-light fringes are straight lines corresponding to very oblique incidence of interfering rays, the spectrum fringes are none the less perfect ellipses.

It is finally necessary to account for the coincidence in adjustment of the center of spectrum fringes and the achromatic fringes, as the latter overlie the coincident white slit-images from which the superposed spectra are produced by the grating. This is easily seen to be referable to the fact that interferences with white light can only be visible if the light in the region of interference, when analyzed spectroscopically, contains but few dark bands. Since the number of bands in the spectrum is least near the center of ellipses, and is further reduced on making them as large as possible, the relation is obvious. In the case of strong, large achromatic fringes, a single fringe virtually occupies the whole spectrum. The light is either white or black.

The displacement of the center of ellipses with the angle of incidence for a given adjustment may be computed from the original equation for centers $N=e(\cos r+2B/\lambda^2\cos r)$, where r is the angle of refraction for the incidence i, e the thickness of plate, and B the dispersion constant. When N and e do not vary it may be shown that (since $\mu=A+B/\lambda^2$),

$$\frac{d\lambda}{di}=\frac{\lambda}{2B/\lambda^2}\,\frac{\sin r\cos i\,(2B/\lambda^2-\mu\cos^2 r)}{\mu\cos^2 r\,(2+\cos^2 r)}$$

To obtain an estimate $2B/\lambda^2=0.026$ may be neglected as compared with $\mu\cos^2 r$ and the equation given the approximate form ($\mu=1.5$)

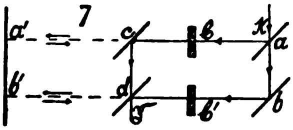
$$\frac{d\lambda}{di}=\frac{\lambda}{0.026}\,\frac{\sin i\cos i}{2.5\,\mu}=10\,\lambda\sin i\cos i,\text{ nearly.}$$

Thus, if $i=45°$, $d\lambda/di=5\lambda$ or $0.09\,\lambda$ per degree of i, which is about 100 times the distance of the sodium lines. If $i=0°$ or $90°$, the shift vanishes.

10. Compensators.—When the fringes are found they may be erected as stated by rotating either pair of the diagonal mirrors of the ray parallelogram towards or from each other, usually on a horizontal axis. The fringes may be enlarged by rotating the paired mirrors on either end of the ray parallelogram and restoring the fringes after each small step of rotation by displacement ΔN at the micrometer. But these processes are tedious and must be very cautiously performed or the fringes are liable to be lost. The same result may be accomplished by the aid of plate-glass compensators, about 0.5 to 1 cm. thick, placed normally in each of the two interfering beams and originally parallel and vertical. (See fig. 7, C and C'.) In addition to rotation and enlargement, these compensators serve with further advantage in equalizing the two beams in intensity. For this purpose it is merely necessary to half-silver lightly the compensator in the stronger beam of the ray parallelogram. If the fringes are more nearly vertical (between

45° and the vertical) they may be erected by rotating either compensator or both around a horizontal axis and enlarged by rotating around a vertical axis. The two compensators should be actuated together in opposite directions if there is danger of the fringes leaving the field; i. e., if the adjustment is considerable. Similarly, if the fringes are more nearly horizontal, and particularly when horizontal fringes are wanted, the fringes may be leveled by rotating the compensators in opposite directions around a vertical axis and enlarged by rotating around a horizontal axis. Horizontal fringes, which climb up and down the broad white slit-image and may be made quite large, are often very advantageous. The illuminated field is much more extensive in the vertical direction.

For slight adjustments it is convenient to have the compensators nearest at hand rotate in the same direction as the fringes. This may be done by working either on one side or the other of the center of fringes. The compensators may easily be manipulated by hand (without tangent screws) and they are most efficient when nearly normal to the respective beams. To pass from vertical to horizontal fringes one would first rotate the compensators in opposite directions around a horizontal axis until the fringes are inclined about 45°, after which the further rotations would be made in opposite directions around a vertical axis. The motion of fringes indicates the proper direction of the rotation of the plates.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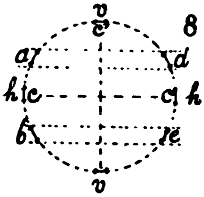
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To account for these apparently complicated effects, it is sufficient to recall that the compensators displace the center of fringes, usually enormously distant outside of the field of view, and besides that invisible with white light; for fringes are visible only in the narrow strip for which the spectrum fringes are very large and centered. Hence the result of rotation around a horizontal axis is to change fringes (fig. 8) of the type *a*, through *c* (vertical) into *b*, while the center moves downward, and vice versa. Again, rotation around a vertical axis changes fringes of the type *a*, through *c'* (horizontal) into *d*, as the center moves from left to right, and vice versa. The effects are necessarily opposite for the two beams. If the fringes are made vertical as at *c*, rotation of a compensator around the vertical axis can have no effect of rotation of fringes; for the center moves in the line of symmetry; but the effective or differential thickness of plates (*e*) is changed and hence the fringes are increased or decreased in size.

In view of the presence of compensators, *C* and *C'*, figure 7, the original adjustment is much simplified; for it is necessary merely that the spots of sunlight on the mirrors at *a* and *b*, figure 7, and at *a'* and *b'*, one or more meters off, be at the same level and at the same distance apart, nearly. The accurate adjustment at *c* and *d* for coincidence horizontal and vertical is then made with the telescope at *T*. When the distances are approximately equal, fine spectrum fringes will nearly always appear. These are enlarged and centered as specified. A broad slit with the spectroscope removed will then show the achromatic fringes, which may in turn be enlarged and rectified. Horizontal fringes, though often convenient, are at fault, inasmuch as they must rotate with the displacement of the micrometer ΔN. This appears at once from figure 8, for the center of ellipses is shifted. Vertical fringes alone are free from rotation in relation to ΔN.

11. Number of fringes visible, etc.—To get the most promising conditions for observing coincidence in case of range finding, the direct and reflected images should be about equally intense. Hence, if α is the coefficient of reflection, the two equal intensities are

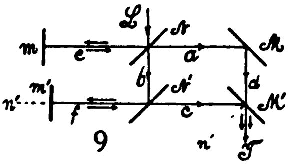
$$(1-\alpha^2)=2(1-\alpha)\alpha^2, \text{ or } \alpha=1/2$$

It is best, however, to have α less than this and to darken the direct beam if necessary by a thin half-silver plate interposed in the beam. If $\alpha=1/2$ the images are too dark and require higher illumination of the foreground than is usually present. As for the achromatic fringes themselves, they may be obtained with clear plate and opaque mirrors almost as well as with half-silvered plate, if the supernumerary images are partially screened off. With optic plate glass they would not appear. The surprising appearance of satellites—i. e., repetitions of the group with increasing faintness—is also common with clear plate.

A series of experiments were made by replacing the half-silver plates with grid-like opaque mirrors. These are easily made by removing the silver along parallel lines (using a T-square) with a sharp wet stick. The slit images were then also gridlike in appearance and the achromatic fringes occurred only on the dark bars. For clearly the superposition of beams takes place on reflection from glass only. In this way the fringes on the supernumerary slit images were identified. These occurred on the bright bars. The two phenomena are therefore complementary.

The reason for different numbers of visible fringes is less easily understood. In the original experiments two achromatic fringes (black or white), with about three green-reddish fringes on either side and rapidly fading out, were alone visible. This narrow grid is very advantageous if displacement interferometry is in question, for the achromatic fringes are easily recognized. Subsequently, however, large numbers of less distinctive fringes (20 or more) were usually obtained. As a small number of fringes is as frequently obtained with clear glass as with half-silvered plate, the occurrence is not attributable to the silver. (§ 13.) A variety of experiments were made with lenticular compensators, convex or concave, in each beam. The fringes, though obtained without difficulty, were usually rounded and irregular and the results without interest.

12. Separate adjustable auxiliary mirrors.—To obtain strong interferences the two component rays *eadT* and *bfcT* of the rectangular interferometer must not only be parallel on entering the telescope *T* (fig. 9), but they must be locally coincident at the mirror *M;* i. e., the two pencils from the collimator *L* (*M′*,*N*,*N′* being half-silvers) finally entering *T*, must very nearly coincide. Otherwise, even when the path-difference is annulled and there is perfect coincidence of the slit image, no fringes may be obtained. This is often a great annoyance when the mirror *M′* is on a micrometer screw (*n*) normal to the face of the mirror; for on continuously displacing *M′* the rays *cM′T* and *dM′T* separate more and more fully and the fringes soon vanish, unless a fresh adjustment for local coincidence is made. It is for this reason that the fringes are often so hard to find. The achromatics are much less sensitive to this disadjustment than the spectrum fringes; but the former are so mobile and easily lost that they have to be found as a rule by the aid of the latter.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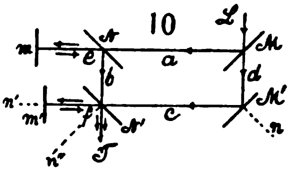
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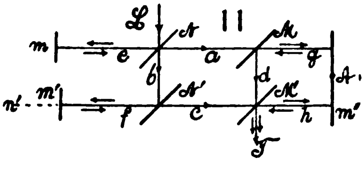
To meet this difficulty there must be one mirror available which reflects the component beam normally and which may be displaced parallel to itself; i. e., whose micrometer screw is parallel to the incident and reflected ray. This condition is most easily secured by separating the auxiliary mirror into the parts *m* and *m′*, each normal to its respective ray, while *m′* only is on a micrometer screw *n′*. Under these circumstances there is no difficulty in finding the fringes after the adjustment for parallel rays and local coincidence at *M′* has been made once for all by actuating the mirror *m′* in one direction or another. Moreover, it makes no difference, within limits, how the parallelism and local coincidence are secured by moving any of the mirrors *M*,*M′*, *N*,*N′*, *m*,*m′*, all of which must be on three leveling screws. Finally, if the mirror *m* and *m′* rotate as a rigid system about a common axis, it is still possible to use *mm′* for the measurement of small angles.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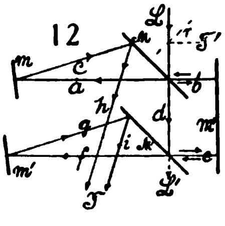
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If the rays *a* and *c*, which may be of any length, are very long, the adjustment shown in figure 10 is preferable, as the observer at *T* is near the micrometer screw *n′*. Here *M*, *N*, *N′* are half-silvers.

For the case, however, in which the body whose small angles of rotation are to be measured is part of another apparatus which does not admit of manipulation, the method may be modified as in figure 11. Here all the plate mirrors *M*,*M'*,*N*,*N'* are half-silvers and the rays from the collimator *L* form the interfering pencils *LemeagdT* and *Lbfm'fct*. The mirror *m* and *m'* meet their rays normally and *m'* is in the micrometer screw at *n'* parallel to *f*. The mirror *m''*, on the axis *A* normal to the paper, rotates, and its small angles of displacement are to be measured. Considerable light is lost in the three penetrations of half-silver films by each ray; but in case of the achromatic fringes the light is usually in excess, so that the diminution of light is an advantage. It is more difficult, however, to find the spectrum fringes, as these require a slit.

The plan of figure 11 is carried out more simply in figure 12, where both reflections take place at the same mirrors *M* and *M'*, respectively, the component rays being *Lbm''bachT* and *Ldem''efgiT*. It is necessary to incline the parallel mirrors *m'* and *m''* on a vertical axis, in order to avoid the entrance ray *L'* into the telescope at *T*. But this separates the component rays *h* and *i* locally, so that means must be employed (compensators, rotation of the other mirrors *m''*, *N*, or *M'*) to obviate this as far as necessary. In both cases of figures 11 and 12 it is therefore not easy to find the fringes, and I did not persevere in the quest because of an eye affliction contracted at the time.

Similarly the system *M*, *M'*,*m''* of figure 12 might be used if a half-silver is placed at *r* and the telescope at *T'* to the right of it. In this case the mirror *m''* must be in two parts, with adequate air-space inserted into the shorter ray *Lb*.

13. Types of achromatic fringes.—The difficulty of obtaining fringes of the strictly achromatic type (i. e., two strong fringes with a black line between and the remaining fringes green-reddish and faint) in the rectangular or other interferometer, has been frequently referred to in the text above. As a rule the fringes found are more or less diffuse, non-symmetric, with large numbers (10 or more) about equally strong. Such fringes are, of course, useless in displacement interferometry. When the sharp fringes needed are obtained, their definition is independent of the particular part of any of the glass plates used, and any plate may be rotated 180° in its own plane without spoiling the sharpness of the fringes. Hence such slight curvatures or wedge-shapes as the plates may possess are without influence on the phenomenon. To further test this I devised a screw-press adapted to push the vertical edges of a plate to the rear and the middle forward, so as to give the plate marked cylindricity. Quarter and eighth inch plates were operated on, in the latter case sufficiently to give the two superposed slit-images quite unequal width; but no essential or useful improvement of the fringes was observed. The type of the fringe was not altered. Again, the symmetrical fringes may be obtained from plates thickly or thinly silvered, without essential difference.

It follows, therefore, that the relative thickness of the glass paths traversed by the interfering beams can alone be of influence in shaping the fringe pattern in the manner in question. This is in consonance with the general theory of achromatic fringes, the result being a superposition of the color phenomena due to the dispersive refraction of the glass and the colors resulting from the wave-lengths of the interfering rays. To test this the apparatus, figure 10, is particularly convenient, as the fringes are easily found. Moreover, both rays, *a* and *c*, from the collimator at *L*, eventually pass through the plate *N'* before reaching the telescope at *T*. It is thus merely the thickness of the half-silvers *M* and *N*, both at 45°, that is here in question. If this thickness is the same, the sharp symmetrical design of but two strong fringes appears. If the difference of thickness is but little over 0.5 mm., many fringes, non-symmetric in distribution, are the rule. If the differential thickness is several millimeters there may be hundreds of fringes. If these are small they may be enlarged at pleasure; but they are always faint and useless for measurement.

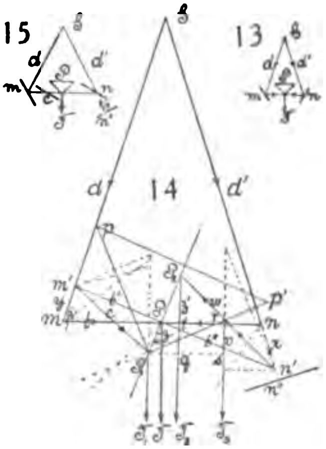
CHAPTER II.

THE INTERFEROMETRY OF SMALL ANGLES, ETC.—METHODS BY DIRECT AND REVERSED SUPERPOSED SPECTRA.

14. Introductory.—It occurred to me that a number of the methods treated in my papers on direct and reversed spectrum interferometry might be used directly for the measurement of small angles and possibly of the distance of the source of light. Such a procedure would have an apparent advantage, at least theoretically, of not calling for the preliminary superposition of two images of distant objects, as the superposition is inherent in the method itself. But there are large constants involved, which make the result very problematical unless these constants can be removed by a compensator. Indeed, it is also very questionable whether such interference can at all occur. A further difficulty which hampers the method is the decrease of size of objects as their distance increases. Nevertheless a progressive investigation with the object of ascertaining to what degree the experiment is feasible is worth while, and as it will be convenient to develop the methods without reference to the ulterior conditions which limit the interferences, this method has been pursued.

15. Method with prism.—Figure 13 is a sketch of one of the methods in which *S* is the distant source of light, from which rays *d* and *d′* strike the mirrors *m* and *n*, and are thence reflected to the silvered sides of the right-angled prism *P*. After leaving it the rays enter the spectroscope at *T* in parallel. If the proper angles are selected the prism *P* may be replaced by one of any angle or by a reflecting grating.

Suppose now the system ***mPn*** is securely attached to a rigid metallic beam or rail capable of rotating around a vertical axis at its center (*P*). This is indicated in figure 14, where the direction of rays and the normals of mirrors have been drawn and where the angle of rotation *α* has placed ***mPn*** into the position ***m′Pn′***. The result is that a part *y* of the ray *d* is cut off on the left side and a part *x* added to the ray *d′* on the right side, so that the path-difference, which may be assumed to have been zero originally, is now appreciably incremented, but not symmetrically for both sides.



It may be shown, however, that the rays ***n′P₂T₂*** and ***m′P₁T₁*** still enter the telescope in parallel and that therefore the conditions of interference have not been disturbed. This is the interesting feature of the method, for the angle α between the two positions of the rigid beam will also be the angle between all corresponding normals of the mirrors, as indicated in the diagram. If we take the case on the left, the angle between incident and reflected ray at P will therefore be $\pi 2+4\alpha$ for the original mirror at P and $\pi/2+4\alpha-2\alpha=\pi/2+2\alpha$ for the new position at P_1. But the angle between the rays reflected at m and m' respectively as $2\ \alpha$. Hence if T_1P_1 is prolonged backwards it must intersect the line mn at the original angle $\frac{\pi}{2}$ and thus P_1T_1 is parallel to PT. Similar reasoning applies on the other side for P_2T_2 and will still hold if the direction of the ray Sn prolonged is *reversed*. Finally, $\pi/2$ may be any reasonable angle.

It will contribute to a more adaptable design of the apparatus for general interferometry if the ray Sn' may also be reversed by reflection (fig. 15, mirror n'') in parallel to itself, allowing a small lateral offset, similar on both sides for clearance of the mirrors. Reflection between fixed parallel mirrors on the left in d and between mirrors set at a reentrant right angle on the right, say at n'', would accomplish this at corresponding distances for the transverse rays. Again, half-silvers may be used at m and n for reflection, which method is probably best. These details will here be disregarded. If small angles are to be measured the direct method is enormously more sensitive.

16. Estimate.—The full expression for the path-difference corresponding to the rotation of rail α will be complicated and of no interest here. It is not sufficient to regard the intercepts y and x as solely contributing to the path-difference, which would therefore be $x+y$ for the direct case and $x-y$ for the case when the ray d' is reversed somewhere at n'' (fig. 15) and returned parallel to itself. It may be shown that for small angles α, if β is the angle between incident and reflected rays originally at m and n and b the distance $mP=Pn$, d the distance $Sm=Sn$,

$$n\lambda=2b\alpha-2b\alpha^2+2b^2\alpha/d \qquad n\lambda=2b\alpha^2-2b\alpha^2/d$$

are sufficiently approximate equations up to the squares of small quantities to meet the interference for the direct and the reversed cases respectively. Hence, if for instance $\alpha=1°=0.0175$; $b=1$ meter $=10^2$ cm.; $d=1$ kilom. $=10^5$ cm; $\lambda=6\times10^{-5}$ cm., the number of fringes corresponding to each of the terms may be computed as

$$\text{(Direct) } n=6\times10^4-10^3+60 \qquad \text{(Reversed) } n=10^3-60$$

In the first case over 61,000 fringes pass per degree of rotation, $\alpha=1°$. This makes about 2.9×10^{-7} or about 0.06 second of arc per fringe. But the method is insensitive as regards distances d, unless the first two terms can be compensated. In the second or reversed-ray case, the method would be relatively much more sensitive as regards d if the first term $2b\alpha^2$ could be compensated. The difficulty lies in the occurrence of α^2 in the term, whereas most compensators would act as the first power of α.

Furthermore, if the angle α is small and S is displaced over an angle α or a distance $r=d\alpha$ to the right, the original triangle may be regarded as restored. Hence the same number of fringes roughly should pass back again. In the second case, supposing $2b\alpha^2$ can be removed by compensation, $r=d\alpha$ and $\alpha=\lambda d/2b^2$, nearly, or

$$r=\lambda d^2/2b^2=6\times 10^{-5}\times 10^{10}/2\times 10^4=30 \text{ cm.}$$

or the object should be located to 30 cm. at a kilometer for each fringe passing. In this case d need not be known, since

$$r=d\alpha=\frac{2b^2\alpha^2+b^2\alpha^3}{n\lambda}$$

and n fringes are observed to pass for the angle of rotation α in the compensated apparatus. The direct rays without compensation would of course give indefinitely better results if d is known; for the angle per fringe has been found as $\alpha=2.9\times 10^{-7}$ when $r=d\alpha=0.03$ cm. per fringe if d is 1 kilometer.

Unfortunately, however, the method of figure 14 can not be rigorously carried out experimentally. For in any practical apparatus the mirrors M and N would have to rotate at a fixed distance from each other, apart from the micrometers; i. e., the two mirrors rotate on a rigid radius or rail and are therefore both rotated and displaced. It is this displacement which is relatively of much importance and by it all terms involving the first order of distance d are wiped out, so that terms of the second order in b/d only remain.

17. Equations.—To derive these equations certain intercepts of the rays, figure 14, in addition to x and y, b and b' may be defined. $P_1\,P_2$ is the trace of the vertical plane of symmetry of the right-angled prism, if rotated at an angle α to the right. In this case the reflected ray $n'P_2q$ on the right corresponds to the reflected ray $m'P_1$ on the left, both terminating in the common wave-front P_1qs before entering the telescope.

Let $n'P_2=c'=b\sin\beta/\cos\alpha\sin(\beta-\alpha)=s^1/\sin\alpha\cos\alpha$
$m'P_1=c\ =b\sin\beta/\cos\alpha\sin(\beta+\alpha)=s/\sin\alpha\cos\alpha$
$P_2t=s'=b\sin\alpha\sin\beta/\sin(\beta-\alpha)$
$tq=s\ =b\sin\alpha\sin\beta/\sin(\beta+\alpha)$
and $nn'=x=b\sin\alpha/\sin(\beta-\alpha)=s'/\sin\beta$
$mm'=y=b\sin\alpha/\sin(\beta+\alpha)=s/\sin\beta$

since the original angles at the ends of the base are β and the rotation α. The angles between incident and reflected rays are respectively $\beta-2\alpha$ at n', $\beta+2\alpha$ at m', $90^\circ-2\alpha$ at P_2, and $90^\circ+2\alpha$ at P_1. Most of the angles are indicated in the figure. The new radii $m'P=b'=b\sin\beta/\sin(\beta-\alpha)$; $n'P=b''=b\sin\beta/\sin(\beta+\alpha)$.

The rays, however, do not reach the planes of symmetry, but are reflected by the faces of the right-angled prism, and this may be sketched in, in the rotated position (angle α) at P_1pp'. The path of the reflected rays from n'

is now $n'rs$ and from m', $m'P_1$ before they meet in the common wave-front P_1qs. Hence the intercepts

$$rs=v=(z+z')\ (\cos\alpha-\sin\alpha)/(\sin\alpha+\cos\alpha)$$
$$rP_2=w=(z+z')/\cos\alpha\ (\sin\alpha+\cos\alpha)$$

will enter in treating the path-differences. On the left the rays have not been distributed.

If we take the direct case first the original path-difference SnP and SmP may be regarded zero or n and m in the same phase. On rotation, therefore (angle α), the path-difference is increased on the right by $x+c'-w+v$ and increased on the left by $-y+c$, so that the total path-difference is equivalent to the equation

$$n\lambda=c'-c-(w-v)+x+y$$

If the above equivalents are inserted, this equation may be reduced to

$$n\lambda=2b\sin\alpha\frac{\sin\beta\cos\beta/\cos\alpha-\sin^2\beta\sin\alpha+\sin\beta\cos\alpha}{\sin(\beta+\alpha)\sin(\beta-\alpha)}$$

in which the three terms in the numerator correspond to the respective intercepts $c'-c$, $w-v$, $x+y$.

Since α and β are small angles, we may write $\sin\alpha=\alpha$, $\cos\alpha=1-\alpha^2/2$, and $\cos\beta=b/d$. Therefore the equation would, for practical purposes, become

$$n\lambda=2b\alpha-2b\alpha^2+2b^2\alpha/d$$

the three terms corresponding to the xy, wv, and cc' effect.

In the case of reversed ray (fig. 2) we may consider the points m' and n' in the same phase. Hence the original path-difference ($\alpha=0$) is $x-y$. The path-difference after rotation $c'-w+v-c$. The total change of path-difference due to rotation is thus given by

$$n\lambda=c'-c-(w-v)-x+y$$

This differs from the preceding by the deduction of $2x$. The rays again terminate in the common wave-front P_1qs to enter the telescope. Hence after reduction

$$n\lambda=2b\sin\alpha\frac{\sin\beta\cos\beta/\cos\alpha-\sin^2\beta\sin\alpha-\sin\alpha\cos\beta}{\sin(\beta+\alpha)\sin(\beta-\alpha)}$$

the terms showing the cc', wv, and xy effects. The approximate form of this equation is thus practically

$$n\lambda=2b\alpha^2+2b^2\alpha/d$$

The wv effect predominates, the cc' effect is intermediate, and the xy effect very small if d is large, as already instanced.

The preceding equations may also be obtained geometrically by letting fall the normal from n (fig. 14) to the prism-mirror and prolonging the ray at s backward. In the isosceles triangle so formed the angle at the base is $45°-\alpha$. Hence in the above notation the path-difference takes the form

$$x+2(c'-w)\cos^2(45°-\alpha)-(z'-z)-(c-y)$$

On inserting the values of the quantities as given above and reducing, an equation identical with the above appears, which for small α is

$$n\lambda = 2b\alpha\,(\operatorname{cosec}\beta - \alpha + \cot\beta)$$

If the prism has its nose at P (nearly), or in the axis of rotation, a small correction is to be added to the preceding expression. The path-difference on the right is increased by

$$z'\frac{1+\cos(90^\circ - 2\alpha)}{\cos\alpha\,(\cos\alpha+\sin\alpha)} + z'\frac{2\sin\alpha}{\cos\alpha - \sin\alpha}$$

and increased on the left by

$$z'/\,(\cos\alpha - \sin\alpha)\cos\alpha$$

Hence the correction is on reduction

$$2b\sin^2\alpha\,\sin\beta/\sin\delta = 2b\alpha^2, \text{ nearly.}$$

This merely wipes out the small middle term, $-\alpha$, of the above equation, leaving

$$n\lambda = (2b\alpha/\sin\beta)\,(1+\cos\beta)$$

When the prism is reduced to reflectors in its plane of symmetry, as treated at the beginning of this paragraph, the equation loses the terms $w-v$ and reduces to

$$n\lambda = x+y+z+z'+c'-c, \text{ or to } n\lambda = 2b\alpha\,(1/\sin\beta + 1)$$

In the practical apparatus the mirrors m and n rotate on a fixed radius b, whereas b in the diagram elongates on the right and contracts on the left respectively to

$$b'' = b\sin\beta/\sin\delta \qquad b' = b\sin\beta/\sin\sigma$$

Hence the mirrors in the apparatus are displaced normally on the right and left by

$$e = (b''-b)\cos\beta/2, \text{ inward, and } e' = (b-b')\cos\beta/2, \text{ outward.}$$

The path-difference thus introduced is the sum of the decrease on the right and increase on the left and its value is $2e\cos i$, when i is the angle of incidence in question. Thus the correction is (after reduction)

$$(b''-b)\,(\cos\alpha + \cos(\beta-\alpha)) + (b-b')\,(\cos\alpha + \cos(\beta+\alpha))$$

The expression may be further reduced to

$$\frac{2b\alpha\cos\beta\sin\beta}{\sin\sigma\sin\delta}(\cos\beta + \cos\alpha)$$

when α is a small angle, or to

$$2b\alpha\,(1/\sin\beta - \sin\beta + \cot\beta)$$

If this quantity is deducted from the above equation for path-difference and direct rays there remains simply $n\lambda = 2b\alpha\sin\beta$. The latter, therefore, is the equation to be used in interpreting the observation. So that generally when $i = \beta/2$ for the micrometer at n

$$2\cos i\,\Delta N/\Delta\alpha = 2b\sin\beta$$

In the case of reversed rays the conditions on the left remain the same as before. But on the right the mirror n is set at an angle $\beta/2$ to the rail and at right angles to its former position. Hence the normal displacement is $e=(b''-b)\sin\beta/2$. The angle of incidence is $i=90°-(\beta/2-\alpha)$. Thus the path-difference here to be deducted is $2\,e\cos i$ or

$$2(b''-b)\sin\beta/2\ .\ \sin(\beta-\alpha)=(b''-b)\ (\cos\alpha-\cos(\beta-\alpha))$$

and the total deduction from both sides is therefore

$$(b''-b)\ (\cos\alpha-\cos\delta)+2(b-b')\ (\cos\alpha+\cos\sigma)$$

This expression when reduced gives for small α

$$2b\alpha\ (\cot\beta-\alpha\cot\beta/\sin\beta)$$

or more simply

$$2b\alpha\cot\beta$$

It is practically as large as the total path-difference for reversed rays found above. If, therefore, the two effects are opposite in sign, the path-difference introduced by rotation would be zero, apart from the change of glass paths and second-order effects which are relatively small. In fact, the experiments show that the rotational effect, $\Delta\alpha$, in case of reversed rays, is relatively negligible as compared with the effect in case of rays not reversed. In other words, if from the equation for direct rays

$$n\lambda=2b\alpha\ (1/\sin\beta+\cot\beta)$$

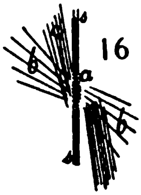
we deduct

$$2x+2e\cos i+2e'\cos i=2b\alpha\ (1/\sin\beta+\cot\beta)$$

the right-hand member vanishes to the second order of small quantities.

18. Observations. Prism-prism method.—In this case (fig. 19 below) a sharp-angled prism at *S*, with its knife-edge vertical, cleaves the beam of white light issuing from a collimator, reflecting the beams *d* and *d'* as described in my earlier papers. The system should be leveled so that all corresponding rays lie in a horizontal plane. By making the strips of light on both mirrors *m* and *n* (figs. 13, 14) coincide horizontally and vertically (using an auxiliary lens, if necessary) and then placing the prism *P* so that the rays *mP* and *nP* all but escape at its edge, the adjustment may be completed by aid of the telescope at *T*. The two slit-images, which should be equally bright, are made to coincide horizontally and vertically by the adjustment screws on *m* and *n*. If now the direct-vision spectroscope (prism grating) is swiveled in front of the objective of *T*, fringes will usually appear when the path-difference is annulled. For this purpose the prism *P* is placed on a Fraunhofer micrometer with the screw in the direction *mn*. The spectrum fringes are as a rule easily found and are quite strong, but they can not be centered in the field of view, for the occurrence of ellipses presupposes the rigorous superposition of the two strips of light on the edge of the prism *P*, which is not possible. The fringes, if too oblique, may be erected by a plate compensator with a horizontal axis, or the prism *P* may be rotated on a horizontal axis. Vertical spectrum fringes are not usually wanted in these experiments, for they are to serve only as an essential aid to finding the achromatic fringes.

It is a curious fact that although the ellipses can not be produced nor the slit much widened, apparently achromatic fringes do occur in white light for a micrometer placement at P such as should produce centered ellipses. Moreover, as the white slit-image is linear, the achromatic fringes are preferably made to run transverse to it. They are then exceedingly brilliant, extending much beyond the slit-image, and they travel up and down it with the motion of either micrometer at P or at n, (ΔN), or with the rotation of the rail ($\Delta\alpha$). As there are but four or six fringes with but one or two strong and brilliant, they make an exceedingly sensitive index for measurement. The occurrence of achromatic fringes may also be detected in the solar spectrum, as all the Fraunhofer lines (homogeneous light) become helical and broad from the cross-hatching due to the fringes. Here with homogeneous light the fringes are indefinite in number and follow each other continuously, whereas with white light but one or two intense black-white fringes appear. Though the achromatic fringes are by far the most brilliant part of the phenomenon, they rarely occur without streamers. The general appearance is roughly suggested in figure 16, where ss is the white slit-image in the telescope and a the achromatic fringes moving up and down ss when ΔN or $\Delta\alpha$ change. In the lateral glare of the field, however, fan-shaped or radiating coarse fringes bb are seen, intersected with very fine hairlike fringes cc. Probably there is also an intermediate group. These streamers are very useful to register the approach of the achromatic fringes, which move so rapidly that they are easily lost.



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A few measurements or rather estimates were made to coördinate the values of ΔN of the micrometer displacement at n and the corresponding rotation $\Delta\alpha$ of the rail necessary to annul this displacement. To do this the achromatic fringes were placed on the cross-hair, or better, on the image of the cross-hair at the slit of the telescope, and both readings were taken. They were then displaced by rotating the rail and restored by moving the micrometer. To measure the rotation an index was placed at the end of the rail (radius 27 cm.) moving over a millimeter scale observed with a lens. The constants of the triangle, figure 13, were

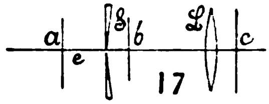
$$b = 20 \text{ cm.} \qquad d = 62 \text{ cm.} \qquad \beta = 71.3^\circ \qquad i = 35.6^\circ$$

Corresponding readings were found as follows in two separate adjustments:

$10^4\Delta N$	$10^4\Delta\alpha$	$10^4\Delta N$	$10^4\Delta\alpha$
0	0	0.0	0
204	9	26.9	7
493	19	76.0	30
Mean $\Delta N/\Delta\alpha = 26.5$ cm./radian		79.8	33
		132.5	56
		178.3	74
		179.3	74
		204.0	85
		Mean $\Delta N/\Delta\alpha = 25$ cm./radian	

Since $\Delta N/\Delta\alpha = \dfrac{2b \sin \beta}{2 \cos i} = 23.3$ the observed data are above the computed values, but not more so than the difficulties of these measurements on an improvised apparatus imply. A much more refined method for finding $\Delta\alpha$ is, of course, essential.

19. Interference from rough surfaces.—The question now at issue is whether the interferences can be retained when the collimator is removed and the light comes directly from a ground-glass surface or a Nernst filament. The spectrum fringes go at once when the slit is widened; not so the achromatic sets. Having produced them clearly with sunlight, I found that a ground-glass screen or a scratched mica film could be placed at *c* or *b* or *a*, figure 17, whereas *S* is the slit and *L* the collimating lens. The fringes should be transverse, as in figure 16, as vertical fringes are too easily confounded with the white slit-image. The slit was now broadened or quite removed; but the fringes, though less prominent from excess of non-interfering light, remained in place distinctly and without other change. On removing the lens, however, the fringes invariably vanished.



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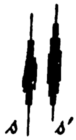
I now replaced the sunlight by the light of a Nernst filament, under the impression that ground glass might to a small degree still behave like plate glass. The same experiments were made, the filaments at *e* (fig. 17) replacing the ground glass. In this case, however, I first removed the lens *L* and it was then seen that the two washed slit-images were not superposed, as is otherwise obvious; but it accounts for the failures of the experiments with sunlight. Superposing the two vague images both out of focus, a position was soon found in which the achromatic fringes appeared brilliantly. The slit could now be widened or removed at pleasure, yet the fringes persisted strongly, but with loss of brilliancy.

It is thus possible to obtain these achromatic fringes directly from the Nernst filament and without a collimator; but they are so mobile, with change of $\Delta\alpha$ and ΔN, that to find them it is necessary first to produce the spectrum fringes with collimator and spectro-telescope; then to find the achromatic fringes on removing the spectroscope; next to remove the lens of the collimator and adjust for superposed images; and finally to remove the slit. These non-collimated achromatic fringes are best seen in a particular focal plane of the telescope and they change their focal plane with displacement ($\Delta\alpha$, ΔN). They practically cover the whole width of the washed slit-image. They usually measure about 0.5° in width, but the streamers may extend laterally five times further, depending on the adjustment. When pronounced, the slit-images may even be separated as in figure 18, while each alone retains the achromatic fringes. This puzzling phenomenon, which I had previously obtained, is probably due to the intersection and interference of rays in a region in advance of the plane of vision. Finally, as the transverse arrow in figure 19 (indicating the right and left side of the slit-images) show, after the reflections at *p m n P*, although the slit-images are not reversed, the superposed rays are reversed; for these constitute the right-hand and left-hand radiation from the filament, separated by the prism *p*. If the slit is widened or removed, there is only one vertical line of rays (coinciding with the position of the slit before removal) which can interfere. The remaining light does not interfere and its admixture robs the phenomenon proper of its brilliancy.



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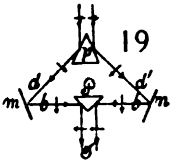
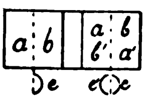


19

A few experiments made on the nature of the achromatic phenomenon here obtained showed that the fringes are probably Fresnellian interferences. To test this the objective of the collimator was removed and strong fringes were obtained by passing the two washed images of the slit over each other laterally, by moving the corresponding adjustment screw on the mirror *n*. It was found that the fringes passed from horizontal maxima in size, gradually to vertical hair-lines, as the images slid horizontally from contact of their nearer edges to the contact of the further edges. The coarse fringes were even strongly present in the narrow gap between slit-images before contact. The telescope was now focussed on the slit, so that sharp linear images appeared. The fringes vanished; but it appeared that the coarse fringes corresponded to coincident sharp slit-images when observed out of focus, and the fine fringes to sharp slit-images far apart. The whole phenomenon thus depends on the distance apart of two lines of light and the interferences are observable before or behind their plane.

20. Reversed rays.—The apparatus was now adjusted for the reversal of the rays *d'* by adjusting a mirror at some place *n''* (fig. 15) on a fixed micrometer and in such a way that the rays on reflection retraced their path. The mirror at *n* being a half-silvered plate, in turn reflected the rays toward the prism *P*. This modification of apparatus introduces very considerable path-difference, 2 *nn''*, on the right, which must therefore be compensated on the left. It is difficult to accommodate the micrometer and leveling devices at *n* and *n''* without an allowance of 5 to 10 cm. of path-excess. In my first experiments, which were merely tentative, the compensation on the left was secured by inserting a glass column about 15 cm. long. With this and the right-and-left micrometer displacement of the prism, or the to-and-fro motion of the mirror *n''*, path-difference was easily annulled and the ellipses found in the spectrum. They are centered as usual by rotating the glass compensator on a horizontal and a vertical axis, till with the occurrence of parallel rays at *T* the illuminated strips on the prism coincide, locally, to the eye.



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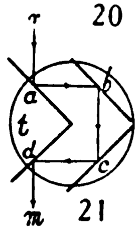


21

In view of the long glass path and therefore of considerable dispersive effect, the ellipses are small and the spectrum is filled with innumerable lines. Moreover, in view of the prism separation (at *p*, fig. 19) the ellipses are throughout half-ellipses, all terminating in the vertical axis. For the two areas or strips of light (*ab*, fig. 20) seen on the face of the grating and entering the spectro-telescope are each single, being one-half of the full area of light rays capable of interference obtained at the collimator. This results in the half-ellipses *e*. If the prism is replaced by a half-silver plate as in the next paragraph, the strips *ab'* and *a'b* are both double, the full areas being superposed; thus the areas *ab* and *a'b'* give rise to the full ellipses *ee'*. Hence, also, the vertical axis in *e*, being at the edge of the prism *P*, is not quite clear. Horizontal lines do not occur. These half-ellipses move with displacement of the micrometer at *n'*, or at *P*, or on rotation of the rail *mPn*, as a body. It is difficult, however, to use them for measurement, as their vertical terminus is not sharp enough. If ΔN is the micrometer displacement corresponding to the rotation $\Delta\alpha$, we may write

$$2\Delta N/\Delta\alpha = 0$$

I did not succeed, however, in obtaining trustworthy results with the half-ellipses.

The achromatic phenomenon can not occur when glass columns are used for compensation from the great number of lines in the spectrum. To obtain large ellipses the dispersion effect B/λ^2 must practically vanish. Hence an air-path compensator is to replace the glass column. This is conveniently made (as shown in fig. 21) of two pairs of parallel opaque mirrors *ab* and *cd*. The pair *ab* are clamped between short lengths of square brass tubing and *cd* similarly and at right angles (nearly) to the pair *ab*. Both are mounted normally to a horizontal brass table *t*, provided with three leveling-screws, capable of being raised and lowered and of rotating around a vertical axis. The path-excess introduced is thus equivalent to *ab* and *cd* and the ray *dm* is collinear with *ra*. This compensator not only introduces path-difference, but since the mirrors are capable of rotating as a whole both around a vertical and a horizontal axis (leveling-screws), the beam *dm* may be moved right and left or up and down without ceasing to be parallel to *ra*. If, therefore, the ray entering *T* (fig. 19), were first made parallel, the ray *d* may be adjusted by the compensator until the strips of light on *P* practically coincide at its edge.

With the use of this air compensator or offset, the fringes were found without much difficulty and enlarged as specified. In view of the reflection at *P*, only half fields are returned; full ellipses or horizontal lines are not obtainable, as explained. But on removing the spectroscope and cautiously advancing the micrometer at *N*, the achromatic fringes eventually appear. In the present experiments these fringes did not take the usual and desirable form, consisting of but few fringes with the middle member in black and white. Probably because of the many reflections at mirrors (fig. 21), none of which was perfect, the fringes were now colored and present in large number without much distinction between fringes. On being made transverse to the white image of the fine slit, they cross-hatched it from top to bottom. Nevertheless, their rapidity of motion is such that they serve quite well for measurement, the datum being more accurate than the measurement of $\Delta\alpha$. The comparison was carried out in the same manner as before, the presence of the achromatics being successively destroyed by rotation ($\Delta\alpha$) and restored by the normal displacement (ΔN) of the mirror at *n*. In this way the following data among many others were obtained. It is necessary to displace the mirrors very carefully; for if the fringes are lost they are extremely difficult to find without beginning with the spectrum fringes all over again.

$\Delta\alpha \times 10^3$	$\Delta N \times 10^3$	$\Delta\alpha \times 10^3$	$\Delta N \times 10^3$
0.	0.0 cm.	×14.4	16.0 cm.
2.6	2.1	18.2	19.0
6.7	7.0	21.8	21.4
8.2	9.4	25.5	25.9
11.5	12.3		

The range of $\Delta\alpha$ is much increased by removing the objective lens of the collimator, and this is done after the observation marked *x* in the table. The fringes are perhaps even more distinct when present in the absence of the lens. The constants of the apparatus were:

$$b = 21 \text{ cm.}; \quad \beta = 70.7^\circ\ ; \quad d = 64 \text{ cm.}$$

From this the rate $\Delta N/\Delta\alpha = 1.05$ was found graphically. In the other series the rates were above 0.9. Approximate estimates of the same value were obtained with the spectrum ellipses and the glass column. This result again differs from the computed value, $\Delta N = 0$. The reason may lie in the fact that the plane of symmetry of the prism *P* (fig. 14) did not pass through the axis of rotation, or was not originally midway between the mirrors *m* and *n*. To test this inference (which will again be treated in the next section) the following experiments were made:

The prism *P* was as carefully as possible centered by the eye, so that its plane of symmetry passed through the axis of rotation. In this case the relative measurements

$10^3\Delta N = 0.0$	1.5	4.5	6.6	8.1 cm.
$10^3\Delta\alpha = 0.0$	2.2	4.8	7.4	9.3

showed a mean coefficient of $\Delta N/\Delta\alpha = 0.87$. Finally the prism was moved to the right, i. e., with its plane of symmetry on the other side of the axis. The results were

$10^3\Delta N = 0.0$	3.0	4.4	7.0 cm.
$10^3\Delta\alpha = 0.0$	3.0	4.4	6.7

giving a mean rate $\Delta N/\Delta\alpha = 1.05$. Thus the shifting of the prism right and left has made but little difference and can not account for the discrepancy.

It is probable that the coefficients found are largely due to the half-silver glass mirror *n* (fig. 15), which rotates with the rail *mn*. To test this a compensator *c* of the same thickness and glass may be placed in the beam *mP* on the left. If *c* and *n* are parallel, both originally at an angle $\beta/2$ to the beams traversing them, it is obvious that the compensation will not be destroyed by the rotation of the rail, provided *c* is fixed while *n* rotates. If *c* is effectively thicker than *n*, the part of the coefficient due to the compensator may become negative. This is apparently the case in the following experiments, in which a compensator was installed as in figure 15 at *c:*

$\Delta N \times 10^3 =$	-0.0	$-.4$	-1.4	-2.8	-4.0	-5.2 cm.
$\Delta\alpha \times 10^3 =$	0.0	2.6	5.9	9.3	12.2	16.0

The rate here is $\Delta N/\Delta\alpha = -0.31$, so that the zero value is exceeded. However, the path-difference in the compensator of thickness *e* at an angle of incidence *i* and refraction *r*, viz, e $(\cos i - \sin i \tan r)$ α, where $i = 90° - \beta/2$, here becomes 0.77 $(0.816 - 0.578 \times 0.618)\alpha = 0.353\alpha$, so that the whole difficulty is not explained away.

Finally, a few experiments were made to compare the effect of displacing (ΔN) the micrometer at n'' (fig. 15) as compared with the effect of a micrometer ($\Delta N'$) which displaces *P* in the direction of its plane of symmetry. The latter ($\Delta N'$) is zero when $\alpha = 0$. Generally if *e* is the normal displacement of the prismatic faces, the path-difference is

$$2e\ (\cos\ (45° - \alpha) - \cos\ (45° + \alpha)) = 2\Delta N \sin \alpha$$

since $e = \Delta N \sin 45°$. Hence, as ΔN is a normal displacement

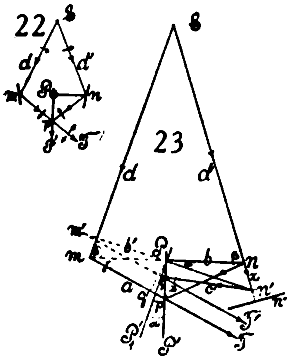
$$\Delta N/\Delta N' \sin \alpha$$

The results obtained were

$10^3\Delta N' = 0.0$	50	100	150	200	225 cm.
$10^3\Delta N\ = 0.0$	1.5	3.4	5.4	7.2	9.2 cm.

Thus the mean rate is $\alpha = \Delta N/\Delta N' = 0.036$ or α is a little over 2°. To obtain these data the achromatic fringes were used as above. When the slit-images seen in the telescope are not quite parallel, they may be made so by slightly rotating the slit on a horizontal axis normal to its plane. The images rotate in opposite directions. A slight angle between the images is, however, of no consequence.

21. Second method.—In view of certain difficulties encountered in the use of reflecting prisms, in particular the loss of rays at the edge, the limitation to half-ellipses, etc., the method of figure 22 enlarged in figure 23 was devised. In this the prism is replaced by a half-silvered plate *PP'*. Hence the rays issuing at *S* and reflected by the opaque mirrors at *m* and *n* are thereafter respectively transmitted and reflected by the half-silvered plate at *p*, and then reach the spectro-telescope at *T* together. When the path-differences are sufficiently equal, elliptic interference fringes will be seen in the spectrum. When first found they are usually very fine straight lines; but they may be rectified by plate compensators in the beams *d* and *d'* or *mp* and *np*, though the operation is not easy. Leaving these details for further consideration, the procedure for angular measurement may advantageously be treated here. For this purpose the half-silver P and one opaque mirror, n, for instance, are mounted on a rigid bar with an axis at P. The other mirror m is to remain fixed. If the bar is now rotated over a small angle α, figure 23, the mirror at n is displaced to n' and the ray Sn prolonged (intercept x) is now reflected from n' to q and thence along T' into the spectro-telescope, parallel to its original direction or to the other ray mp. Hence the interferences remain intact, but many fringes pass during the transfer. The persistence of parallelism is easily seen, because the angle between the incident and reflected ray at n is decreased by 2α when n passes to n', but is again increased by 2α owing to the rotation of PP^1_1 to PP' over the angle α.



To control the fringes either the mirror at n (or at m) may be displaced on a micrometer screw normal to itself, or the half-silvered plate at P may be displaced parallel to itself. If the angle of incidence at n is i and the normal displacement of n is e, the path-difference introduced will be $2e \cos i$. Similarly if the normal displacement of the plate P is e' and the angle of incidence i', the path-difference will be $2e' \cos i'$.

As in the preceding experiment, the mirror at n may be a half-silver, so that the ray d' passes through it and may then be returned in its own path by a mirror at n'' on a fixed standard. The displacement of this mirror over a distance e, parallel to itself, introduces the path-difference $2e$, so that the cosines are avoided. But a much more important result is the fact that the rays np or $n'g$ now are stationary. The strips of light originally at p do not therefore travel over each other while one passes from p to q and the interferences are kept at full intensity throughout. This is a great advantage. Moreover, the half-silver plate at n compensates the half-silver Pp, which is a further advantage, since both paths within are glass paths with high dispersion coefficients. It is obvious that the path-excess nn'' on the d' side, must be separately compensated on the d side. The method of doing this by an air compensator (fig. 21) will presently be considered, as a long glass compensator would not in general be desirable because of the sluggish motion of the small ellipses thus produced.

22. Equations.—The equations for this case are apparently very complicated. If in figure 23, m and n are in the same phase and Pp is symmetrical, there will be no path-difference at p. When Pn is rotated over an angle α into Pn', the path on the right becomes $nn'+n'q+qs$ while (ps wave-front) the path on the left remains mp as before. The path-difference is thus the difference of these quantities, to which, however, the increased glass path at PP' would have to be deducted, and the surface PP', must pass through the axis P.

If the angle SnP is β and Pnp is γ, the values of the branch paths may be found to be (since $nP=mP=b$), if $\beta-\alpha=\delta$ and $\gamma-\alpha=\tau$,

$$np=mp=b/\cos\gamma$$

$$nn'=b\sin\alpha/\sin\delta$$

$$nq=b\sin\beta/\sin\delta\sin\tau$$

$$qs=\frac{b}{\sin\delta\cos\tau\cos\gamma}\left\{\begin{array}{l}\sin\alpha\sin\beta\sin\tau+\sin^2\gamma\sin\delta\cos\tau\\ \qquad\qquad -\sin\beta\sin\gamma\sin\tau\cos\tau\end{array}\right\}$$

Hence the path-difference is equivalent (after some reduction) to the equation

$$n\lambda=\frac{b}{\sin\delta\cos\tau\cos\gamma}\left\{\begin{array}{l}\sin\alpha(\cos\gamma\cos\tau+\sin\beta\sin\tau)+\sin\beta\cos\gamma\\ \qquad -\cos^2\gamma\cos\tau\sin\delta-\sin\beta\sin\gamma\sin\tau\cos\tau\end{array}\right\}$$

If $\alpha=0$, then $\beta=\delta$, $\gamma=\tau$, and the right-hand member becomes zero, as it should. I have not succeeded in putting this equation in a much more convenient form.

If α is very small, so that differential expressions may be introduced, the rigorous equation, to an approximation of the second order in α, may be reduced to

$$n\lambda=b\alpha\frac{\cos\gamma(1+\cos(\beta-\gamma))}{\sin\delta\cos\tau}=b\alpha\frac{1+\cos(\beta-\gamma)}{\sin\beta}$$

If β is nearly 90° and if the distance Pp is p, then

$$n\lambda=b\alpha(1+\sin\gamma)=b\alpha+p\alpha\cos\gamma$$

The same expression may be obtained geometrically by prolonging $n'p$ and $T'q$ to m'. The triangle $n'qm'$ is isosceles. Hence if we draw the wave-front Pq', normal to pT, we may deduct the common length pq' from both rays, or add it to both. Hence the path on the right will be $x+b'\cos(\gamma-\alpha)+p\sin\gamma$, where b' is the line Pn' and on the left as before, $b/\cos\gamma$. Hence (if $b=p/\tan\gamma$) the path-difference is

$$b\left(\frac{\sin\alpha}{\sin\delta}+\frac{b'}{b}\cos(\gamma-\alpha)+\tan\gamma\sin\gamma-\frac{1}{\cos\gamma}\right)$$

which, as above, also reduces to

$$b\alpha\left(1+\cos(\beta-\gamma)\right)/\sin\beta$$

There is a source of discrepancy which enters when the face PP' does not pass through the axis P, but is eccentric. In such a case, if e is the distance of the plate from the axis, a correction equivalent to $e(1-\cos\alpha)\cos\gamma=e\alpha^2\gamma$, nearly, will have to be supplied. Again, if there is lack of symmetry from such a cause, the base-lines will be b and b' and the angles γ and γ', so that a modified equation is suggested. Finally, from all these expressions the changes of glass path on the left with α, if not compensated, must be deducted. As the method admits of a good achromatic phenomenon of reversed slit-images, it is theoretically interesting and I have given it some study.

For the case where the ray Sn prolonged returns on itself, as from n'' in figure 23, the mirror n being a half-silvered plate, the quantity $nn''=2b\alpha/\sin\varphi$ must be deducted. This makes the first term negative, so that the equation for reversed rays is, apart from sign,

$$n\lambda=b\alpha\,\frac{1-\cos(\beta-\alpha)}{\sin\varphi}$$

One may notice that when $\gamma=0$ these expressions coincide with the equations for the prism-prism methods above, except for the factor 2.

It is now necessary to apply the correction for the occurrence of a constant radius of rotation, whereby the mirror n is both rotated and displaced. This correction is, for a normal displacement e and an angle of incidence i, $2e\cos i$. If the distances $Pn=b$ and $Pn'=b''$

$$e=(b''-b)\sin(90^\circ-(\beta-\gamma)/2)=(b''-b)\cos(\beta-\gamma)/2$$

The angle of incidence is now $(\beta+\gamma)/2-\alpha$, so that the path-difference in question becomes

$$2(b''-b)\cos(\beta-\gamma)/2\,.\cos\left((\beta+\gamma)/2-\alpha\right)=(b''-b)\left(\cos(\beta-\alpha)+\cos(\gamma-\alpha)\right)$$

Since $b''=b\sin\beta/\sin(\beta-\alpha)$ if α is a small angle, this may be written $(b''-b)=\alpha b\cos\beta/\sin\delta$. Hence the correction is

$$b\alpha\,\frac{\cos^2\beta+\cos\beta\cos\gamma}{\sin\beta}$$

Subtracting this from the first equivalent of $n\lambda$, and reducing, the equation to be used for non-reversed rays becomes $\lambda n=b\alpha(\sin\beta+\sin\gamma)$ when α is small. Except for the factor 2 this result is again identical with the above for two prisms, if $\gamma=0$. As the angle of incidence at the micrometer at n is $i=(\beta+\gamma)/2$, and if ΔN is the displacement

$$2\Delta N\cos(\beta+\gamma)/2=b\,(\sin\beta+\sin\gamma)\,\Delta\alpha$$

For reversed rays, $e=(b''-b)\sin(\beta-\gamma)/2$ at once, and the path-difference is then for small α

$$2b\alpha\cot\beta\sin\frac{\beta-\gamma}{2}\cos\left(90^\circ-\frac{\beta+\gamma}{2}+\alpha\right)=b\alpha\cot\beta\left(\cos(\gamma-\alpha)-\cos(\beta-\alpha)\right)$$

since the angle of incidence is now $90^\circ-(\beta+\gamma)/2+\alpha$. Subtracting this path-difference from the above equivalent of $n\lambda$, the practical equation for reversed rays becomes, after reducing,

$$n\lambda=b\alpha\,(\sin\beta-\sin\gamma)$$

and since $i=0$, then

$$2\Delta N=b(\sin\beta-\sin\gamma)\Delta\alpha$$

Both equations contain the distance d of a remote object in $\sin\beta$.

23. Observations.—In the first experiments a sharp-angled prism was placed at S, reflecting the two rays d and d' of white light from a collimator beyond. The method of figures 22 and 23 with a micrometer at n was first used, the screw being in the direction of the bisectrix of the angle $\beta+\gamma$. The collimator is convenient, as otherwise the coincident slit-images are liable to separate when the micrometer mirror moves. Moreover, quite a narrow silt is preferable, so that all the light reflected by the prism is that diffracted by the slit and issuing in parallel rays from the objective. Any further light escapes interference and blurs the fringes. The adjustment is not easy, as has been shown in the earlier papers. It is necessary that the two rays *mp* and *np* pass through the same vertical at *p* and additionally enter the telescope at *T* in parallel. When this is the case and the path-difference is annulled, the fringes are very black on the yellowish background of the spectrum near the *D* lines. In proportion as the rays separate into *pT* and *qT*, the fringes become fainter and finally vanish. Near *p*, however, they may be seen until they vanish at either elongation from smallness. Another reason for this marked tendency to vanish is the fact that the slit-images are mirror-images of each other. Hence if the slit-images are widened, even when the strips of light seen at *p* coincide, there can only be a narrow vertical region of actual coincidence of light of like origin. It follows, moreover, that the achromatic fringes proper can not be produced with white light by this method, though it is possible to produce the linear phenomenon with white light when the ellipses are vertically centered. The experiment, which is at first very difficult, will presently be treated. Movable fringes are sometimes seen on the white slit-images.

When first obtained the spectrum fringes are usually very fine parallel lines. To bring the center of ellipses into the center of the field a plate-glass compensator, capable of rotation on a horizontal axis, should be placed either in the ray *d* or *b*, or on the other side, as the case may be. When the compensator is set at the proper angle very dense black ellipses may be brought into the center of the field by the micrometer screw or by the rotation of the system *Pn*, figure 23. In fact, on using both of these displacements in a contrary sense (i. e., annulling the effect of slight displacements of the micrometer by a corresponding counter-displacement of rotation), the two beams may be made to pass through *p* together and without path-difference. In this case the ellipses are very strong. If both the mirrors at *n* and *P* are displaced contrariwise but parallel to themselves this may also be done. There is no such difficulty with the method of reversed rays.

The following measurements were made as a rough test of the equations given above. The constants of the apparatus were (fig. 4)

$d=62.3$ cm. $p=Pp=10.6$ cm. $\beta=71.3°$ $\gamma=27.1°$ $b=20$ cm.

the prism angle therefore being about 18.7° and the divergence of rays from it 37.4°. A much *sharper* prism than this would have been desirable, so the *d* could have been larger, but none was at hand. The angle of incidence here is $(\beta+\gamma)/2=49.2°$. Hence

$$\frac{2\Delta N\times 0.653}{\Delta\alpha}=20\ (0.947+0.455)=28.$$

The procedure for the test of the equation consisted in establishing the ellipses with the micrometer at *n* (reading *N*), rotating the rail over a small angle (reading α), and re-establishing the ellipses again at the micrometer (N'). To find α, an index was attached to the end of the rail (radius of rotation 23.6 cm.) and the motion of this index over a glass millimeter scale was read off with a lens. For really refined work it would have been necessary to adjust a micrometer tangent screw to find α; but this did not seem called for here. The results were in case of two series ($2 \cos i = 1.306$):

ΔN	$2\Delta N \cos i$	$\Delta\alpha$	ΔN	$2\Delta N \cos i$	$\Delta\alpha$
0.0 cm.	0.0 cm.	0.0	0.0 cm.	0.0 cm.	0.0
0.1357	0.1777	0.0076	0.0607	0.0793	0.0034
0.0577	0.2531	0.0101	0.0541	0.1530	0.0068
0.0462	0.3135	0.0139	0.0664	0.2397	0.0098
0.0718	0.4073	0.0160			
Mean $\dfrac{2\Delta N \cos i}{\Delta\alpha} = 24.5$			Mean $\dfrac{2\Delta N \cos i}{\Delta\alpha} = 24.0$		

These coefficients (which are here below the computed value) were found graphically. The discrepancy for the correction due to increasing obliquity of the plate Pp is but (e thickness, μ index of refraction, r angle of refraction, the incident being $i=\gamma$)

$$e\,(\sin \gamma - \cos \gamma \tan r)\Delta\alpha = 0.14$$

per unit of α, if $e = 0.775$ cm., $\mu = 1.55$, and $\gamma = 27.1°$.

A further discrepancy may be sought for in the fact that if the surface Pp (fig. 23) does not pass through the axis of rotation, this plate is both rotated and displaced. The error so introduced may be either positive or negative. If the displacement of plate is $e' = e\,(1 - \cos \alpha)$, the equation should read

$$\frac{2\Delta N \cos \pm i2e' \cos \gamma}{\Delta\alpha} = b\,(\sin \beta + \sin \gamma)$$

where e is the distance of the plate from the axis. The following experiments were therefore made with some consideration to greater symmetry of apparatus. The constants were: $\beta = 70.5°$, $\gamma = 27.1°$ and therefore $i = 48.8°$, or $2 \cos i = 1.32$. The data found were in different adjustments;

$\Delta N \times 10^3$	$\Delta\alpha \times 10^3$	$\Delta N/\Delta\alpha$	$2\Delta N \cos i/\Delta\alpha$
110.9	5.2	21.2	28.0
203.7	9.6		
246.5	11.6		
102.9	4.8	20.5	27.1
208.4	10.0		
47.6	2.6	20.7	27.3
80.1	4.1		
170.4	8.5		
223.6	10.7		
257.6	12.7		

These results agree much better with the theoretical equation than the former, and may be considered as coinciding with it.

In the present case the attempt to get interference from rough surfaces was not at first successful. The slit-images are reversed, as indicated by the transverse arrows in figure 22. Hence if the white slit-images are wide there can be coincidence only in a single vertical line. Fringes with white light will occur as a case of the interference of fine slit-images. To produce them it is first necessary to obtain the spectrum fringes with the ellipses, or else with horizontal fringes in the field. If now the spectroscope is removed and the white slit-images put out of focus, the phenomenon indicated in figure 24, where *s* is the superposed, washed slit-images will usually appear, or may be found on cautiously moving the micrometer screw. Within the slit-image the fringes are coarse and colored, but they send out fine oblique streamers into the field of diffuse light or glare, on both sides of *s*. When the slit is widened these fringes are liable to vanish just as the spectrum fringes vanish, except perhaps at the edges of the images. These achromatic fringes climb up and down the slit-image with motion of the micrometers (ΔN, $\Delta\alpha$) with extreme rapidity and are easily lost, as there are not usually more than 10 or 20 of them. If the spectrum ellipses are huge, the white fringes are almost too coarse to be seen and too mobile to be controlled.

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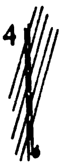

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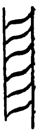


I next removed the objective of the collimator. The fringes, though much changed in appearance, practically black and white, were not destroyed. In such a case the slit-image shrinks vertically. To obtain a long strip a highly illuminated ground-glass screen (sunlight and weak condenser) should be placed in front of the slit as a source of very diffuse light. In such a case this long white post (as it were) is covered from top to bottom with sharp blackish and usually oblique lines, which vanish at once, up or down, on moving the micrometer. No fringes are seen if the slit is in focus. When considerably out of focus (as in case of the diffraction patch in fig. 25,) strong, sharp-colored cross-markings are present, which would be quite available for measurement. However, in this experiment, when the slit was widened or removed, the fringes apparently vanished. The phenomena as a whole seem to me to be fringes of the two white slit-images, and seen either behind or in front of their focal plane, like the complementary fringes described elsewhere. This is confirmed by experiments presently to be described.

24. Reversed rays.—The apparatus was now adjusted for a reversal of rays by putting a half-silver plate at *n* and an opaque mirror on a micrometer (with the screw normal to its face) at some position *n''* (fig. 23) fixed independently of the rotation. In this case, therefore, the intercept *nn'* changes sign. Moreover, the angle of incidence at *n''* is 0°. Hence the equation should be

$$2\Delta N/\Delta\alpha = b\,(\sin\beta - \sin\gamma)$$

The constants of the apparatus were

$$b=20 \text{ cm.} \qquad \beta=71.3^\circ \qquad \gamma=34.9^\circ$$

Thus

$$2\Delta N/\Delta\alpha=20(0.947-0.572)=7.5$$

To obtain the ellipses a thick plate-glass compensator may be placed in the d ray to provide for the elongation $2x$ in d'. About 14 cm. of glass column were necessary. This makes it very easy to center the ellipses and to obtain them intensely black on a colored ground by rotating the compensator on a horizontal and vertical axis until the two strips of illumination at p quite coincide when the rays T, T' are parallel. But on the other hand, because of the thickness of glass used, the small ellipses obtained move relatively sluggishly with displacement of the micrometers. The sensitiveness decreases proportionately to the thickness of glass path.

Experiments, of which the following data are examples, were made by alternately restoring the center of ellipses to the D lines of the solar spectrum first by the micrometer (ΔN) and thereafter by the rotation of rail ($\Delta\alpha$). The adjustments were very different.

$\Delta N \times 10^3$	$\Delta\alpha \times 10^3$	$\Delta N \times 10^3$	$\Delta\alpha \times 10^3$
0.0 cm.	0.0 rad.	0.0 cm.	0.0 rad.
53.2	13.7	24.3	1.9
66.1	17.0	33.3	4.4
58.1	14.8	—	—
44.1	11.5	43.3	7.4
21.1	5.2	57.4	10.7
		68.8	14.0
Mean $2\Delta N/\Delta\alpha=7.6$		Mean $2\Delta N/\Delta\alpha=6.8$	

The second series changed its rate enormously, almost one-half, owing to necessary intermediate adjustments (inserting a new zero). Otherwise the observations are as good as the apparatus permitted; but the computed $2\Delta N/\Delta\alpha=7.5$ is above the observed value, usually, possibly owing to an eccentric position of the plate Pp relative to the axis of rotation. To test this point of view the plate Pp was displaced eccentrically toward the left of the axis. The result should be a modified coefficient, but the following data obtained in the same way as before fail to bear this out, however:

$$\Delta N \times 10^3 = 0.0 \qquad 19.9 \qquad 40.2 \qquad 56.8 \text{ cm.}$$
$$\Delta\alpha \times 10^3 = 0.0 \qquad 5.9 \qquad 12.2 \qquad 16.7 \text{ rad.}$$

The rate, $2\Delta N/\Delta\alpha=6.6$, does not differ essentially from the above. With these small ellipses there can not, of course, be an achromatic phenomenon. To obtain large ellipses the glass-path difference (i. e., the dispersion) must be abolished on both sides and an air-path difference introduced, preferably in a way which has been shown above in figure 21. As such experiments are so much more trustworthy and sensitive, I did not pursue the glass-column work further.

25. Fringes from rough surfaces.—Experiments were now made with the use of the air-path compensator (fig. 21,) placed in the *d* rays when these rays were reversed. A magnificent set of achromatic fringes were here found, only about five in number, with the central members in black and white. Tests similar to the above showed that they are Fresnellian interferences. To prove this the objective of the collimator was removed and even more brilliant fringes were found on placing the washed slit-images in contact. If these patches of light were slid over each other horizontally, by moving the adjustment screw for rotating the micrometer mirror on a vertical axis, the fringes rotated nearly 180°, passing from vertical hair-lines, through a maximum of coarseness for the horizontal fringes, back to hair-lines again. On focussing the telescope on the slit it was then found that the large horizontal fringes corresponded to coincident slit-images in focus, whereas for the very fine fringes the focussed slit-images are far apart. No fringes appear on the slit-images in focus, in any case. They lie in front of and behind the image plane. This is exactly the case found above, except that here the edges of the washed slit-images are exchanged.

The endeavor to obtain the fringes without the slit was next tried. For this purpose a ground-glass screen illuminated by sunlight (*a*, fig. 17) was first placed in front of the slit *S* in the absence of the objective (*L*) of the collimator. The fringes were still very prominent, though the light was darker. The slit *S* was now also removed. The fringes could then no longer be seen; but on narrowing down the illuminated ground-glass screen *a* to a vertical strip of light 1 to 2 mm. broad, they were unquestionably present. In such experiments, therefore, the chief function of the slit *S* is to cut off the light which does not interfere, so that the fringes are lost in the glare. In the absence of such excess of light the fringes are quite visible and therefore certainly always present. By aid of the offset air compensator huge achromatic fringes may be easily produced; but they are so sensitive as not to be manageable in an improvised apparatus.

A number of measurements were now made with the achromatic fringes set at convenient (small) size by the air-compensator. In this work the plate *PP′* (fig. 23) was moved in three steps over about 1 cm. For each step a set of data was investigated. The results were in succession

$$(1)\left\{\begin{array}{llllll}\Delta N\times 10^3=0.0 & 1.9 & 6.1 & 12.4 & 17.8 & 21.9\text{ cm.}\\ \Delta\alpha\times 10^3=0.0 & .4 & 1.2 & 2.3 & 3.3 & 3.9\text{ radian}\end{array}\right\}\frac{2\Delta N}{\Delta\alpha}=10.4$$

$$(2)\left\{\begin{array}{llllll}\Delta N\times 10^3=0.0 & 7.1 & 12.2 & 19.2 & 26.6 & 32.6\text{ cm.}\\ \Delta\alpha\times 10^3=.0.0 & 1.1 & 2.2 & 3.7 & 5.2 & 6.3\text{ radian}\end{array}\right\}\frac{2\Delta N}{\Delta\alpha}=9.6$$

$$(3)\left\{\begin{array}{lllll}\Delta N\times 10^3=0.0 & 9.3 & 19.2 & 27.7 & 37.7\text{ cm.}\\ \Delta\alpha\times 10^3=0.0 & 1.8 & 3.7 & 5.9 & 7.4\text{ radian}\end{array}\right\}\frac{2\Delta N}{\Delta\alpha}=10.2$$

The coefficients so obtained are practically identical; and they agree as nearly as may be expected with the equation, since the angle B and γ are not easily specified with accuracy. They were

$$\beta = 71.3^\circ \qquad \gamma = 28.4^\circ \qquad b = 21 \text{ cm.}$$

so that theoretically

$$2\Delta N/\Delta\alpha = 21(0.947 - 0.476) = 9.9$$

26. Direct interferences without cleavage prism.—The next step in advance was made by dispensing with the sharp prisms heretofore used for cleaving the rays issuing from a collimator (or the slit simply) in the endeavor to obtain two rays capable of interference. The assembly of apparatus is shown in figure 26, where S is the slit (to be replaced by a Nernst filament or a tungsten filament), m and n the opaque mirrors, pp' the half-silvered plate. The rays dd', diffracted at S, pass after reflection into c and c' and may be observed by spectro-telescopes placed either at T or T'. In the first experiments the distance Sp' was about 4 meters and the distance mn 10 cm. The mirrors n and pp' were on micrometers with the screws normal to their respective faces. The distance mn must be within the limits of the wedge of light from the slit and is therefore small, unless d is very large. Both pp' and n are on the rotating rail (as above), whereas m is fixed. The apparatus was also adjustable for reversed rays by attaching an auxiliary mirror, normal to the rays d' prolonged through n. S being distant, this slit must be long, as otherwise the spectrum band will be a mere horizontal line and the fringes difficult to detect. A doublet of lenses, each about 10 cm. in diameter and of the same focal power (1.60 cm.), but respectively convex and concave and having a combined focal distance of about 5 or 6 meters, is of advantage for focussing a large solar image, 1 to 2 inches in diameter, on the slit. The Nernst or tungsten filament gives the same advantages at once; but the former is too thick, at least for the initial experiments at shorter distances.

The fringes are exceedingly difficult to find in spite of the brilliant spectra. It was not until after about three days of searching, in which (besides sunlight) the filaments as well as the methods of direct and of reversed rays were used, that the experiment ultimately succeeded with sunlight. The filaments are much less gracious. To obtain the fringes calls not only for very accurate adjustment for horizontal and vertical spectrum coincidence, but the fringes lie quite sharply in a definite focal plane, usually between that of the slit-image and the principal focal plane; the rays must interpenetrate at the plate and finally path-difference must be nearly annulled. And there are other conditions presently to be stated. After being found they are quite strong elliptic spectrum fringes, but when lost nevertheless difficult to rediscover. The slit may be broadened till the spectrum lines vanish, perhaps to more than a millimeter, before they disappear in a uniform spectrum band.

The achromatics which coincide in adjustment with horizontal spectrum fringes and are seen with the slit-image out of focus are also difficult to find because of the short length of the slit-image. As first obtained they lacked brilliancy and were not easily observed. Similarly, experiments with filaments failed to show the fringes, although made in parallel with the successful result with sunlight.

A considerable assistance in finding the fringes is an opaque screen with a vertical slit 2 mm. to 4 mm. wide placed just in front of the objective of the spectro-telescope, in the best position as to symmetry. This screen cuts out rays which do not interfere and makes the fringes stronger, even though the background is darker. Fringes are frequently found in this way when they are all but invisible in the full spectrum.

In addition to the regular fringes, a much larger vague set seems to be present in another focal plane. They also rotate, etc., like the regular fringes, but the experiment led to no decision with regard to them, as they appeared in the field erratically and could not be produced at will. They may be shadow interferences of the principal set.

An attempt was made to register slight lateral displacements of the slit in terms of the displacement of fringes, but as the slit-images are thrown out of coincidence when the slit moves, trustworthy numerical data can not be obtained. One may estimate that as a first approximation

$$x = \frac{d}{c}\lambda$$

if x is the lateral displacement of the slit and c the distance between mirrors m and n. Hence

$$x_0 = \frac{400}{10} 6 \times 10^{-5} = 0.0024 \text{ cm.}$$

should have been equivalent to the passage of one fringe in the given apparatus, or generally

$$2\Delta N \cos (\beta+\gamma)/2 = cx/d$$

If $(\beta+\gamma)/2 = 70°$, $\Delta N = 10^{-4}$ cm., and $d/c = 40$, then

$$x = 2 \times 10^{-4} \times 40 \times 0.34 = 2.7 \times 10^{-3} \text{ cm.}$$

could have been registered. Incidentally it appears that two vertical lines of the slit, $x_0/2 = 0.0014$ cm. apart, would wipe out each other's interferences; but this is not the case, as much greater slit-widths are admissible. To the right and left of the line of the slit capable of producing interferences the parallel lines either cease to produce parallel rays, or parallel rays come from symmetrical but different lines.

After completing these experiments, the distance between slit S and the mirrors m and n was increased to about 9 meters. The same lens doublet, focussing a large solar image on the slit, was used as before. With the aid of the slotted screen in front of the telescope and the micrometer distances from the preceding experiment, the fringes were found without difficulty.

In fact, in view of the longer distance *d*, the slit could be opened to over a millimeter of breadth before the fringes quite vanished from the spectrum; but on using a somewhat stronger condensing system (concave lens of doublet preceding the convex lens) and consequently more oblique rays, a very fine slit was needed to show the fringes. They are thus more easily found when the rays are more nearly parallel.

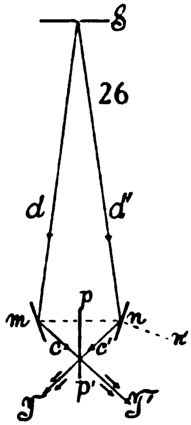
With artificial light, again, I obtained no results, even after long searching.

Operating with two successive slits at about 9 meters from the interferometer, one of which received the light through the other, I found that two independent sets of fringes very different in size and inclination could be put in the field together. The further investigation eventually showed that the size and inclination of the fringes is essentially dependent on the degree of parallelism of the two slit-images. When the images are parallel, the fringes are of maximum size and vertical. When the images are not quite parallel (they incline in opposite directions when the slit is slightly rotated in its own plane from the vertical), the fringes rapidly grow smaller and rotate. With parallel slit-images the spectrum ellipses are centered in the field; otherwise they are very far out of center. The adjustment for actual (not X-like, coincidence must therefore be made with precision if large fringes are wanted.

Further work was also done with sunlight to obtain more pronounced achromatics. For this purpose a compensator was inserted to equalize the glass path in the half-silvered plate. Huge spectrum ellipses were obtained in this way and their centers were placed above the telescopic field, so that the fringes seen were large horizontal bars. On removing the spectroscope and placing the slit-images out of focus, brilliant achromatics were in fact obtained, of the concentric hyperbolic type, vividly colored and broad between the apices, and diminishing to hair-lines laterally. With these it was possible to enlarge the slit to at least 3 mm., without destroying the fringes, though they became more vague. It is necessary that the slit-images, when in focus, should be quite parallel, otherwise any broadening of the slit will wipe out the achromatics. It was possible to place a plate of ground glass on the far side of the slit without destroying the fringes, but not on the side towards the interferometer. In other respects the behavior was as described in the case of achromatics in the earlier experiments with a cleavage prism.

Finally, the spectrum fringes and the corresponding achromatics were obtained with the light of a Nernst filament, at first by focussing an image of it with a strong condenser lens on the slit. The experiments, however, are very difficult. The spectrum fringes are often weak, out of focus, and extremely sensitive to small disadjustments in the horizontal and vertical coincidence of the slit-images. They require a fine slit. When well produced the achromatics are also obtainable on removing the spectroscope when the spectrum fringes are horizontal bars. The achromatics may also be obtained brilliantly without the condenser lens, but the adjustment must in such a case be made first with sunlight, as the spectrum from the Nernst filament is too feeble for detecting fringes so elusive as the present. The achromatics, however, are strong and brilliant even here (Nernst filament).

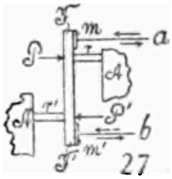
An interesting result is obtained in case of the achromatic fringes by narrowing one of the beams, for instance that coming from the mirror *m* (fig. 26), by a screen with a vertical slit about 2 mm. wide. In such a case the slit-image (out of focus) is correspondingly narrowed. It may be passed from side to side of the broad washed slit-image coming from the mirror *n*, by moving its adjustment screws (vertical axis). The fringes then appear only in a particular position of the narrow image in the field of the broader; but when they do appear they spread far beyond the margins of the narrow image on both sides. Interference thus apparently occurs where but one beam is present. The phenomenon is like those instances above (figs. 18, 24, Chapter II) and means, as I understand it, that the beams have met in some other focal plane, though one is tempted to conclude that interference is stimulated by resonance, in particular as it is often impossible to find a plane in which they have met. The achromatics may sometimes be seen before and behind the principal focal plane, but more frequently either in the one or in the other region only.



26

CHAPTER III.

THE ELASTICS OF SMALL BODIES.



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27. Introductory method.—At the request of Professor W. G. Cady, who was in need of Young's modulus in case of certain crystals used in experiments in which he is interested, the endeavor was made to adapt the above interferometer for measuring small angles with an auxiliary mirror for this purpose. The project seems feasible and apparently simple in execution when the method of end-thrust indicated in figure 27 is used. Here F is a rigid metallic bar subjected to the force couple P,P', carrying the coplanar mirrors m,m' and capable of rotating slightly in a horizontal plane. The mirrors m and m' receive the corresponding rays a,b of the interferometer. The couple P,P' is resisted by the resilience of the rods r,r' to be tested, as these push respectively against the ends of the bar F and against the rigid abutments A and A' of the apparatus. If the couple P,P' changes, the bar F rotates correspondingly and the component rays a,b of the interferometer will register the amount of rotation by the methods given in Chapter I. Thus, if E is the traction modulus, l the elongation of each rod of length L and right section A under the force P,

$$E=\frac{\Delta P/A}{\Delta e/L}$$

Δ being a differential symbol. If the distance apart of the rays a,b is $2R$ and that of the force couple is $2R'$,

$$2\,R\Delta\alpha=\Delta N\ \cos i$$

if the bar F rotates over an angle $\Delta\alpha$, in consequence of the increment ΔP of thrust, and if ΔN is the displacement of micrometer mirror (angle of incidence is i), needed to restore the interference fringes to their original position. But

$$\Delta l=R'\Delta\alpha=R'\,\frac{\Delta N\cos i}{2R}\ \text{nearly;}$$

so that after reduction

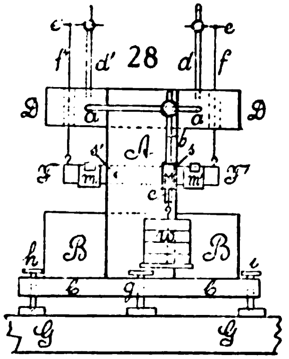
$$E=\frac{2LR}{AR'\cos i}\cdot\frac{\Delta P}{\Delta N}$$

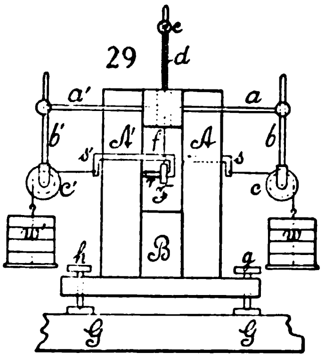
Since $i=45°$, $$E=2\sqrt{2}\ (L/A)(R/R')(\Delta P/\Delta N)$$

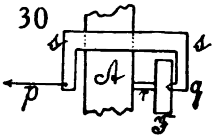
The method will not of course be very exact, because for rods less than an inch long the quantities involved, particularly ΔN, are so small. Any flexures or slight dislocations of parts of the apparatus are of relatively great consequence. But there is a much more serious consideration. In long rods the stresses distribute themselves equally throughout the sectional area; but in short rods this is probably not the case. There will be lines of longitudinal stress and part of the area A will be relatively unstressed. Hence the values of E will come out too small and the question is rather to what degree such a method can be made trustworthy. With the optical method there will be no difficulty, if the achromatic fringes are used. The observed displacement of these is adequate if a reasonably thin rod is used and fringes need not be counted. It is not even necessary to make the method very sensitive. Fringes of moderate size suffice.

Should the method of end-thrust fail, the method of flexure is more liable to be successful, since the measurement of the sag at the middle of two rods placed at r and r' parallel to FF offers less serious difficulties. But such flexure involves continuously decreasing stresses from the middle to the ends of each rod and is thus fundamentally less interesting. For very short rods, moreover, theoretical difficulties similar to the preceding would be encountered.

28. Apparatus.—The apparatus used is shown in front (normal to the interferometer rays) and side elevation in figures 28 and 29. F is the bar carrying the mirrors m,m', already mentioned, and compressing the rods r,r'







to be tested. The forces are applied (shown in detail in figure 30) by means of a set of weights ww', the pulleys cc' and the rigid brass offset ss acting on the cone at q, which fits into a depression in F. The lines pq pass through the axes of the rods r or r'. In this way any flexure of the bar F is obviated.

The rigid part (abutments of the system) is made up of the cast-iron bricks A, A', B bolted together firmly. These rest upon the plate C provided with three leveling-screws g,h,i, g,i being parallel to the bar F. These leveling-screws are of special importance in controlling the size and inclination of the achromatic interference fringes, gi being the horizontal axis. The whole apparatus may be rotated around a vertical axis on the horizontal base GG, which is itself movable on slides nearer or further from the interferometer.

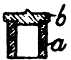
Size of fringes is controlled by rotation around the vertical. The iron bricks A,A',B were each $10\times 2\times 3.5$ cubic inches in my apparatus, certainly rigid enough for the purpose. The inner faces should be smooth and parallel. Between them at the top the wooden bar DD has been bolted in place which carries the rods of the railing aa', etc., with clamps for the support of the forks bb' of the pulleys cc'. It also carries the standards dd' and arms ee', to which two silk threads ff' are adjustably attached for the bifilar support of the bar FF', when not in use.

The stresses on the system thus seem balanced throughout, the ultimate tendency being a compression of A and A' in a horizontal direction, which is of course negilgible. The only discrepancy which may be effective is the possible flexure of the plate C by the varying weights ww'. It is for this reason that the weight of A,A',B was made excessive as compared with ww.

29. Preliminary observations.—The first experiments were made with hard-rubber tubes each $L=2.2$ cm. long and $A=1.46$ cm. in sectional area. The offset ss (fig. 30) was not at first applied and the beam F showed definite flexure, inasmuch as the white slit-images separated. With this tube of hard rubber a value of the order of $E=6\times 10^9$ was obtained. With the offset (used throughout the following work) the apparent modulus increased to $E=9\times 10^9$ and the white slit-images remained in contact. Thus far horizontal spectrum fringes had been the criterion of measurement. It was found just as easy and more accurate to use the achromatic fringes.

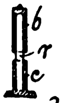
In three series with the same tube the values came out as

$$10^{-9}E=7.8,\quad 7.8,\quad 8.1$$



31

All these data are apparently much too low. Consequently the hard-rubber tubes a were provided with thick brass caps b, as shown in figure 31, the small conical projection fitting a reentrant cone in the beam F. Nevertheless the value now found ($E=6\times 10^9$) was even lower. There is thus no doubt that the section of the tube is not uniformly strained in a short solid, whereas in a long slender body the stresses are soon equalized. Part of the sectional area of the short solid may in fact be quite free from strain. Hence the device shown in figure 32 was next tested, where a relatively thin rod of hard rubber r is surrounded by brass caps b and c with closed ends. The caps fit the rod loosely and room is left between them for compression. For hard-rubber rods of the dimensions $L=1.7$ cm., $A=0.24$ cm.^{2}, and in three series of experiments the moduli $10^9E=6.9,\ 7.5,\ 7.5$ were computed, showing therefore no marked change from the preceding data, in spite of the greatly diminished sections. In the next experiments shaped rods like figure 33 were used directly. For the dimensions $L=2.2$ cm., $A=0.41$ cm.^{2}, the moduli were found to be



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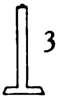


33

$$E\times 10^9=9.6,\quad 9.6,\quad 8.4,\quad 8.4$$

in four series made after different periods of loading.

In all this work the initial loads of 1 kg. each were not removed. The elongations between 1 kg. and 4 kg. were consistent, though naturally with definite evidence of apparent hysteresis. For instance, on the last series with brass caps the individual contractions were

$\Delta P=$	4	3	2	1	2	3	4	kg.
$10^4\Delta N=$	120	94	55	0	40	78	141	cm.

and in the last series with the mushroom-shaped solid,

$\Delta P=$	4	3	2	1	2	3	4	kg.
$10^4\Delta N=$	89	68	38	0	34	64	89	cm.

As the values ΔN are proportional to the decrements of length, the rod is shorter on unloading and longer on loading, *cæt. par.* In other words, the hard-rubber rod is influenced by its immediately antecedent history. The curved lines obtained may, however, also be influenced by gradual dislocation or decreased firmness in the seating of the rod. Thus the modulus in the last example decrease from about $10^9E=9$ between the two highest loads (3 to 4 kg.) to $10^9E=6$ between the two lowest loads (1 to 2 kg.). Of the two the former (high loads) is unquestionably the more trustworthy and least influenced by imperfections of apparatus.

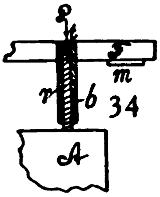
Results of the same nature were obtained for brass, though here, from the much greater rigidity, the effect of dislocation at small loads is much more apparent. The first experiments were made with a relatively thick rod, of the dimension $L=2.2$ cm., $A=0.70$ cm.², for which $E=4.5\times10^{10}$ was obtained. This is enormously too low and the result is a mere indication of the yield of the apparatus or inequalities of stress. The rod was now turned down on the lathe to a mushroom-shaped solid (fig. 33), the dimensions $L=2.1$ cm., $A=0.113$ cm.². With this the values $E\times10^{11}=1, 2, 5$, were found, according as the rod was unloaded or loaded from 4 to 3 kilograms. Clearly this is a case of dislocation of parts of the adjustment. The rod was then further diminished to diameter of but 0.2 cm., so that $L=1.8$ cm. and $A=0.035$ cm.² now obtained. With this, for the highest loads, values $E=1.4\times10^{11}$ followed under like conditions. This is no improvement and shows that the apparatus yields seriously for moduli as large as that of brass (say 10^{12}). Thus in the series with largest values of E the individual contractions were

$\Delta P=$	4	3	2	1	2	3	4	kg.
$10^4\Delta N=$	1.6	1.4	0.9	0	.9	1.5	2.1	

30. Rods in metallic sheath.—In the data thus far obtained the value of E is throughout too low, showing that the section of the rod has not been uniformly stressed. A final modification was therefore made as shown in figure 34, in which the rod *r* (to be tested) is rather loosely surrounded by a rigid metallic tube or sheath rigidly screwed into the bar *FF* of the interferometer. This tube is closed at *t* with a tightly-fitting screw-plug provided with a conical depression to receive the thrust *P* from the point *q* of the offset *ss* (fig. 30). The rod *r* is thus compressed between *q* and the rigid abutment *A* of the apparatus, and projects but slightly (1 or 2 mm.) beyond the tube *b*. Special sheaths *b* are provided, fitting neither too tightly nor too loosely, for different diameters of rod *r*; or a number of coaxial tubes, *b*, may be telescoped for the purpose.

This device gave more satisfactory results at once, and with such bodies as hard rubber showed the change of modulus with stress, the occurrence of hysteresis, and viscous deformation. It is particularly interesting, inasmuch as it gives the apparent value of the modulus under each of these conditions.

The successive contractions (ΔN) and modulus values for hard rubber as found in several successsive series are exhibited in table 2 and figures 35 and 36. In the first series the relatively large micrometer displacements, ΔN, are probably due to crushing or to fitting the unstressed rod to the abutments of the apparatus under increasing pressure. Thereafter these large contractions do not again occur. The moduli obtained from triplets of observation between 3 and 4 kg. gradually increase to a fixed value. In the second series the rod which had been loaded for some time (see table) with about 40 kg. per centimeter shows the limiting value of moduli found, $E = 4.4 \times 10^{10}$. We may contrast with this the small modulus when the load is but 1 to 2 kg.



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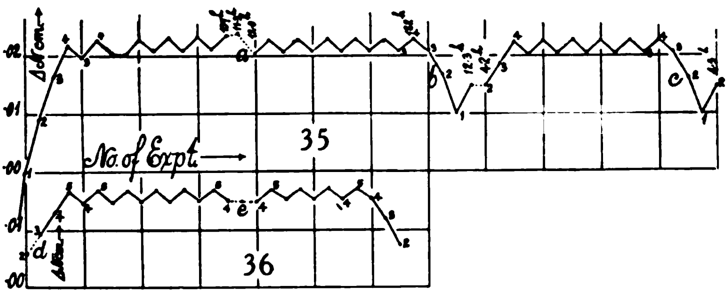

35

36

In the third series, beginning with a rod but slightly stressed, a low value of the modulus at first appears, but it soon reaches the limiting values again. In figures 35 and 36 the contractions (the numerals show the loads) are given in succession. With the exception of the necessary break at the beginning of the second series, the work is continuous without modification of adjustment. As contractions are positive the rod is gradually becoming shorter and more viscous. The data are throughout consistent, much more so than was expected. For instance, the effects of the removal of weights at *b* and *c* are practically identical. The triplets in figure 35 all show an upward slope or continuous viscous contraction of the rod under large end-thrust. In series

TABLE 2.—Modulus of hard rubber. End-thrust apparatus with sheathed rods; $L=2.45$ cm.; $2r=0.367$ cm.; $A=0.106$ cm.^{2}; $i=45°$; $2R=10.3$ cm.; $2R'=7.0$ cm. Permanent load 1 kg. each. Achromatic fringes.

P	$\Delta N\times10^4$	$E\times10^{-10}$	P	$\Delta N\times10^4$	$E\times10^{-10}$	P	$\Delta N\times10^4$	$E\times10^{-10}$
kg.	*cm.*		*kg.*	*cm.*		*kg.*	*cm.*	
10^h30^m 0.5	−50		12^h0^m **3	105		4^h10^m 2	49	
1.0	0		4	128	4.41	3	87	
2	90		3	108		4	124	3.32
3	164		4	129		3	103	
4	219	2.53	3	108	4.41	4	126	
3	198		4	130		3	105	4.47
(1) 4	228		(2) 3	108		(3) 4	127	
3	206	3.70	4	130	4.47	3	105	
*3	204		3	109		4	127	4.37
4	230	4.18	4	129		3	105	
3	209		3	109		4	127	
4	233	4.13	4	129		3	106	4.43
3	210		3	109		4	128	
4	234		2	116		3	106	
3	211	4.13	1	0	1.67	2	62	
10^h45^m 4	235		12^h16^m 2	48		1	0	1.77
					(b)	4^h30^m 2	46	(c)
11^h20^m 4	240	(a)						
†2	47		5	167		***4	149	
3	85		4	148	5.12	5	169	5.34
4	120	3.41	5	167		4	153	
3	99		4	149		5	170	
			5	168	5.20	4	153	5.43
‡2	58	(d)	4	150		(7) 5	171	
3	94		5	168		4	153	
(6) 4	131		4	150	5.25	5	170	5.81
5	165	3.63	5	168		4	153	
4	146		4	151		3	120	
						2	73	

* Slight adjustment. ** After adjustment for larger fringes. *** 2 hours later.
† Next day. ‡ Two days later.

6 and 7 a special collection of data was investigated two days later at the highest loads which the apparatus admitted, 4 to 5 kg. These are shown after *d* (fig. 35) and with a further lapse of time after *e*. It is interesting to note that whereas the contractions under 5 kg. are relatively stationary, the contractions at 4 kg. increase in the successive loadings and hence E also increases. Otherwise the behavior, allowing for the fact that the rod had been stressed for several days, is about the same, *cæt. par.*, as before.

In figures 37 and 38 results[1] with special reference to hysteresis are shown as a whole, indicating the three successive loops terminating at r, s, t, for loads between 1 and 3, 1 and 4, and 1 and 5 kg. on the 0.106 sq. cm. of section. It is difficult to interpret the individual data, because clearly their nature is exceedingly complex. There may be some instrumental error; but it is interesting to note that in figure 37 the rod subjected to variations of loads between 1 and 4 kg. shows gradual expansion at the lower pressures. Just as the loops are evidences of hysteresis, so the gradual upward trend of successive loops between the same end-loads is evidence of viscosity.

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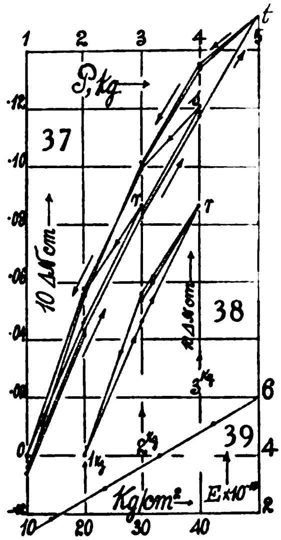
[1] As the data are sufficiently reproduced by the curves, the numerical tables have been removed.



Finally I collected the values of the modulus E, obtained under different loads P in the successive series of experiments. In each series the stable value was reached between given alternating end-pressures, gradually, as already explained. The mean values are

Loads =	14.2	23.6	33.0	42.4 kg/cm².
$10^{-10}E$ =	1.81	2.82	4.00	5.11

results which are exhibited in figure 39. They happen to lie on a straight line. Although these mean values are unequally influenced by the character and number of the experiments made, the result as a whole disarms suspicion. It is improbable, in other words, that data which have shown such detailed consistency as appears in figures 35 to 39 should be seriously influenced by imperfections of apparatus or of method, though it is possible that at the higher pressures the sides of the hard-rubber rods may have expanded into and been sustained by the walls of the rigid sheath b (fig. 34). In the above estimates the rate R at which the modulus E increases per kg./cm.² of pressure would be, nearly,

$$R = 10^{10} \frac{5.1 - 1.8}{42.4 - 14.2} = 0.12 \times 10^{10}$$

The rate R is excessive as compared with subsequent values.

31. Same. Thinner rods, hard rubber.—The suspicion left in the preceding experiments that the marked increase of E was possibly due to the lateral expansion of the hard-rubber rod, so that it more or less filled the rigid sheath at the highest loads, induced me to repeat the work with slightly thinner rods. The former case would make E approach the bulk modulus. The rods last used were therefore turned down on the lathe until their dimensions were

$$2L = 4.90 \text{ cm.} \qquad 2r = 0.35 \text{ cm.} \qquad A = 0.098 \text{ cm.}^2$$

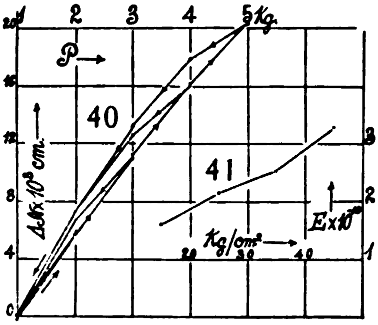
They were then tested on the interferometer for cyclically increasing and decreasing loads P, as shown in table 3, where ΔN is the micrometer equivalent of elongation. A few extra triplets were added. The results are also given in figure 40 and show a beautiful case of hysteresis with gradually increasing loops. This hysteresis is in no way less accentuated when compared with the preceding cases.

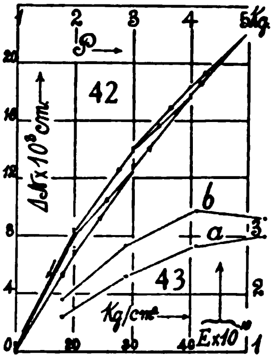
TABLE 3.—Hard rubber. Thinner rod. $2L=4.90$ cm.; $2r=0.353$ cm.; $A=0.098$ cm.^{2}; other data as above. Load kg. permanent. $R/R'=10.3/7.0=1.47$.

P	$\Delta N\times10^6$	$E\times10^{-10}$	P	$\Delta N\times10^6$	$E\times10^{-10}$	P	$\Delta N\times10^6$	$E\times10^{-10}$
kg.	cm.		kg.	cm.		kg.	cm.	
1	0		2	725	1.60	3	1150	
2	565	2.15	1	5		4	1660	3.06*
3	1103		2	585		5	2070	
2	670	1.62	3	1135		4	1800	
1	−20		4	1620	3.18*	5	2225	3.76**
2	575		5	2035		4	2590	
3	1125	2.52	4	1795		5	2200	
4	1605		3	1335		4	2460	2.53
3	1260		2	730		3	2920	
			1	15	1.58	4	2555	
			1	0				
			2	605				

* Loads 4, 5, 4 kg. ** Loads 5, 4, 5 kg.

The values of the modulus E were then computed from the triplets in succession. One may note that the succession 5, 4, 5 kg. gives a much larger value of E than the contrasting succession of loads 4, 5, 4, kg. One can not, therefore, expect a smooth march of values unless this difference is systematically included, while it would be exceedingly difficult to even conjecture a rational method of quantitative interpretation. With this conceded, the mean values were





Load =	15	25	35	45 kg./cm.^{2}
$10^{-10}E=$	1.60	2.15	2.53	3.32

These data are shown in figure 41. From them the mean rate $R=\Delta E/\Delta(P/A)$ $=0.053$ may be deduced. This is in fact less than one-half the preceding ratio R, so that the suspicion that the rigid sheath containing the rod loosely contributes to the high E values at high loads is not removed. It is necessary to reduce the thickness of rods further.

In figures 42 and 43 cyclic data are given for the same rod turned down further to the following dimensions:

$$2L=4.90 \text{ cm.} \qquad 2r=0.33 \text{ cm.} \qquad A=0.085 \text{ cm.}^2$$

The cycles remain, though they are more slender than before. The general trend of the observations, indicating a continual slow viscous yielding throughout, superposed on the hysteresis, is the same as before. The change of the modulus with the load, i. e.,

$P/A=$	18	29	41	53	kg./cm.^{2}
$10^{-10}E=$	1.64	2.30	2.83	3.00	

is in the first three cases not very different from the preceding. The fourth datum is low. One may write $R=\Delta E/\Delta(P/A)=10^{10}\times 0.05$. It is questionable, therefore, whether lateral support received from the rigid sheath can account for the increment of E with the load. The variability of R is rather due to the occurrence of hysteresis, whereby the datum for E is very variable unless found in triplet observations between two definite loads. A few additional triplets (3 or 4 for each step of loads, added to elucidate this question) follow:

$P=$	1 to 2	2 to 3	3 to 4	4 to 5	kg.
$P/A=$	18	29	41	53	kg./cm.^{2}
Mean $10^{10}E=$	1.88	2.84	3.44	3.32	

These data are all larger than the preceding set, owing to the different method of observation (triplets confined to two fixed pressures and not to pressures varying cyclically over a large interval); but the general trend of results (see fig. 43*b*) is the same as before. As E is a maximum in the interval between 3 and 4 kg., it does not seem probable that the possible sustaining effect of the walls of the sheath can have had any important influence. More probably complications of vicosity, hysteresis, and possibly temperature, or even of adjustment, account for the erratic behavior observed. As such I have refrained from pursuing it further.

32. The same. Brass.—To estimate how far the above work may be in error, owing to inadequate rigidity of apparatus, the hard-rubber rods were replaced by brass rods of about the same size. Their constants were

$$L=2.35 \text{ cm.} \qquad 2r=0.375 \text{ cm.} \qquad A=0.110 \text{ cm.}^2 \qquad R=10.3 \text{ cm.}$$
$$R'=7.0 \text{ cm.} \qquad i=45^\circ$$

An example of cycles obtained in this way (after initial loading) may here be given:

Load	1	2	3	4	5	4	3	2	1	kg.
$10^4\Delta N$	0	6.5	11.0	12.0	12.5	10.5	9.0	7.5	1.5	cm.

One may infer that below 2 kg. there is some dislocation, but very little is shown by the residual loop above 2 kg. Moreover, the loop is not a case of hysteresis, as is clear from the inverted march of values. Hence the mean value $\Delta N/\Delta P=0.00016$ cm. may here be accepted, from which $E=5.5\times 10^{11}$ may be computed. This result is too low; but with a micrometer reading to but 10^{-4} cm., limiting the displacement interferometer, a value even as good as this is hardly expected. It indicates the degree of deficient rigidity of the apparatus which is not adapted for brass rods as thick as this, as well as the sectional irregularity of stress. If x be the true micrometer displacement and x' that due to apparent yielding of the apparatus and sectional discrepancies, and if C is a constant and 10^{12} the modulus of brass, we may write (since $\Delta N = x + x'$)

$$E = 10^{12} \frac{C/\Delta N}{C/x} \text{ or } x = E\Delta N/10^{12} = 0.00009 \text{ cm. per kg.}$$

while the yield, etc., of parts is thus equivalent to 0.00007 cm. per kg. In any case, therefore, the individual fringes would have to be used in a rigid apparatus for rods of this relative thickness (0.375 cm.). But there is, of course, no difficulty in inserting thin brass rods, taking advantage of the convenience of displacement interferometry. We may also conclude that the insufficiency of the apparatus lies well within the smallest division (10^{-4} cm.) of the micrometer.

Further experiments were now made by loading the rod with 2.5 kg. and making observations in triplets between 2.5 and 4.5 kg. The results came out equally unsatisfactory, being

$10^6 \Delta N/\Delta P =$	18	26	25	26
$10^{-11}E =$	5.1	3.4	3.5	3.4

Between these and the preceding results there may have been some dislocation, such that a smaller part of the sectional area was strained in the latter case, resulting in a decrease of E; but an error in reading of 10^{-4} cm. would also account for it.

To test the preceding values experiments were now made on a hard drawn-brass rod much thinner than the preceding. Its dimensions were

$$L = 2.5 \text{ cm.} \qquad 2\gamma = 0.205 \text{ cm.} \qquad A = .033 \text{ cm.}^2$$

To accommodate this rod a sleeve was turned, telescoping into the larger sheath *b*, figure 34, and holding the new brass rod within loosely. From triplets of observations between loads of 2.5 and 4.5 kg. the values of E came out as follows:

$10^6 \Delta N/\Delta P =$	149	161	123	120
$10^{-11}E =$	4.2	3.9	5.1	5.2

As this is but half the usual modulus of brass, and as $\Delta N/\Delta P$ is relatively large, the discrepancy must be attributed to yielding or other dislocation within the apparatus.

Tests made for hysteresis showed no effect beyond the possible errors of observation. The mean results obtained from rising and falling loads were, for instance,

$\Delta P =$	1.5	2.5	3.5	4.5 kg.	1.5	2.5	3.5	4.5 kg.	
$10^6 \Delta N =$	137	128	121	114	136	127	121	114 cm.	
$10^{-12}E =$		0.33	0.44	0.52		0.34	0.47	0.48	

The low value of E when the smaller pressures are applied is again the result of irregular or insufficient contact of the rod with the abutment of the apparatus. Much greater pressures are apparently needed to secure an adequately fixed seat of so rigid a body as the brass rod.

33. The same. Glass.—Glass rods of about the same size as the sheath were next tried, the dimensions being

$$L=2.3 \text{ cm.} \qquad 2r=0.36 \text{ cm.} \qquad A=0.102 \text{ cm.}^2$$

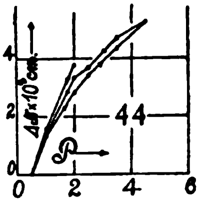
Observations were made in triplets for loads between 2.5 and 4.5 kg. The following results are examples:

$10^5\Delta N/\Delta P=$	70	75	65	69	69 cm.
$10^{-11}E=$	1.4	1.3	1.5	1.4	1.4

A film of pitch was then placed between the end of the glass rod and the cast-iron abutment. The apparatus was heavily loaded for some time to squeeze out all superfluous cement. The triplets measured between 2.5 and 4.5 kg. of load showed the same order of value, viz,

$10^5\Delta N/\Delta P=$	77	70	72	69	65	65 cm.
$10^{-11}E=$	1.2	1.4	1.3	1.4	1.5	1.5

The results throughout in very different adjustments are thus remarkably consistent, but they are all enormously too low, probably not more than about one-third to one-quarter of the true value of E. These unsatisfactory results were not expected, because the modulus is only about half that of brass. The sectional distribution of stress is thus even less uniform in case of glass.



Experiments were also made in cycles, but the early results, though markedly looped, were not quite trustworthy. The following values are of later date and better, remembering that the micrometer reads to but 10^{-4} cm.:

$\Delta P=$	0.5	1.0	2.0	2.5	3.5	4.5	3.5	2.5	2.0	1.0	0.5 kg.
$10^5\Delta N=$	0	130	285	345	440	530	475	373	335	155	20 cm.

They are given in figure 44 and show both in the curved loci and irregularity of detail, that the true elastics of the glass are masked by the incidental discrepancies.

The values of E thus obtained for glass, though consistent, are necessarily too small. It appears, therefore, that a brittle solid like glass does not conduct stress uniformly if the sectional area is relatively large as compared with the length. Thinner rods were next provided, loosely fitting the sleeve already used for brass rods. The new dimensions were:

$$2L=4.84 \text{ cm.} \qquad 2r=0.185 \text{ cm.} \qquad A=0.027 \text{ cm.}^2$$

The rods were placed in the interferometer and the compressions observed, as usual in triplets, between loads of 2 and 4 kg. Most of the individual contractions were again remarkably consistent. Consecutive mean results for instance are:

$10^5\Delta N/\Delta P=$	152	156	153	135	132	137	168	168	152	147	cm.
$10^{-11}E=$	2.5	2.4	2.5	2.8	2.8	2.7	2.2	2.2	2.5	2.6	

As the rods were drawn and not ground true and fitted the sleeve loosely, the occurrence of some dislocation in the data obtained in the three different series is perhaps inevitable. Again the maximum difference of ΔN is within 4×10^{-4} cm. and with a micromter reading to 10^{-4} cm. some of this is observational error. The results, however, show conclusively that even when the glass rod is apparently thin enough as compared with its length, the actual value of the modulus is not obtained. Some outstanding discrepancy has escaped detection.

34. Same. Steel.—A final test of the degree of insufficiency of the apparatus was made by using steel rods loosely fitting the original sheath. Their dimensions were:

$$2L=9.2 \text{ cm.} \qquad 2r=0.37 \text{ cm.} \qquad A=0.107 \text{ cm.}^2$$

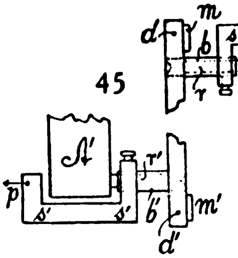
Such rods are, of course, far too rigid and thick for the apparatus and the displacement per kilogram would be but

$$\Delta N/\Delta P=10^{-6}\times 44 \text{ cm.}$$

less than the mean wave-length of light, while the micrometer registers but 10^{-4} cm. It would be necessary, therefore, to work with fringes in any case, even if the apparatus were sufficiently rigid, etc., to guarantee such small displacements. In the best results from triplets between 3 and 4 kg., $10^5\Delta N/\Delta P=22$ cm., equivalent to $10^{-12}\ E=0.4$, could not be improved, and yet this is about 5 times too small. Steel rods less than a millimeter thick would have to be used if a trustworthy value of E were aimed at.

35. Modifications of apparatus.—The above apparatus failed in case of rods of high rigidity and insufficiently reduced sectional areas. Brass and steel rods 2 cm. long will have to be at least as thin as 1 mm. in diameter if the data for E are to be trustworthy. There is, in other words, a source of error in the apparatus itself, by which $\Delta N/\Delta P$ is incremented; and this is to be sought in the method by which stresses are applied. Though each rod is supported at one end by the friction at its contact with the abutment A, figure 28, and at the other by the thread of the bifilar suspension, this support is not guaranteed to the extent required. The vertical bifilar stress yields in its relation to the much larger horizontal stresses of the loads, so that the bar F undergoes independent slight rotations around vertical and horizontal axes of its own. This is evidenced by the fact that the fringes sometimes change their inclination, or the two white slit-images may to a small degree lose their full coincidence (flexure).

The occurrence of the hysteresis loops in case of hard rubber may be anticipated, *a priori*, although the amount of this quality is also modified by the yield of the apparatus. If hysteresis is marked, the moduli E would naturally increase with increasing loads as the top ends of the loops become relatively more horizontal. One would expect, however, that the curve with continually increasing loads should be straight, and the curvature (concave downward) of the above graphs is probably introduced by the apparatus.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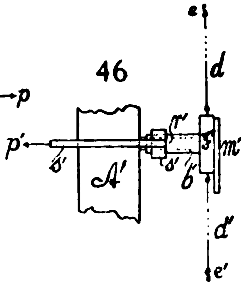
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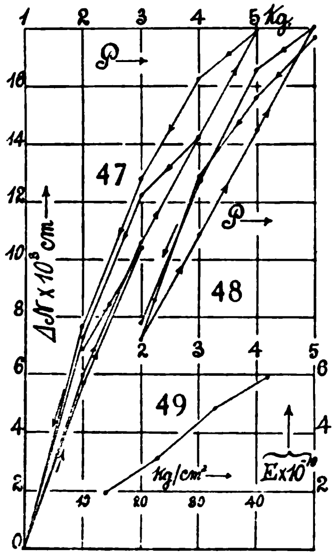
I therefore modified the apparatus as shown in figure 45 (plan) by attaching the offsets ss and $s's'$ firmly to the sheaths b and b' (holding the rods r, r'), by means of set screws. The pull of the pulley strings p and p' is to be coaxial with the respective rods r, r'. FF' is the bar holding the auxiliary coplanar mirrors m, m' and is supported by the bifilar threads attached at d and d'. This bifilar is now also to be advantageously modified (fig. 46, side elevation) by doubling it and attaching the tense fibers d,d', above and below at e and e' to corresponding standards. There are such pairs of fibers at either end of FF'. Thus the bar is held in place in the absence of the stress along p', and this should be applied in such a direction as not to disturb the reflection from the mirrors m', m.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36. Observations.—Cycles obtained with the new apparatus are given in figures 47 and 48. They are in many respects satisfactory, showing the hysteresis loops and the gradual yielding of the viscous solid to stress. In spite of all precautions, however, the slit-images separated very slightly, proving that the rigid attachment of the offset ss (figs. 45, 46) promotes flexure as the result of small cross-torques. The loose offset which can not convey flexural torque is in this respect preferable. Of the values of the modulus computed from the pressure—ascending branch of the cycles—the following data may be recorded by way of example (hard-rubber rod, $2L = 5.05$ cm.· $2r = 0.37$ cm.; $A = 0.107$ cm.^{2}).

P	$10^5\Delta N/\Delta P$	$10^{-10}E$
kg.	*cm.*	
2–4	362	2.7
3–5	360	2.7
1–3	530	1.8
2–4	412	2.4
3–5	367	2.7
1–3	522	1.9
2–4	427	2.3
1–3	520	1.9

The modulus thus increases definitely with the stress, except in the first case; but not so much as in the case of the measurements made in triplets. These were also carefully observed and appeared as follows:

P	$10^5\Delta N/\Delta P$	$10^{-10}E$	Kg./cm.^{2} (load)
4–5–4 kg.	280	3.5	42
5–4–5	173	5.6	
4–5–4	165	5.9	
4–3–4	315	3.1	33
3–4–3	200	4.9	
2–3–2	310	3.1	23
3–2–3	310	3.1	
1–2–1	485	2.0	14
2–1–2	495	2.0	

The rod, therefore, grows apparently more rigid as a result of this alternation of stress, particularly at high loads. The same result is seen in the individual readings of the triplets. If the last results are taken, the curve (fig. 49) results. They make a series much like figure 39 for similarly shaped hard-rubber rods, and the following ratio is of the same order as the above:

$$R=\frac{E}{\text{kg./cm.}^2}=10^{10}\times 0.14$$

With a total change of apparatus the successive increments of rigidity for the same step of loading were again investigated with similar results, as follows:

P	$10^5\Delta N/\Delta P$	$10^{-10}E$	Kg/cm.^{2}	P	$10^5\Delta N/\Delta P$	$10^{-10}E$	Kg./cm.^{2}
4–4–4	275	3.5	42	2–1–2	600	1.6	14
5–4–5	173	5.6		1–2–1	470	2.1	
4–5–4	160	6.1		2–1–2	455	2.1	
5–4–5	165	5.9		1–2–1	465	2.1	

37. Apparent yield within the apparatus.—It remains to determine how far the above apparatus may be made rigid. With such an end in view, steel rods of the dimensions

$$2L=5.03\text{ cm.}\qquad 2r=0.37\qquad A=0.107\text{ cm.}^2$$

were put in the sheath. In this case (P in kg.), if $C=98$ is the constant of the apparatus,

$$10^{-6}E=C/(\Delta N/\Delta P) \quad \text{or} \quad \Delta N/\Delta P=49\times 10^{-6}$$

about one-half the smallest division of the micrometer, or a little over one fringe (1.2), is all that may be looked for. The results with the *fixed* offset were larger, 3 to 4 fringes; or at the micrometer

$$P=4-5-4 \text{ kg.} \qquad 10^{5}\Delta N/\Delta P=10 \text{ to } 30 \qquad 10^{-12}E=0.5 \text{ to } 1.0$$

Thus E is about one-quarter of the true value and three-quarters of the displacement is apparently in the apparatus. Under lower loads the conditions are not essentially different. I obtained in the triplets, for instance,

$$P=1-2-1 \text{ kg.} \qquad 10^{5}\Delta N/\Delta P=33 \text{ to } 50 \qquad 10^{-12}E=0.2 \text{ to } 0.3$$

largely influenced by errors in setting the micrometer, and the increased uncertainty of the seat of the rod.

The fixed offsets (ss, figs. 45, 46) were now removed and replaced by the loose offsets with conical plugs and sockets (figs. 28, 34). Observations were made in triplets as before, showing

$$\begin{array}{lll} P=4-5-4 \text{ kg.} & 10^{5}\Delta N/\Delta P=33 \text{ to } 34 & 10^{-12}E=0.23 \text{ to } 0.30 \\ \quad 1-2-1 & \qquad\qquad 27 \text{ to } 40 & \qquad\qquad 0.24 \text{ to } 0.36 \end{array}$$

These results are identical and substantially the same as the preceding set. In a series of decreasing pressures I found

$$\begin{array}{rccccccccc} P= & 5 \text{ kg.} & & 4 \text{ kg.} & & 3 \text{ kg.} & & 2 \text{ kg.} & & 1 \text{ kg.} \\ 10^{5}\Delta N/\Delta P= & 195 & & 155 & & 125 & & 75 & & 0 \\ 10^{-12}E= & & 0.24 & & 0.32 & & 0.20 & & 0.13 & \end{array}$$

so that it is not until all but 2 kg. are removed that the apparatus registered a yield or dislocation of seat. In case of the loose pulley there was no flexure of the bar F; the slit-images remained in coincidence, and the fringes strong and clear at all loads, without changing their inclination. Hence, since the above mean $\Delta N/\Delta P=0.00037$ cm., and as $x=E\Delta N/10^{12}=0.000098$ cm. is to be ascribed to the steel, 0.00027 cm. per kg. may be considered as yielding coefficient of the apparatus; i. e., about 0.0003 cm. should be deducted from all deflections per kilogram of load.

The endeavor was now made to ascertain where this yield is to be located. It was found that by loading the pulley standards b,b' (figs. 28 and 29) no appreciable displacement of fringes was produced, provided the load was the same on both sides b and b'. This is the case in the above experiments. On loading b or b' alone, however, the yield was quite marked, showing $10^{5}\Delta N/\Delta P=25$ in case of loads of 1 and 2 kg. But this non-symmetrical method of loading is never used.

In a further examination of the apparatus it was found that vertical or longitudinal slight pushes on the bed-plate (tripod) were ineffective, but that a cross-push, even if a mere touch, as this directly tends to change the angle α of the bar F, was very appreciable. Somehow, through the interaction of the bifilar and the load, stress or spurious torque about the vertical (hence $\Delta\alpha$) is introduced; but it is extremely difficult to state just how when all loads are symmetrically vertical. To obviate this the permanent parts of the apparatus should be cast in one piece.

An interesting corroboration of these observations is given by the pendulum oscillations of the loads. These produce (even for loads as small as 1 kg.) marked vibration of fringes if the load vibrations are transverse to the rays, while vibrations are ineffective if longitudinal (in direction of the rays). Noticing the behavior of fringes for the first time, I supposed that the centrifugal accelerations introduced by the vibration might be the cause. In fact, if the length of the compound pendulum (load w, figs. 28 and 29) treated as a simple pendulum is L and its mass M, the centrifugal force at any displacement s, corresponding to the displacement velocity v, is

$$F=\frac{Mv^2}{L}$$

But if $s=A\sin\omega t$, then $v=A\omega\cos\omega t$, and thus

$$F=MA^2g\cos^2\omega t\,/L^2$$

since $\omega^2=g/L$. This force is a maximum at $t=0$ sec., or $F_0=Mg\,.\,A^2/L^2$. If $A=1$ cm. and $L=15$ cm., F_0 is but $\frac{1}{225}$ of the weight of the load Mg, say a maximum not exceeding 25 grams of weight. Since the whole displacement is but a few fringes, this small fraction could not be discernible with a body like the steel rod above. Thus the alternating transverse stress produced by transverse swinging alone can account for the observed effect.

38. Ocular micrometer. Collimator micrometer.—These methods of measuring the displacement of fringes have been discussed in Chapter I, § 4, 5. It was of interest to test them here. The scale on the ocular plate inserted divided the field width into 100 parts, the division being in 0.1 mm. The distance apart of the achromatic fringes was a little more than this (1.4 cm.) The correspondence could be made exact by rotating the auxiliary plate (bar F, fig. 28) about a vertical axis slightly, but the adjustment was not necessary here. These excessive tenth millimeters thus correspond roughly to an elongation $2\Delta l$ of both rods, equal to the mean wave-length of light, or more accurately,

$$2\Delta l=\frac{R'}{R}\lambda$$

where $2R$ is the distance apart of the interfering rays ab (fig. 27) and $2R'$ the distance apart of the rods rr'.

If the size of fringes is known on the ocular micrometer, they may be counted by their displacement along it, since the central achromatic fringe is always distinguishable and serves as an index. But the width of fringes, if the laboratory is not quiet, is hard to measure, for they quiver or vibrate. It is easier to express the displacement Δe of fringes in the ocular in terms of ΔN, the corresponding displacement of the micrometer of the interferometer.

By direct comparison the following values were found:

$$\begin{array}{rcccl} 10^3\Delta e = 460 & 480 & 405 & 460 & \text{cm.} \\ 10^5\Delta N = 145 & 165 & 125 & 155 & \text{cm.} \\ 10^4\times\text{ratio} = 32 & 34 & 31 & 34 & \end{array}$$

The mean value is $\Delta N/\Delta e = 0.00328$, or $\Delta e/\Delta N = 305$. The variable Δe is thus over 300 times larger than ΔN.

The cyclic experiments with the steel rods previously used showed a lack of coincidence of the ascending and descending graphs which must here be spurious. I obtained, for instance,

$$\begin{array}{rcccccccl} \Delta P = 1 & 2 & 3 & 4 & 5 & 4 & 3 & 2 & 1\ \text{kg.} \\ 10^3\Delta e = 0 & 90 & 170 & 250 & 330 & 275 & 190 & 110 & 0\ \text{cm.} \end{array}$$

As the ascending graph is fairly regular, the apparent modulus may be found from it. Three such series gave mean values of Δe, from which ΔN, Δl, and E were computed as follows:

$$\begin{array}{rccl} 10^3\Delta e/\Delta P = 85 & 81 & 86 & \text{cm./kg.} \\ 10^5\Delta N/\Delta P = 28.1 & 27.0 & 28.4 & \text{cm./kg.} \\ 10^5\Delta l/\Delta P = 14.6 & 14.0 & 14.8 & \text{cm./kg.} \\ 10^{-10}E = 0.35 & 0.36 & 0.34 & \end{array}$$

To find Δl, the elongation of each rod, the equation

$$\Delta l = \frac{R'}{R}\,\frac{\Delta N \cos i}{2} = 0.52\ \Delta N$$

suffices, since $R'/R = 1.47$ and $i = 45°$. The values of $E = 97.8/(\Delta N/\Delta P)$ are of the same low order, scarcely one-fifth of the true value already found; i. e., they merely indicate the deficient rigidity of the apparatus.

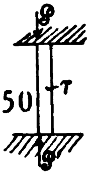
In the last series of triplets made under different circumstances by aid of Δe, the mean values were

$$\begin{array}{lccccl} \text{Mean load} & 15 & 25 & 35 & 45 & \text{kg.} \\ \text{Mean } E\times 10^{-12} & 0.28 & 0.35 & 0.46 & 0.51 & \end{array}$$

a steady progression, with remarkable consistency throughout; but it would require a load of 20 to 25 kg. to reach the true modulus for steel. So also E increases in successive triplets to a limit; as, for instance, at the loads 4 to 5 kg.,

$$E\times 10^{-12} = 0.40,\quad 0.49,\quad 0.54,\quad 0.52$$

39. Summary.—It has been found that a spurious displacement of 3 to 6 fringes per kilogram of the load, apparently within the apparatus, has not only not been overcome in the above form of construction, but the cause of this residual discrepancy could not be definitely ascertained. Very slight flexures or torsions of the bar F (fig. 28) by the stresses would account for it; but in such a case the slit-images should lose coincidence, and this has not been typically the case. A slight rotation of the whole apparatus around a vertical or a horizontal axis normal to the rays from the interferometer would also contribute to the error in question. Yet the method of loading does not seem to admit of such stress, unless the weight supported on the single leveling-screw *g* of the base (figs. 28, 29) nearer the interferometer, is at a disadvantage as compared with the load supported on two screws remote from the interferometer. Moreover, this apparent yield is consistent, increasing proportionately to the load, so that it is difficult to separate it from the true strains. The only way of counteracting these difficulties is to remodel the apparatus, casting it as a single massive piece of metal, and providing other means (precision slides) of applying stress. The pulley-offset device used must be regarded as improvised, for it is here that fictitious strains undoubtedly enter.



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Apart from these considerations and in view of the consistent presence of the discrepancy, its cause might be sought in the unequal sectional distribution of stress from end to end of the rod. Such an effect, moreover (i. e., any unevenness of the seat of the rod *r*, fig. 50), is not unlikely to produce flexure. Thus the stresses *PP'* in the figure, acting on the left edge, would tend to bend the rod to the right at the middle. This would also be a purely elastic deformation and behave as such. Again, if in figure 50, *P* and *P'* act at diagonally opposite corners, the rod is subject to shear.

There is still another possible explanation of the discrepancy in terms of friction, which may be adduced. Appreciable friction effects at the pulleys are improbable; but at the conical points of the offsets, as they engage the corresponding conical sockets, displacement involving marked friction is not excluded. If, therefore, c is the coefficient of friction and W the weight applied, and if we write $N = CW$, where C is a constant, the effect of additional weight dW may be written

$$dN = C(1-c)dW \text{ or } \Delta N = C(1-c)\Delta W$$

when the load is increased; and similarly,

$$-\Delta N = C(1-c')\Delta W$$

when the load is being gradually decreased. If the effective coefficients are not equal, the observations would therefore correspond to two lines of different slopes and the passage from one to the other would invariably be hysteresis-like, even if there is no such quality appreciable in the elastic solid under observation. Also, the phenomenon would increase in magnitude as the deformations are greater, giving a result very similar to the above observations. True, whether the friction effect occurred in a symmetrical or regular fashion or not, it would not have been eliminated in the triplets of observations made. It is quite conceivable that if a weight W added has produced cW less stress than is implied, the removal of that weight would deduct $c'W$ less stress than implied, or even for larger c' deduct no stress at all. Hence in such triplets ΔW would be much too large or ΔN much too small and therefore E too large. The effect would be a marked increase of E with the load such as occurs above.

As a result of the residual difficulties enumerated, rods whose modulus is of the order of 10^{12} can not be expected to show trustworthy behavior, unless their diameters are less than a millimeter for a length of about 2 cm. in a suitable protecting sheath. Rods whose moduli lie markedly below 10^{11} may be used when the diameter is 3 or 4 mm. It is for these materials, i. e., the large class of well-developed organic solids, that the apparatus has been devised. From them, moreover, in view of the accentuated deformations, interesting information bearing on the elastics of bodies as a whole may be expected. Viscous phenomena in their entirety, including hysteresis, have a close analogy to the condensation of a vapor. To condense the instabilities to molecular aggregates of small volume takes more pressure than is necessary to release them; i. e., to evaporate them, as it were. If one considers the viscous deformation (*cæt. par.*) as decreasing at a very rapid rate through infinite time, the hysteresis may be considered as an integral part of the viscous phenomenon. The viscous after-effect may then be explained as due to condensation of configurations made unstable or evoked by the heat motion within the body, whereas all the instabilities present at a given time in relation to the applied stress are swept away in the hysteresis phenomenon.

A great many experiments were now made to endeavor to locate the seat of the yielding within the apparatus. Thus the bifilar was variously attached independently of the weighting appurtenances, the base was clamped and screwed down in different ways, a new rigid bar FF was constructed, etc.; but all these attempts failed to eliminate the discrepancy. Pull on the framework and distribution of weights upon it produced no displacement of fringes. Only when weights are placed on the scale pan does yielding (real or apparent) occur; so that it must in some way be connected with the offsets. Replacing the conical ends by sharp darning-needle points was not advantageous.

Somewhat improved results appeared when the bifilar threads were replaced by brass strips about a foot long, a half inch broad, and one-sixteenth inch thick, care being taken to insert them without stress. An example of a cycle (e referring to the ocular micrometer) with steel rods may be given ($2L=5.03$ cm., $2r=0.37$ cm.).

$P=$	5	4	3	2	1	2	3	4	5 kg.
$10^3\Delta e/\Delta P=$	85	75	60	38	0	25	50	70	85 cm.
$10^6\Delta N/\Delta P=$	170	150	120	76	0	50	100	140	170 cm.

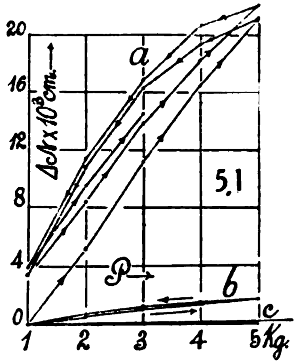
The results of many similar experiments all consistently indicated the same order of yield within the apparatus as before. Data would have been much smoother but for the air-currents in a steam-heated room, which caused the fringes to vibrate. The yielding is usually excessive at the lower loads.

A final test was made by replacing the slit of the collimator by a glass scale (Chapter I, § 5). This is distinctly seen in the telescope and supplies an even more sensitive micrometer, with the advantage that the telescope may be shifted, there being no fiducial line within it. If $\Delta\epsilon$ refers to displacement of fringes measured along this scale, the standardization showed that $10^3\Delta N/\Delta\epsilon = 101$. Triplets for the loads 1-2-1 kg. gave a mean value of $\Delta\epsilon/\Delta P = 0.25$ cm., whence $10^4\Delta N/\Delta P = 2.51$ cm. and $E = 0.39 \times 10^{-12}$ as above. The change is cyclic, the graphs are curved. The mean E, even above 3 kg. of load, will not exceed 0.3×10^{12}. Triplets gave in succession

$P =$	1—2—1	2—3—2	3—4—3	4—5—4	kg.
$10^3\Delta e/\Delta P =$	27	17	13	9	cm.
$E \times 10^{12}/\Delta P =$	0.20	0.29	0.38	0.54	cm.

which is no marked improvement over what has preceded.

There seems to be little hope of further increasing the effective rigidity of the apparatus. Measurements for E will therefore have to be made differentially. For this purpose the constants of the apparatus (here with elastic brass bifilar) are to be determined by a relatively thick steel rod, as has just been done. An example of this has been put in figure 51, *a*, showing the behavior of a fresh, hard-rubber rod treated in successive cycles of loads between 1 and 5 kg. Hysteresis and viscous deformations appear very clearly, as usual. The mean moduli for the ascending branches will not much exceed 2×10^{10}, and in triplet observations they ran from this to about 6×10^{10} for loads up to about 50 kg. per cm.², also in the manner found above. As a contrast to these results the behavior of the steel rod of the same dimensions ($2L = 5$ cm., $2r = 0.37$ cm., nearly), for which data have just been given, is also inserted in the lower curve *b* of the same figure. Both curves are cyclic, but on an enormously different scale, even though the true steel moduli can not be approached to more than one-quarter. The true steel line is shown at *c*. We admit that the hysteresis in curve *b* is possibly in the apparatus, which is a compound elastic body; but it is hardly plausible that the hysteresis of curve *a* can be similarly explained away. The apparatus discrepancy is given in *b*.



CHAPTER IV.

EXPERIMENTS IN GRAVITATION.

I. GRAVITATIONAL ATTRACTION.

40. Introduction.—The ease with which the rectangular interferometer admits of the measurement of small angles induced me to adapt an apparatus with reference to it, for the measurement of the Newtonian constant. In addition to the usual system suspended in air, I also tested a floating system. Accurate work is scarcely to be expected in this laboratory, where temperature variations and the agitation of the room conflict with the condition of its obtainment. But the trial is nevertheless interesting and the final work may be done elsewhere.

41. Equations.—The old bifilar system has more recently been superseded by the quartz fiber of Boys. We should have in systems of the same length L under like torque T per radian in the two cases, the modulus

$$\mu = T/\theta = mg\ ll'/L = \pi n r^4/2L$$

where mg is the weight of the needle, ll' the spacing (above and below) of the bifilar, n the rigidity, and r the radius of the quartz fiber. If $n = 5 \times 10^{11}$, $g = 981$, this relation reduces to

$$m l l' = 8 \times 10^8 \times r^4, \text{ nearly}$$

If $r = 10^{-2}$ cm., $m l l' = 8$, so that for $l = 0.1$ cm. and $l' = 0.5$ cm., $m = 160$ grams would give equivalent elastic efficiency. The bifilar would be superior, as the quartz fiber could not carry this load. If $r = 10^{-3}$ cm., $m = 0.016$ grams only could be used in equivalence, quite apart from the torsion coefficient of the silk fiber of the bifilar. Here the quartz fiber would be superior, as so small a load implies a correspondingly small gravitational force.

In relation to the gravitational experiment, if μ is the modulus in either case, we would have in succession

$$f = \gamma Mm/d^2 \qquad \Delta\theta = \frac{\cos i}{2R}\Delta N \qquad T = fl'' = \mu\Delta\theta$$

where $i = 45°$ is the angle of incidence and $2R$ the distance apart of beams of light, ΔN the micrometer displacement of the interferometer, fl'' the force couple of the torsion balance. Hence

$$\gamma = \mu \frac{d^2}{Mml''}\frac{\cos i}{2R}\Delta N$$

If we take the first case ($l = 0.1$, $l' = 0.5$, $L = 50$) for the bifilar $\mu = 2 \times mg$

$\times 10^{-3}$ (since the load of the bifilar is $2m$ if m is the mass of each ball) and put $M = 10^3$ grams, $m = 1$ gram, $2R = 10$ cm., $l'' = 30$ cm.,

$$\gamma = \frac{d^2}{30 \times 10^3 \times 1} 2 \frac{0.71}{10} \Delta N = 4.8 \times 10^{-6} \times d^2 \times \Delta N, \text{ nearly}$$

or

$$\Delta N = 6.7 \times 10^{-8} / 4.8 \times 10^{-6} \times d^2 = 0.015 d^2$$

If d is estimated as 3 cm., then $\Delta N = 0.0016$ cm. With the given interferometer and reasonable estimates as to the other magnitudes, one should therefore obtain nearly 40 achromatic fringes (even with the bifilar as stated) for the attractions of 1 kg.

There would be no gain, in case of the bifilar, by increasing the mass m at the ends of the needle; for the modulus of the bifilar increases as m, which would therefore cancel the m in the denominator of the expression for γ. But if the system is floated in water, m may be increased with advantage indefinitely, while the load of the bifilar is kept constant by providing a corresponding float. If this bifilar load is m', we therefore have

$$\gamma = \frac{d^2}{Mml''} m'g \frac{ll'}{L} \frac{\cos i}{2R} \Delta N$$

Inserting the data of the apparatus of the next section,

$d = 5$ cm. $l'' = 30$ cm. $M = 10$ g. $m = 30$ g. $m' = 2$ g.
$ll' = 0.05$ cm.^{2} $L = 50$ cm. $i = 45°$ $2R = 10$ cm.

$$\gamma = \frac{25}{30 \times 10^3 \times 30} 2 \times 10^3 \frac{0.05}{50} \frac{0.71}{10} \Delta N = 3.9 \times 10^{-6} \Delta N$$

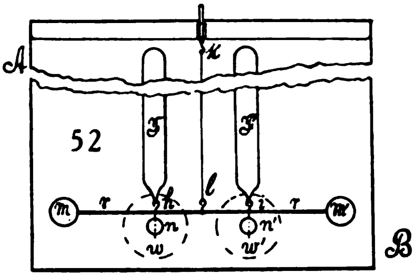
or

$$\Delta N = 0.017 \text{ cm.}$$

per attracting kilogram. With a reasonable size of float there is no difficulty in increasing the attracted mass m to over 60 grams and the attracting mass (with a slight increase in d) to 10 kg., so that values of ΔN of the order of a millimeter are not out of the question.

The difficulty with the method lies in the simultaneous increase of the period of vibration of the needle, and this seems fatal; but I thought it worth while, nevertheless, to give the method a trial.

42. Observations. Floating system.—Figures 52 and 53 represent the floating needle submerged in the narrow trough AB provided with two windows of plate glass w w', through which the interfering beams enter and leave, nearly at right angles to the coplanar mirrors n and n' attached to the needle. This consists of a tube of aluminum, about 30 cm. long and 6 mm. in diameter, with balls at the ends m and m', pairs of 30 grams to 60 grams each being admissible. The two hooks h and i carry the floats F, F', test tubes as much as 15 cm. long and 2.5 cm. in diameter, but to be changed in capacity with the balls m, m' used. The stems of the hooks, screwed and sealed into the tube rr, carry the mirrors n, n'. The needle is suspended from a torsion-head by the bifilar of silk fiber kl, also completely submerged in the water-bath. The hooks k and l are provided with a thin flat sheet-metal link, by which the bifilar may be appropriately spaced.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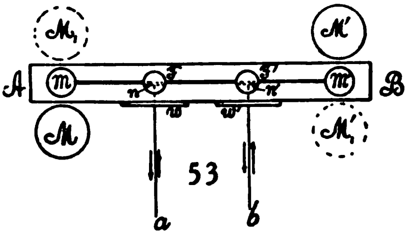
The needle system is so adjusted as just to float. It is then weighted by 1 or 2 grams to sink it. The weight in water may be measured at K. In view of the weight of needle, the mirrors n, n' may be rigidly connected by a very narrow strip of this plate glass, which facilitates adjustment.

The attracting weights M and M' up to 5 kg. were used in pairs M, M_1 and M', M'_1, on each side suspended by a steel belt from a pulley overhead in such a way that when M, M' are in place M'_1, M_1 are raised out of effective reach.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The chief difficulty was encountered in floating the needle. When this was done the whole tank was fastened in place on the interferometer, as the torsion-head at k is attached adjustably to the tank.

The fringes were found without much difficulty; but they were in incessant motion, owing no doubt to eddy currents produced by temperature differences. After many trials I concluded that measurements would be untrustworthy and further trials were therefore abandoned. The experiment is in fact too difficult for a single observer and would be feasible only in an environment of perfect quiet and constant temperature.

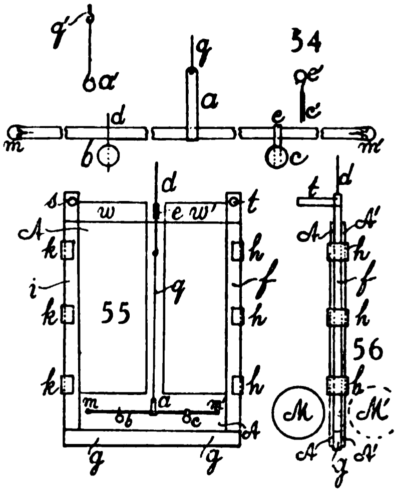
43. Expeditious fringe detection.—In work like the present it is necessary to find the fringes quickly under considerable disadjustment of parts. In this case, if the auxiliary mirror m (fig. 4, Chapter I) can be manipulated, the two images in the telescope T are first made coincident by adjusting M', in which case the rays are parallel as they enter T. The mirror m is then rotated around the vertical and the horizontal axis, until, to the eye, the spots of light coincide locally on the face of M'. Fringes when found by moving the micrometer here are then strong. If mm can not be interfered with, the rays are first made parallel as before by the coincidence of images in T. Thereafter the mirrors M and M' are rotated around a horizontal and a vertical axis *in parallel* (i. e., successively or alternately retaining their parallelism) until the spots of light on M' again coincide, locally, to the eye, or better, when caught objectively on a screen. It is clear from figure 2, Chapter I, that the remoter ray from M is displaced on the screen more rapidly than the nearer ray from M' and therefore parallelism with local coincidence of rays on M' is generally possible. If the parallel rays T_1 and T_2 which coincide in T are too far apart, no fringes can be found, even when the path-difference is annulled.

44. Heavy needle in air.—The needle was now deprived of its floats and other appurtenances, provided with somewhat lighter balls ($m=18$ grams each), and suspended in the same float-vessel in air. Since in this case ΔN is independent of m, and as the dimensions were about of the same order given in the example above ($ll'=0.05$ cm., $L=50$ cm., $M=10^3$ g., $2R=10$ cm., $l''=30$ cm.) with a somewhat larger $d=6$ cm., $\Delta N=0.0004$ cm. per kilogram of attracting load was to be expected; i. e., about 10 achromatic fringes.

The adjustment is quite difficult, since the small mirrors must not only be approximately parallel, but must be so spaced and inclined as to receive the component beams and to reflect them through the mirrors of the interferometer. This was accomplished with some patience and the achromatic fringes were found. But here again they proved to be in incessant and relatively rapid motion, so that they swept continually through the field. Though observed for some time, on different days, they were never found sufficiently quiet to admit of the above measurements. It was impossible, in other words, to eliminate the air-currents within the shallow envelope sufficiently to warrant counting at the rate of 10 fringes per attracting kilogram. In the summer I think this would have been feasible, as there was no other drawback militating against the completion of the experiment.

45. Light needle in air.—In view of the failure of the heavy needle, I went to the opposite case of a relatively light needle, weighing when loaded but 1.49 grams. This was made of a rigid shaft of straw (mm', fig. 54) 25 cm. long, the ends being slit slightly into four symmetrical segments each, which receive the two shots m and m', additionally secured with a little wax. The two light mirrors b and c were differently mounted; b on a fine pin d, snugly fitting corresponding perforations in the straw, was thus capable of rotation around the vertical axis (d) and moving up and down slightly. The mirror c, however, was mounted on a thin elastic strip of aluminum, clasping the shaft as shown in section at $e'c'$, and thus capable not only of rotating on a horizontal axis, but of being placed at different distances (moving right and left) from d to accommodate the rays of the interferometer, to which b and c are to be normal. The needle is swung from a quartz fiber q and a strip or hanger of elastic aluminum a (see a', q'), which clasps the shaft. Hence the latter may also be moved endwise to be balanced and rotated around a horizontal axis till b and c meet the rays normally. These operations are completed by trial before the needle is definitely hung, preferably in a broad beam of sunlight. No great accuracy is required.

The shallow case is made of two plates A, A' (figs. 55, 56) of $^1/_8$-inch plate glass about 32 cm. broad and 45 cm. high, spaced by two strips *i f*, 3 cm. wide, of thick (9 mm.) plate glass, reaching not quite from top to bottom. Six steel clips *h, h, h, k, k, k*, (such as are used for binding pamphlets) held these strips in place and also clasped securely on the outside two wooden strips of about the same width and one-half inch thick. At the top of the wooden strips, two nipples, *s* and *t*, of ¼-inch gas-pipe, projecting normally to the strips, served for the hanging of the case and needle from a firm wall-bracket. It would have been preferable to use the wood strips (without the glass strips) clasped between the glass plates *A*, *A'* as spacers and hangers at once, and this was eventually done. The bottom of the case is subsequently to be closed from below by a strip of felting *gg*. To diminish the space within the case two thin cloth-covered wooden boards *w*, *w'* are inserted from the top. The quartz fiber *q*, to carry the needle *mm*, hangs from a long $^1/_8$-inch brass rod *d*, which may be raised or lowered in view of the sleeve *e*, held by a separate adjustable arm without. The rod *d* must fit the perforation in the cork nicely, so that the former may be smoothly raised or lowered and held in any position by virtue of friction. To swing the needle this is first placed on cork Y's below the opening of the case *A*, *A'*, the felting *g g* having been removed. The quartz fiber is then lowered on the long stem *d*, until the lower hook on the fiber is in position to grasp the clasp on the needle *mm*, still below the case. The needle is then cautiously lifted by raising *d* until it has the required position relative to the interferometer, about as shown in the figure. After this the felt strip *gg* is inserted to close the case, and the necessary adjustment made at *e* and *d* to swing the needle freely in the restricted space provided for it.



Observations were made with the needle at some length. The quartz fiber used was $L = 17$ cm. long; the distance *d* apart of attracting weights *M* and attracted weights *m* was 6 cm. Hence for $M = 10$ gr.

$$\gamma = \frac{36}{23 \times 10^3 \times 75} \frac{5 \times 10^{11} \pi}{2 \times 17} r^4 \frac{0.71}{10} \Delta N = 6 \times 10^6 r^4 \Delta N$$

Thus, if $r = 10^{-2}$ cm. $\Delta N = 1.1 \times 10^{-6}$ cm. per kg. of M $T = 4.6$ sec.,
$r = 10^{-3}$ cm. $\Delta N = 1.1 \times 10^{-2}$ cm. per kg. of M $T = 465$ sec.

The first case would then correspond to but one-fortieth of a fringe; in the second case there should be 250 fringes per attracting kilogram.

If the tenacity of quartz be taken as 1.5×10^3 kg. per sq. cm. the latter filament ($r = 10^{-3}$ cm.) should still hold 4.5 grams, or much more than the weight of the above needle (within 2 grams.)

As the moment of inertia of the needle is $2 \times 0.75 \times 13^2 = 253$, and if the modulus of torsion be computed from the slide modulus $n = 5 \times 10^{11}$, the periods would be as given above, the second being nearly 8 minutes. The period of the above needle was estimated at about 2 minutes. This would give a modulus of torsion about 0.7 and make $\Delta N = 67 \times 10^{-5}$ cm. per attracting kilogram; i. e., about 17 achromatic fringes per kilogram were to be expected.

Having mounted the needle as stated, the fringes were found without much difficulty and the image of the wide slit (i. e., the reflected beam seen in the telescope) was almost quite stationary, the light needle being thus adequately damped. But within this virtually stationary slit-image, the fringes (preferably made horizontal) continually wandered up and down, showing that micrometric vibration had not been eliminated. The experiment is a very impressive one, but as the drift is still much larger than the 17 fringes per kilogram to be anticipated, any attempt at measurement is again idle. Whether this drift is due to temperature or to the tremors of the laboratory would be difficult to state. Cutting down the intensity of the interfering rays (which need not be strong) made no difference.

When the case is open above, there is no difficulty in finding the position of equilibrum of the needle symmetrically to the glass walls of the case. This position is assured by the parallelism of the images of the needle in both faces with the needle itself, and their distance from it. When, however, the wooden boards are inserted, the needle does not seem to be in stable equilibrium in the symmetrical position. It tends to move either into one or the other extreme of oblique positions at which the balls touch the plates of the case. Believing the phenomenon to be electrical, I placed a radium tube in the vicinity of the case for some time, but this made no difference. Damp cloth did not change the result. This was particularly true in the earlier experiment, where ¼-inch plates were used for the case in the absence of thin plates, and in which one plate was thicker than the other. One would thus be inclined to interpret the instability as possibly due to the gravitational attraction of the residual disk. Considering the case as that of a point mass confronting an infinite disk, the potential would be $\gamma(C \pm x) 2\pi\sigma$ and the force $2\pi\gamma\sigma$ per gram attracted, which is a little above the mass of the ball. Thus, if $\sigma = \rho t$ (σ being the density and $t = 0.1$ cm. the residual thickness of plate corresponding to the surface density σ), the force should be

$$f = 6.3 \times 6.7 \times 10^{-8} \times 3 \times 0.1 = 1.3 \times 10^{-7} \text{ dyne.}$$

Since the lever arm is 13 cm., this makes the torque 1.7×10^{-6} dyne cm.; and if the modulus of torsion of the quartz fiber is 0.7, as estimated above, the deflection should be

$$\theta = 1.7 \times 10^{-6} / 0.7 = 2.4 \times 10^{-6} \text{ radian}$$

i. e., about half a second of arc and therefore quite ineffective so far as instability is concerned. Neither does it seem plausible that the needle striking the glass under the influence of the γ rays of radium or in a damp atmosphere can be charged. Yet the phenomenon which shows itself as an accelerated drift toward either plate is very decided, so that the free position of equilibrium to be obtained by gradually adjusting the torsion-head can not be found.

In view of the importance of this question, I installed a thin quartz fiber of about the same length $L=17$ cm. and found its moment of inertia by attaching to the fiber a brass cylinder $l=3$ cm. long and $m=2$ g. in weight. The period of vibration was found to be 50 to 52 seconds. Hence, as the moment of inertia is 1.5 $\mathrm{g}\times\mathrm{cm.}^2$ the modulus of torsion is $n=0.024$. But even this (by the above method of computation) would not be appreciably attracted

$$(\theta=1.7\times10^{-6}/0.024=7.1\times10^{-5}\text{ rad.})$$

by the glass plates; and yet this tendency is marked.

One may suppose, therefore, that each end of the needle is attracted by the nearer glass plate independently. This will give a superior limit enormously larger than the preceding estimate; for the force is now

$$f=2\times2\pi\gamma\sigma=4\times3.14\times6.7\times10^{-8}\times1.8=1.9\times10^{-6}\text{ nearly, if }\sigma=3\times0.6=1.8$$

the mean thickness of plate bearing 0.6 cm. Hence the torque is

$$10^{-6}\times1.9\times26=4.8\times10^{-5}$$

since the length of needle is 26 cm., and finally

$$\theta=4.8\times10^{-5}/0.024=2\times10^{-3}\text{ radian}$$

Even this excessive estimate, however, fails to account for the result; for the deflection is but a little over a half degree. Moreover, the short brass cylinder in question (length 3 cm.) showed a similar tendency to take oblique positions not corresponding to the torsion-head, as the long needle.

46. Summer experiments.—It is obvious that a fair trial of the apparatus can not be made in an artificially heated room. For this reason experiments in a semi-subterranean room of the laboratory were reserved for midsummer.

In the summer installation in a subcellar at constant temperature, with a few improvements of apparatus (the mirrors being readjusted, etc.), the needle was without difficulty made to take a stable position midway between the glass plates, subject to the torsion of the fiber. Some trouble was experienced in finding the fringes, owing to incidental causes. The adjustment is made difficult in view of the definite distance apart of the small mirrors to which the breadth of the ray parallelogram must conform. With the micrometer at 45° the latter is very limited in its displacement. I later attached a special micrometer with three identical pairs of parallel V-mirrors (the latter at 90°) similar to the design shown in figure 21. The middle V-mirror is movable in a micrometer. This, when the mirrors are parallel, has the additional advantage of being independent of slight changes of inclination in the micrometer. The displacement of mirrors is now virtually parallel to the rays and no difficulty in finding the fringes need occur. Naturally the mirrors must be good, there being now four additional reflections in each ray; and this V-micrometer must be accurately adjusted for parallelism of mirrors.

With the fringes found, there is now no difficulty in showing the attraction of gravitation. In fact, an iron brick moved on a small truck, near the shot at one end of the needle, grips these balls very much like a magnet acting on the pole of a magnetic needle. By approaching and withdrawing the iron mass on one side, the fringes could be put in regular and uniform vibration over enormous (relatively speaking) arcs as measured by fringes. Thus in an incidental experiment the micrometer reading was 0.255 cm. with an iron mass near and slow vibration and 0.026 cm. (eventually) with iron mass remote.

Throughout the whole of the experiment the fringes were under the perfect control of the micrometer.

A more systematic experiment was then made by testing the attraction of a lead ball 5.43 cm. in diameter and weighing about $M=950$ grams for the shot (at the end of the needle) weighing $m=0.61$ gram. M was moved on a circular track with stops to a distance of $R=4.24$ cm. (between centers of balls) from the ball of the needle, alternately. The position of the large ball M was reversed every 10 minutes, but the period of the air-damped needle can not have been less than 18 minutes. The case is, then, that of a forced vibration under constant force and a large logarithmic decrement. The observations are given in table 4, the reading being made every minute, beginning with the equilibrium position (M in the neutral position). If these data of the displacement x of the mass m are constructed graphically it will be seen that the motion of the needle is nearly dead-beat. The successive arcs of vibration increase, and from the limiting distance between elongations the attracting force could be computed, if the torsion coefficient of the quartz fiber and the logarithmic decrement were known. The limiting arc was not reached, owing to incidental reasons. From static experiments made during hour intervals this elongation was found to be about 0.116 cm., or a departure of the shot m at the end of the needle from its position of equilibrium of 0.058 cm. in response to the attraction of M. If l is the semi-length of the needle (between centers of shots), the micrometer displacement ΔN and the displacement Δx of the mass m are given by the equation

$$\Delta x = l\Delta N \cos i/b = 0.89\,\Delta N$$

where b is the breadth of the ray parallelogram and $i=45°$ the angle of incidence of the interferometer. Thus the micrometer displacement is of the same order as the displacement of m, and if the latter is 0.116 cm., we should have

$$\Delta N = 0.13 \text{ cm.}$$

TABLE 4.—Gravitational attraction. $M=950$ grams, $m=0.61$ gram. $R=4.2$ cm. Period (damped) about 18 min. Length of needle, 25.2 cm.; weight (total), 1.9 grams.

Adjustment	Time.	$x\times10^3$	Adjustment	Time.	$x\times10^3$	Adjustment	Time.	$x\times10^3$
	min.	*cm.*		*min.*	*cm.*		*min.*	*cm.*
Ball *M* right (from equilibrium)	0	26	Ball *M* right	21	50	Ball *M* right	41	94
	1	31		22	60		42	101
	2	35		23	69		43	111
	3	40		24	79		44	126
	4	46		25	90		*45	139
	5	54		26	98			
	6	58		27	105			
	7	64		28	110			
	8	68		29	117			
	9	72	Ball *M* turned	30	124			
Ball *M* turned	10	77	Ball *M* left	31	124			
Ball *M* left	11	78		32	123			
	12	77		33	120			
	13	74		34	117			
	14	69		35	113			
	15	64		36	108			
	16	61		37	105			
	17	56		38	100			
	18	53		39	96			
	19	49	Ball *M* turned	40	93			
	20	48						

* Out of field of telescope.

As the micrometer reads to 10^{-4} cm., 1/1300 part of the attraction between M and $m=0.61$ gram could therefore be detected; i. e., the attraction of $950/1300=0.73$ gram, or per interference fringe well within one-third of this, for the given quartz fiber, which was not specially selected, and distance R. This is equivalent to the attraction of two tenth-gram masses per centimeter of distance per fringe.

Apart from the measurement of the torsion coefficient of the fiber, there is, however, a real difficulty involved, and that is the occurrence of marked drift in the needle. It is only incidentally that the fringes are found at rest. The chief contributory cause of this is no doubt the occurrence of motion of air around the needle provoked by small differences of temperature, resulting (for instance) from illumination. If the possible accuracy of deflection measurement is to be of any value, therefore, the apparatus must be kept in the dark, except during observation. Fortunately, the achromatic fringes require little light. Even then a closed case which can be exhausted of air is essential, for such radiometer forces as may enter would in any event be differential, seeing that the mirrors are symmetric and the illumination is not subject to alternations like those in the table. It is probable that in case of the above regular method a thicker quartz fiber and a greater distance R would conduce to the best results, since ΔN is the least difficult quantity to determine.

Measurements could not be attempted in time for the present report, but the question may be asked whether it is not possible in the present case to determine the attractions in terms of the mere acceleration of balls resulting. With an ocular micrometer this would not be difficult, as the fringes move slowly enough so that the position can be sharply specified; but with a needle of long period in vacuum, the screw micrometer would also be available. If there is no damping we may write

$$\gamma Mm/R^2 - ax = 2m\alpha$$

where α is the acceleration, x the displacement of m, and a the torsion coefficient, referred to the displacement of m, t the time, supposing the needle starts from rest, and the gravitational force is applied at $t=0$. If at the outset we may put $x = vt/2 = \alpha t^2/2$

$$\gamma Mm/R^2 = x(at^2 + 4m)/t^2$$

an equation whose interesting feature is that if t is kept very small (which should be possible with an ocular micrometer and the achromatic fringes, a fine quartz fiber presupposed), the term involving t may be neglected and the experiment interpreted as a case of uniformly varied motion, in which

$$\gamma = 2R^2\alpha/M$$

For instance, if $R=5$ cm., $M=10^3$ grams, and $\gamma = 6.7\times 10^{-8}$, and if $t=100$ sec. is admissible, $\alpha = 1.3\times 10^{-7}$ cm./sec.^{2} and the distance traversed in 100 sec. would be 0.0065 cm., well measurable on the interferometer, quite so if the work is done reciprocally and the interference fringes are used individually. The theoretical error will be a minimum if m is as large as the fiber can safely carry and t as small as possible. On the other hand, x is independent of m, and if t is to be kept small, the result may be compensated in a large M/R^2. The measurement is thus to consist in keeping the fringes at zero by moving the micrometer screw for the small interval t during which the weight M acts. The constant would then follow from the micrometer reading M and R only, all other quantities entering secondarily as corrections. The experiment seems well worth while.

II. USE OF THE RECTANGULAR INTERFEROMETER IN CONNECTION WITH THE HORIZONTAL PENDULUM.

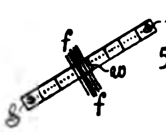
47. Introductory.—In 1915 and in the reports of the Carnegie Institution of Washington, No. 229, Chapter I, part 2, pp. 30 *et seq.*, I adduced a method for the application of the displacement interferometer to the horizontal pendulum with a graphic exhibit of the results obtained during a series of months. The concave-mirror design by which the spectrum interference ellipses were made available showed a very satisfactory performance, in spite of the fact that deformations of the pier to which the pendulum was attached were local disturbances and excessive in amount. The attainable accuracy was such that for moderate constants in the installation of the pendulum, an inclination of 3×10^{-4} second of arc should have been registered per vanishing interference fringe (ellipse), or about 10^{-3} second per 10^{-4} cm. of displacement of the micrometer. The inclination of the line of suspending pivots was here about 1° to the vertical. A smaller angle would have correspondingly increased the sensitiveness.

The apparatus, however, required a space about 2 meters long between the extreme mirrors for its installation. This is in a measure a disadvantage, since small changes of temperature in the brackets and supports, as well as in the pier, would interfere with the full realization of the precision of the method. In this respect the rectangular interferometer with an auxiliary mirror is to be preferred; for here all the necessary parts may easily be placed within a distance of 1 foot from the wall of the pier carrying the horizontal pendulum. If the achromatic fringes are used, these are straight and intense, so that photographic methods are available, while for visual observation a gas flame would give sufficient light. The sensitiveness under similar conditions would be slightly smaller, but not enough to cancel the advantages specified.

48. Apparatus.—The old horizontal pendulum formerly described was again used. It was made of thin steel tubing, and in this respect, since its plane was nearly in the meridian, may be subject to change of the earth's magnetic field; but as my object here is merely the trial of a method, these annoyances are of slight consequence.

Figure 57 gives a sectional plan of the pendulum installation and figure 58 a front view of the pendulum *HH* alone, on a somewhat reduced scale. Its general shape is that of an isosceles triangle and the distance from the line of pivots *tt'* to apex *B* about 110 cm., while the distance between pivots was 97 cm. The pivot supports *SS'* are fine screws ending in hard-steel points, which enter a glass-hard steel socket (below) and a steel groove (above). The line tt' of the horizontal pendulum can thus be given any inclination to the vertical, while the rods p,p' which receive the screws S,S' may be moved mormally to the wall of the pier PP', inward or outward, and clamped to secure parallelism between the pier PP' and pendulum HH'. The apex B of the pendulum is also provided with a clamp, holding vane D submerged in an oil-vat v for damping.

The whole pendulum is inclosed by a flat case CC' of tin plate provided with a plate-glass window at g, through which the auxiliary mirror m of the interferometer may be seen. This is attached to one or two vertical tubes h,h' of the pendulum, adjustably, so that it can be moved up or down, and rotated slightly above a vertical and a horizontal axis.



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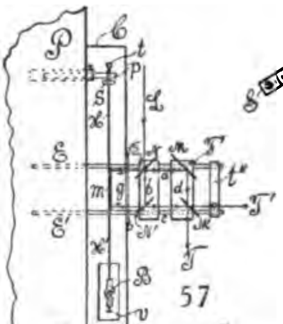

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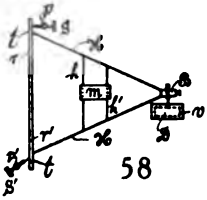

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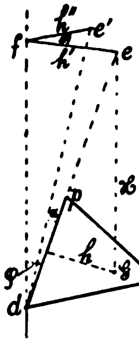
The interferometer consists essentially of the 4 plate-glass mirrors M,M', N,N', all but M being half-silver, the collimator (beyond L) and the telescope at T or T', t'' being a telescope support. The collimated white beam L is thus separated into the component rays $LNmadt$ and $LbN'mcT$, to be observed at either T or T'. M' is on a micrometer slide with the screw normal to the face of the mirror. All mirrors must be capable of slight rotation about horizontal and vertical axes and the silvered faces all lie towards m for compensation of glass paths. The rays leaving M' for T must not only be accurately parallel, but locally (visible as spots of light) nearly coincident, as specified above (Chapter IV, § 43). Otherwise the fringes will be weak or invisible.

The telescope T should be provided with an ocular micrometer (centimeter divided in tenth millimeters) standardized by aid of the sliding micrometer at M', since the main purpose here is the measurement of small angles. Moreover, the image of the wide slit of the collimator adapted to the use of the achromatic fringes should be placed at right angles to them, with the ocular micrometer so placed as to read from end to end of the slit-image. A very fine wire beam across the slit gives the fiducial line relative to the ocular micrometer. Figure 59 shows the general arrangement, SS' being the wide, oblique slit-image, ff' the achromatic fringes, w the image of the fiducial wire across the slit, and ss' the ocular scale. Of course the fringes may be made horizontal or vertical; but this requires much adjustment or else compensation, and is therefore an unnecessary complication of the preliminary work. With this fiducial mark at the collimator (which is permanently out of reach), if the telescope is accidentally shifted, or temporarily removed, it may be replaced without difficulty. It is the telescope, however, which contains the ultimately fiducial scale, and like the collimator it should be held on a standard t'' suitably attached to the pier. Similarly the mirrors M and M', N and N', fixed in pairs to slides or carriages F,F', are clamped to two parallel horizontal tubes E,E' (¾-inch gas-pipe smoothed, for instance) anchored in the pier. The highest attainable rigidity in the placement of the mirrors M, M', N, N' and of the telescope is essential. At the outset of the work the viscous yielding of standards and braces is quite apparent. When, as in the present paper, the observations are made at T' and not at T, the telescope is conveniently attached to the slide rods E,E' joined in front at t''.

49. Equations.—In figure 60 let pBd denote the horizontal pendulum in the plane of the diagram and dpe the line of pivots prolonged, terminating in e vertically above the center of gravity G. Let the inclination of de to the vertical be φ, a constant of the apparatus, and suppose a perpendicular h' is let fall from e to the vertical df through d. If, in consequence of a change in the inclination of the pier, the line of pivots passes to de', over a nearly vertical angle α, h' will pass into h'' over a horizontal angle θ. Thus the measurement consists in finding α in terms of the interferometer angle θ. Since these angles are all very small, we may write (Δ being a differential symbol), as shown in the preceding paper,

(1) $$\Delta\alpha = \varphi\Delta\theta$$



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But in the rectangular interferometer with an auxiliary mirror, if the distance apart of the rays a and c (fig. 57) be $2R$,

(2) $$4R\Delta\theta = 2\Delta N \cos i = n\lambda$$

where $i = 45°$, ΔN the displacement of micrometer to bring the achromatic fringes back to the fiducial line, and n the number of fringes which pass that line. Hence

(3) $$\Delta\alpha = \varphi\frac{\Delta N \cos i}{2R} = \varphi\frac{n\lambda}{4R}$$

The smallest angle, $\Delta\alpha$, which can thus be measured depends essentially on $2R$, the breadth of the ray parallelogram. There would be no difficulty in making this as long as the line from tt' to B of the horizontal pendulum, i. e., over a meter; but this would necessitate two mirrors m, one at each end. For the present purposes I preferred to use apparatus which I had at hand, in which $2R$ was but 10 cm. and a single mirror could be used at m. Nevertheless, if $n = 1$, the limit of angles measurable, if $\varphi = 0.0175$ radian, or 1° is

$$\Delta\alpha = 0.0175\,\frac{6\times 10^{-5}}{4\times 5} = 5\times 10^{-8}$$

radian per fringe; i. e., about 0.01 second of arc. With an ocular micrometer and well-produced achromatic fringes there is no difficulty in estimating one-tenth fringe, so that the limiting angle here is to be a few thousandths of a second, even if $\varphi = 1°$, which may also be reduced. By making $2R = 100$ cm. one should therefore be able to reach 0.0001 second per tenth-fringe breadth if φ is $1°$.

Similarly, if ΔN reads to 10^{-4} cm., $\varphi = 1°$,

$$\Delta\alpha = 0.0175 \frac{10^{-4} \times 0.71}{10} = 1.2 \times 10^{-7} \text{ radian}$$

or 0.025 second of arc, with the opportunity of passing to 0.0025 if R is a meter.

Finally, on using the ocular micrometer for moderately sized fringes of say one scale part (0.1 mm. fringes in the ocular), the case is equally promising. A comparison of the two displacements ΔN at M' and Δe of the fringes in the ocular showed

$\Delta e =$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9 cm.
$10\Delta^5 N =$	1	35	75	120	155	190	225	265	305 cm.

Fluctuations are due to the motion of the pendulum. Thus the mean value is

$$\frac{\Delta N}{\Delta e} = 0.0038 \text{ or } \frac{\Delta e}{\Delta N} = 265$$

Hence

$$\Delta\alpha = \varphi \Delta e \cos i \,/ (265 \times 2R)$$

If $\Delta e = 10^{-2}$ (one scale part) and $2R = 10$ cm., etc., as above,

$$\Delta\alpha = 0.01 \frac{10^{-2} \times 0.71}{265 \times 10} = 2.7 \times 10^{-8} \text{ radian}$$

or 0.005″ per scale part of the ocular micrometer. A few tenths of this may be estimated on the scale. The sensitiveness would be 10 times greater if $2R$ were a meter.

50. Observations.—The interferometer was installed with rather smaller fringes than instanced above and therefore with less sensitiveness, as the inclination of the pier in a heated laboratory would probably run into seconds of arc in the lapse of time. For this reason R was also satisfactory at its small value of $2R = 10$ cm. The angle φ was directly measured, as the inclination of the line joining the points of the pivots to the plumb-line. Under these circumstances the constants given at the head of the table suffice. Since $\Delta\alpha = 0.9\ \Delta e$ seconds, roughly, the tenth millimeters of the ocular scale are about 0.01 second of arc in relation to $\Delta\alpha$ and the fringes were of about the same size. There would have been no difficulty in making them much larger and therefore more sensitive, as they were clear and strong. As it was, there should have been no difficulty of estimating within 10^{-2} second.

The end of the compound pendulum was damped in lubricating oil. This is probably too viscous for refined work, but the purpose here is merely to try out the method.

The earlier observations were discarded, but even after January 14, after which time the apparatus worked comparatively smoothly, instances of displacement within the apparatus required readjustment. These betray themselves in a lack of coincidence of the two wide slit-images (fig. 59) or of the cross-wire *w* (the slit is really superfluous except as the collimator lens may be so placed as to widen the illuminated field). This is probably referable to the two supports $E, E,'$ which change their parallelism with marked changes of temperature in the room; or it may have been within the apparatus at $M, N, N,' M'$. It is difficult to allow for it, and a reconstruction of apparatus is the only resort.

The illuminant was an electric arc at a distance of about a meter from the interferometer. This was chosen for convenience solely, as the achromatic fringes can be adequately seen with a Welsbach lamp closer at hand.

The observations will for convenience be given graphically. I merely recall that the breadth of the interferometer rectangle was $R = 10$ cm.; the inclination of the pivots of the horizontal pendulum about $\varphi = 0.01$ radian; the angle of incidence of rays $i = 45°$. Hence the change $\Delta\alpha$ of inclination α of the pier will be (if ΔN is the displacement of the mirror micrometer and Δe of the ocular micrometer)

$$\Delta\alpha = \varphi \Delta N \cos i/2R = 5.9 \times 10^{-3} \varphi \Delta e \cos i/2R$$

or

$$\Delta\alpha = 4.2 \times 10^{-6} \Delta e \text{ rad} = 0.86 \times \Delta e \text{ seconds of arc}$$

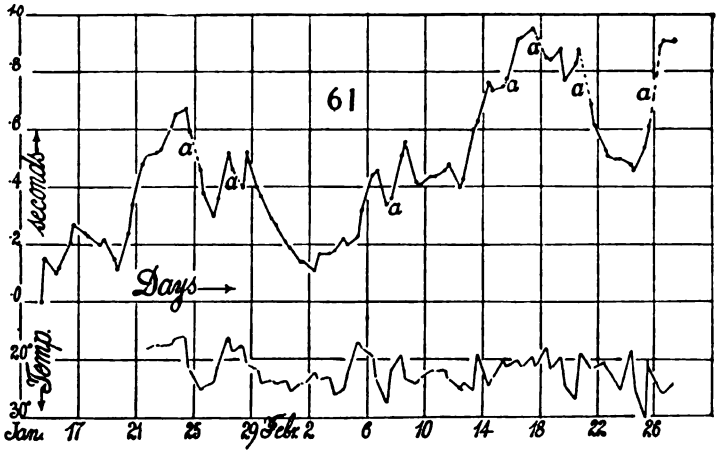
Observations were made at about 10 a. m. and at 6 p. m. Variations of α might easily have been recorded during the day, but these are not of interest in their bearing on the present paper. It seemed premature, moreover, to attempt the installation of photographic apparatus, as this would interfere with the visual observation, which is the chief purpose here.

In the graphic figure 61, the observations at which adjustment of slit-images was necessary are marked *a*.

a. This effect was always in the same direction; i. e., indicating that if no readjustment had been needed the curve between January 13 and February 27 would have risen as a whole more rapidly than the data actually inscribed show. As a mere matter of convenience the curve has been made continuous irrespective of readjustments (on the average a rise of about 0.1 second each). In this way the whole change of inclination within the 45 days of observation did not exceed a second of arc and should therefore be comprised within the scale of the ocular micrometer. For secondary reasons, however, the mirror micrometer was moved in several cases, but as the amount of shift is registered on the ocular micrometer, no essential discontinuity is introduced in this way.

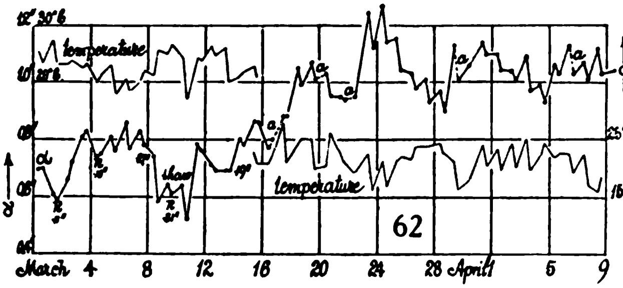
In addition to the reading *e*, the very variable temperature of the laboratory was taken and the data are inserted in the lower curve, with the values increasing downward. With this arrangement the two curves show considerable general resemblance up to about February 10. One may suppose that these are actual changes in the inclination of the pier as a result of non-uniform heating; if the viscous deformations of apparatus were accentuated with rise of temperature, the two curves should not be in opposition and there does not seem to be any means by which mere thermal expansion could produce this result.

After February 10 the effect of the temperature of the laboratory to be inferred in contrasting the two curves is no longer apparent. In its place, however, is an interesting indication of the effect of external meteorological conditions on the inclination of the pier. Thus, while the curve has in a general way been falling toward the time of the intense cold spell culminating in February 5, the marked thaw following soon thereafter is accompanied by a rise of curve as far as February 18 and later. Then, with a second cold spell, the curve falls again to February 24 and in turn rises with the next thaw as far as February 27. It seems hardly probable that so marked a general behavior can be a mere coincidence, and I have regarded it probable that the hill on which the laboratory stands is undergoing similar inclinations.



In conclusion, therefore, one may regard the general trend of the curve upward as following from some yield within the parts of the apparatus. On this is superimposed the effect of the warping of the pier from internal causes, (chiefly change of temperature) and the change of inclination of the laboratory as a whole, due to external climatic conditions.

51. Observations continued.—The apparatus was now taken down for repairs and some modifications. A brace was added at t'' (fig. 57), supporting the end of the interferometer platform, an addition to which I was at first averse; but if thermal expansion is equal throughout the iron framework, it should not produce discrepancies. The new data (figs. 62 to 65) contain observations between February and August, a period of about 5 months. They are constructed in the same way as the preceding (fig. 61) and places where readjustment was needed are marked *a*. Moreover, the adjustment screws of the mirrors after March 16 were relieved from strain as far as possible, an operation which required several subsequent trials, after which the stability of adjustment (persistently coincident slit-images) was much improved. The (internal) temperature (degrees centigrade) of the laboratory has also been added, here laid off positively upward. In May and June the external temperature, as reported by the Weather Bureau in degrees Fahrenheit, is inscribed, and the precipitation in inches (marked *R*) is often indicated.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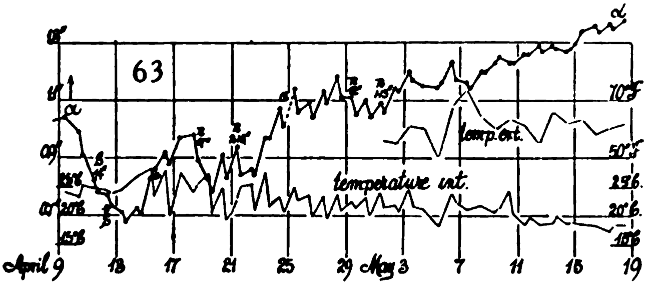
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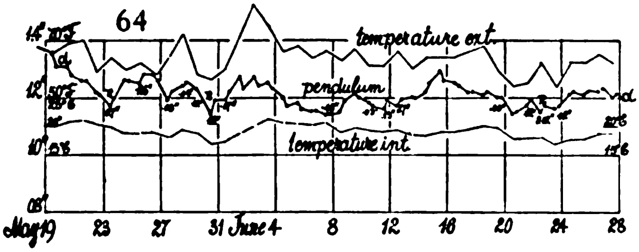
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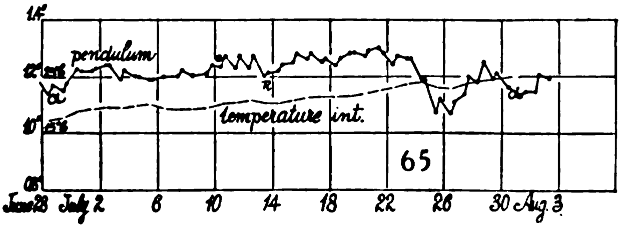
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If we examine the curves as a whole, the marked variability of the amount of inclination, α (seconds of arc), in March, April, and May, is in contrast with the relative quiescence in the latter part of May, in June and July.

This is at once an evidence of the discrepant effect, direct and indirect, of the heating of the laboratory. In a general way, moreover, rise of internal temperature is more apt to be associated with a fall of the α curve, though the similarity is far from consistent. In June and July the continuous general rise of temperature associates itself with a rise of the α curve. Thus the temperature relations, which undoubtedly exist, are very complicated, as is to be expected. During March and April the α values fluctuate between $\alpha = 0.6''$ and $\alpha = 1.3''$, but in May the curve rises and remains permanently above $\alpha = 1.0''$. In fact, this trend toward large α values may be said to begin in the middle of April. A comparison with the external temperatures (degrees Fahrenheit) leads to no consistent results. The effect of the thaw about March 9 is apparently well-marked, but the pocket of the α curve may here also be attributed to the ascent of the temperature curve. Subsequent thaws are not indicated. There are a number of rain-pockets in the α curve, but they do not bear a definite relation to the amount of precipitation. Rains in summer also usually involve changes of temperature. After the period of comparative quiescence of the α values in May, June, and July the fall recorded after July 24 is peculiar. Nothing local was detected to account for it and it is temporary.

So far as the observations were made to test the availability of the apparatus, they may be regarded as quite satisfactory, but it is obvious they can be used as evidence only during the summer months in the absence of internal heating and that marked temperature changes are menacing under all circumstances. I had hoped that this would not be the case. The conclusions already drawn above thus hold in the sequel. It is my purpose to install this sensitive apparatus under fit surroundings at some future opportunity. It should, then, contribute substantially to geophysical investigation.

CHAPTER V.

THE INTERFEROMETRY OF VIBRATING SYSTEMS.

52. Introductory.—The high luminosity of the achromatic interferences and the occurrence of but two sharp fringes make it possible to utilize them even in cases when the auxiliary mirrors vibrate. Experiments of a similar kind have been previously tried with telephones and the spectrum ellipses;[1] but these fringes do not easily admit of being drawn out into a ribbon and there is usually deficient light. The endeavor to distinguish the phases of vibrating telephonic systems was partially successful, but the marked importance of synchronism or resonance in these systems is the chief outcome of the research.

53. Telephonic apparatus.—I began the work with two similar telephones, as shown at *t,t'*, in figure 66. Small mirrors were rigidly attached to the centers of the diaphragms and each of the telephones secured on a standard which admitted of adjustment around vertical and horizontal axes. The intermittent current was supplied at *a, b*, by a small induction coil with a rheostat in circuit. Four clamp-screws at *c, d* were available for putting the telephone bobbins in series or in parallel. One telephone could be reversed in action by the commutator *K*. White light *L* from a collimator was reflected or transmitted by the half-silvered mirrors *M, M', N, N'* of the interferometer and from *m,m'* on the telephones, as indicated by the arrows. *M'* was on a micrometer with the screw on the direction *n*. To facilitate the finding of the fringes one of the telephones, *t'*, should also be on a micrometer with the screw normal to *m'*. The use of *n* requires special precautions stated below. The fringes when found are observed by the vibration telescope at *T*. It is sometimes difficult to catch the fringes even when using the spectro-telescope, owing to the accentuated quiver of the system, and the work is simplified by first supporting the diaphragm against vibration.

The vibration telescope is shown in vertical section in figure 67, with the ocular at *E* and the objective originally at *e*, the tube being supported on the standard *d* and clamp *cc*, admitting of raising and lowering and slight rotation around the horizontal axis *b*. The objective *A* has been removed and is now supported by a flat steel spring *s, s*, in front of its former position. Hence the ocular holder is a simple tube which can be thrust far inward and clamped in any position by three screws at *a*.

In order that the ocular may vibrate parallel to the fringes, and as these appear in all angles of altitude, the special vibratory system *f, g, k, s* has been devised. The rod reaching downward (about 10 to 15 cm. long) is attached to a ring *rr*, capable of revolving with friction around the tube of the objective and of being fixed for any angle in altitude of the rod. The horizontal clamp *g*, adjustably attached to the rod *f*, supports the spring *s*. This passes through a vertical crevice in *g* and is fixed by the vertical set-screw *p* and the two oblique adjusting strips *k* on either side of *g*. These pieces, *k*, are clamped by the screws at *h*, and serve as holders of the two coaxial set-screws at *i* on either side of *sk*, so that the objective *A* may be centered relative to the tube *Ee* for any oblique position of *f*, which must be normal to the fringes. The vibration may frequently be considerably changed by sliding *f* and *s* in *g* and reclamping the system. If the objective is to be fixed, a screw at *n* may be depressed for the purpose. The vibration may be started and stimulated from time to time manually. It has not, thus far, been necessary to add an electromagnetic vibrator.

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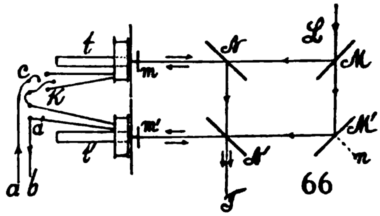
[1] Carnegie Inst. Wash. Pub. No. 149, Part III, 1914, pp. 208-213.



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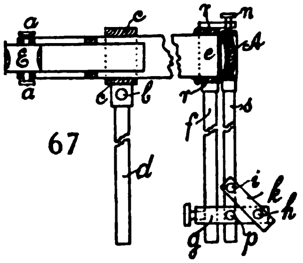

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To find the fringes a spectro-telescope will usually first have to be used. This is then replaced by the apparatus in figure 67, for observation. If not too far displaced, the fringes of a vibrating system may thereafter be found by the vibration telescope even when they can not be seen with the fixed telescope, as they overlap during vibration. A fine slit, so long as it supplies sufficient light, gives the sharpest wave-curves. The angle of altitude of the fringes is of no consequence, since *f* is set in altitude normal to them; but it is of advantage to rotate the slit also, until its image in the telescope is nearly normal to the fringes.

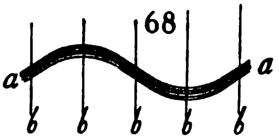
Under all circumstances the two spots of light representing the slit, if caught objectively on a screen at *T*, must be nearly coincident, horizontally and vertically, when the rays at *T* are parallel. If these spots are too far apart the fringes will be very small or even absent. A search for them by any method is then useless. To meet this preliminary but essential condition, the spots are first made coincident by rotating *N* (horizontal and vertical axes) and thereafter rotating *N'* until the images coincide in the telescope at *T*. One or two adjustments of this kind usually suffice, since perfect coincidence is non-essential and in fact undesirable, because the fringes are then too large for convenience.

54. Observations.—The use of two telephones soon showed itself to be unsuitable for the present purposes; for the diaphragms oscillate not merely fore and aft, as is here desirable, but locally around horizontal and vertical axes as well, particularly when the vibration is relatively intense. Hence the coincident slit-images of the silent telephone periodically separate or pass through each other when the diaphragms vibrate. This shows itself in a peculiar manner in the field of the vibration telescope, as indicated in figure 68. The whitish field carrying the fringe-waves *aa*, due to the fore-and-aft motion of the mirrors *m*, *m'* (fig. 66) on the diaphragms of the telephones, is intersected by nearly equidistant vivid lines *b*, *b*, normal to the waves. The latter are apt to be broken and appear only as traces between the vertical lines. The wave-length of *a*, *a* and the distance apart of *b*, *b* will depend on the maximum speed of the objective of the vibration telescope, but the distance *bb* and the form of *aa* usually have some simple relation to each other, so that frequently the wave form is lost entirely and merely oblique lines with the same inclination are seen between the lines *b*, *b*.

To account for these occurrences of the lines *b*, it is sufficient to recall that the originally coincident identical slit-images are in vibration through each other in some direction relative to the lengths of the slits, effectively therefore normal to this direction. The amount of this displacement is small, but it is greater than the breadth of the fine slit-images. Hence these images will be seen clearly only at the elongations when the slit-images are temporarily stationary and at a maximum distance apart. These apparitions are the lines *bb* and they belong to a system with higher frequency than the objective. Hence, also, the waves *a*, *a* (fig. 68) are absent at *b*, *b*; for here the slit-images are so far apart as to eliminate the interferences.

It made little difference how the two telephones were connected. In fact, one telephone may be thrown out of circuit. Vibration is thus communicated mechanically similarly to the case when the fingers drum lightly on the table. If a fine wire is drawn across the slit, the shadow remains straight unless the vibrations are very intense; then beating waves in trains run along the black line of shadow.

The telephone diaphragms were now removed and replaced by the two long strips of steel made from hack-saw blades 20 cm. long and about 1 cm. broad and 0.06 cm. thick, rigidly attached to the body of the telephone by the arch *cd* (fig. 69.) The mirror *m* was cemented to the middle of this strip, balanced by a piece of iron on the other side. To approach this as near the magnet as possible, forcing screws, *e* and *f*, were provided at a little distance from the end of the strip. In this case the vibration of the strip *ab* and the mirror *m* at its middle was fore and aft only, and as a consequence the lines *b*, *b'* in figure 69 vanished completely. Here also the arrangement of telephones, whether in series or parallel, made a decided difference in the amplitude of the waves *a*, which could be increased many times the breadth between successive fringes before the waves became turbulent and broke up.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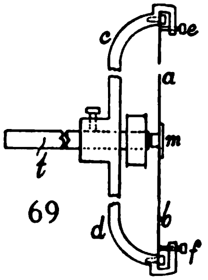
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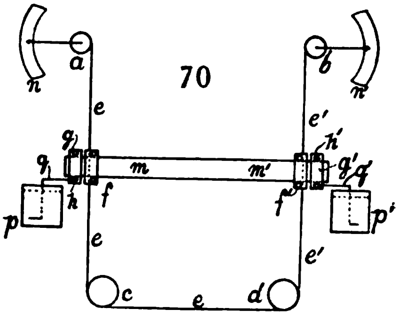
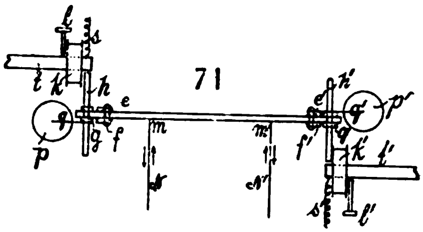
The telephonic system was now put in place of the galvanometer of a Wheatstone bridge, the small induction-coil being used as a source of current. It was then found that the adjusted resistances of the bridge could be changed by no more than a few tenths of 1 per cent before the even band of fringes changed appreciably to the wave-form *aa* (fig. 68). But as a dynamometer the instrument was still much inferior to the audible telephone. Currents of an order less than an average 10^{-4} ampere would be difficult to detect.

55. Bifilar systems.—To utilize such a system as figure 69 to full advantage it would be necessary to attune the springs *ab* of the two telephones to the same period, which should then be as nearly as possible identical with the period of the source of intermittent or alternating current. As an earth inductor or a small magneto inductor (single magnet rotating in a flat coil) would have to be used in the latter case, it seemed best to convert the apparatus into a bifilar vibrator as shown in figures 70 and 71 in elevation and plan. Here ***mm'*** is a strip of thin mirror plate-glass, about 32 cm. long, 1 cm. broad, and 2 mm. thick, horizontal and in a position to receive the rays *NN'* of the interferometer (compare fig. 66). Motion of ***mm***, parallel to itself, fore and aft, will therefore produce no effect on the fringes; but any rotation around a vertical axis will be immediately apparent, as indicated in the above methods for small angles.





This strip of glass is supported by the bifilar system *ee*, *e'e'*, made of a single thin wire of brass, 0.2 mm. in diameter. The ends of *ee'* are wound around the horizontal screws *a*, *b*, which rotate with friction and are supplied with an index and scale *n*, *n'*, so that any tension may be imparted to the wire. This passes below under the pulleys *c*, *d*, as nearly free from friction as possible, with the object of securing the same tension throughout *ee'*. Flat clamps *ff'*, of fiber and screws, attach the strip *mm* to the wire at any height, but necessarily near the middle of the vertical threads, where it receives the rays *NN'*.

The telephonic system consists of the soft-iron horizontal screws *hh'*, similarly attached to *mm'* by the flat fiber clamps *g*, *g'* and the telephones *t*, *t'* (omitted in fig. 70). These were made of large flat files, each provided near its end with an appropriate bobbin, *kk'*, of fine telephone wire, the ends of which are attached to clamps, as already shown in figure 66, with one of the telephones provided with a commutator for reversing its current. The resistance of each bobbin was about 140 ohms. The screws *ll'* are used to approach the telephone magnets *tt'* as near the soft-iron armatures *hh'* as possible without overstepping the unstable position, in view of the tension of the tense wire *e*, *e'*. To prevent the sticking of *h* to *t*, which is very annoying, small rubber buffers may be placed between. It is not usually practicable t approach *h* to *t* by more than 1 or 2 mm. and keep *h* perfectly free. To give the vibrating system adequate damping, thin wires *qq'*, less than a millimeter thick, bent, and dipping into lubricating oil in small vats *p*, *p'*, suffice. To change the damping the latter may be lowered or even removed. The fibers *e*,*e'* were about 45 cm. long and their distance apart about 29 cm. Their period and that of the vibrating telescope were made about the same, on the average about 0.2 sec., and this was for convenience nearly the same as the period of the vibrating telescope and of the induced alternating current.

It is convenient to insert an extra telephone (resistance about 100 ohms) in circuit, in order to insure against breaks of contact or other discrepancy, when the perturbation of fringes ceases.

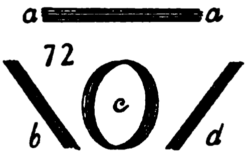
As a generator an earth inductor with a coil of wire 60 cm. in diameter was at first used. It was turned by a small motor, and by putting a sliding rheostat in circuit the period could be varied from about 0.19 to 0.26 second. To measure the average intensity of current a Siemens precision dynamometer was installed; but though indicating currents as low as even within one-tenth of an average milliampere, it was not influenced by the earth inductor, though at maximum speed. This was therefore replaced by a small magneto consisting of a bar-magnet about 6 inches long, rotating in an oblong coil. By aid of the rheostat and appropriate pulley-wheels large differences of speed could be obtained and maintained at any value, so that periods from about 0.1 to 0.3 sec. were available. With a period of 0.2 sec., moreover, it showed a deflection on the dynamometer and, being in general lighter and more easily controlled, was preferable to the earlier instruments.

The three vibrating systems (mirror, telescope, alternator) thus all admit of an adjustment of their periods, and these should be nearly the same if the elliptic system of Lissajous curves are to be obtained, which is the preferable case. A change of the tension of the wires *ee'* in figures 70 and 71, or any adjustments at the telephones, calls for a fresh search for fringes; but this is not difficult if the spectro-telescope is first used and the admonitions relative to the objective coincidence of pencils entering the telescope, as well as their parallelism, as above explained, are given consideration. These difficulties do not enter when the mirrors can be displaced normally to the incident rays.

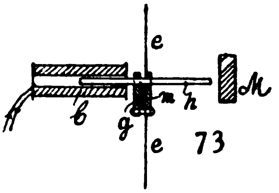
In addition to the telephone *tt'* in figure 71, coils in great variety were used. The telephones were also placed within and without the rectangle of wires *e*,*e'* and in the same or on opposite (as in fig. 71) sides of *mm'*. But the phenomena in such cases were not dissimilar nor advantageous.

For general purposes the mass of the vibrating system *mm'* should be diminished, as it could easily be; but the straight blade of glass (in preference to two small mirrors) is a convenience in adjustment and here suffices.

56. Further observations.—Without synchronism in the two vibrating systems (current and telephone), the motion of fringes obtained is practically inappreciable when average currents within the order of milliamperes are treated. This is a curious and at first a disappointing result. As soon, however, as approximate synchronism is established, the sensitiveness of the apparatus increases enormously. It is best for this purpose to vary the period of the motor of the alternate-current generator by the slide rheostat. If the fringes are horizontal and the objective therefore vibrating horizontally across the vertical slit-image, the motion of the fringes is vertical. Hence the horizontal band *a* (fig. 72), in the absence of current, at once takes the form *b,c,d,c,b*, in succession, with the opposition of rotation in *c* quite visible. The continuous change of these may be most conveniently accelerated or retarded by controlling the motor of the alternator with the slide rheostat. Since the lines *b* and *d* are of different inclination, they will usually show a difference of breadth. Circles appear when the amplitude of the objective has sufficiently decreased, and it is advantageous, as a rule, to keep this amplitude small, for the phenomenon is then more luminous and brilliant. Very large fringes are not usually desirable, as they are too mobile and may pass out of the field. The smaller fringes are quite satisfactory and more easily obtained. When the tension of the fibers *e*,*e'* (fig. 70) is too small the fringes drift, showing that with the varying magnetization there is no persistent position of equilibrium. It is annoying if they leave the field while executing their gyrations, though they may always be restored by moving the micrometer. For mean tensions the higher Lissajous curves 2:3, 3:4 may be obtained, both for the alternator moving at smaller and at larger periods than the vibrating mirror. To obtain them the motor running at maximum speed is gradually slowed down by means of the rheostat, when the forms appear in succession, passing through the elliptic series at mean speeds. This in fact is the best tension for practical purposes. The size of the curves and the brilliancy of the whole display is increased by decreasing the damping or lowering the cups *p*, in figure 70.



A beautiful phenomenon is observed when the magnets *t*,*t'* hold the armatures *h*,*h'* to the intervening rubber cushions and the fringes are fairly large. The slightest vibration anywhere in the vicinity will then cause the even band *a* (fig. 72) to change to magnificent large roof-shaped or violin waves. This loose contact device could, no doubt, be made useful for observational purposes. Without the vibration telescope such fringes would not be visible, as they overlap during vibration.



The Lissajous curves continue to be very marked when additional resistances as high as 1,000 ohms are put into the circuit of the alternator. Indeed, they do not vanish appreciably, even for an additional 10,000 ohms, if well produced. They do not, however, increase in size in proportion to the decrease of variable resistance, a result attributable to the amount of resistance and inductance necessarily in circuit, the effect of which is relatively large when the additional resistance is small.

To obtain some idea of the smallest average current appreciable, the Siemens dynamometer may be put in circuit, though unfortunately it has a resistance as high as about 1,000 ohms. With the magneto inductor running at a speed of $T=0.17$ sec., the Siemens showed a deflection of but 0.02 cm. on the given scale, owing to this resistance.

Estimating the average current i as $i=C\sqrt{\varphi}$, where C is the dynamometer constant and φ is the deflection in centimeters, the value of C as found by Clarke's cell and resistances was $C=5\times10^{-4}$, so that the average magneto current corresponding to $\varphi=0.02$ cm. is $i=5\times10^{-4}\times\sqrt{0.02}=7\times10^{-5}$ ampere. The resistance in circuit was here about 1,500 ohms. If an additional 10,000 ohms is inserted and the magneto reduced in speed to $T=0.25$ sec., the current is still appreciable at the interferometer and would be of the average value of $i=6\times10^{-6}$ ampere. Although no account has been taken of self-induction, it is improbable that the smallest average current here observable by the interferometer apparatus in the earlier form could have been much within a microampere. In this respect the device was somewhat disappointing.

In a later and more refined adjustment the ellipses obtained with an insertion 10,000 were found to fill (as to their vertical or current axes) fully one-quarter of the field of the telescope. As little as one-tenth to one-hundredth of this would be easily appreciable with certainty, so that the minimum average current capable of detection may be estimated as a few 10^{-7} amperes. It must be remembered, however, that the above mirrors (mm') and appurtenances are unnecessarily heavy, and the bifilar too robust. The improvement would therefore consist in constructing a very light needle and delicate bifilar.

If the dampers p,p' are removed, the ellipses, even at high resistance, are apt to pass out of the field of the telescope. Even if the principles of forced vibrations are applicable, the system in the absence of current is too mobile for convenience.

When the magneto was run at maximum speed (without pulleys) a deflection of about 0.12 cm. was obtained on the dynamometer, corresponding to an average current, therefore, of about $i=1.5\times10^{-4}$ ampere. In this case the even band *a* (fig. 72) takes the definite shape of a train of waves (fig. 68) of short wave-length, with amplitude of but 10 or 20 fringe-breadths. These also admit of the additional insertion of several thousand ohms before they are reduced to the linear band.

If one of the telephones is reversed, the fringes showing marked vibration (*bcd*, fig. 72) in the first position frequently cease to show any vibration (*a*, fig. 72) until the ellipses in the former case are very large (small resistance in circuit). The results in such a case are uniformly consistent. Thus, for the commutator positions I and II and resistances R in circuit, the results were, for instance:

I	Circuit open.	II	R
Band	Band	Strong ellipses	100 ohms
Band	Band	Strong ellipses	500
Band	Band	Smaller ellipses	1,000
Band	Band	Ellipses just seen	2,000
Band	Band	Band	5,000

The reason of this is apparent, for when the telephones are joined in series the mirror *mm'* (fig. 70) is periodically rotated and released around a vertical axis and the displacement of fringes is proportional to the small angular amplitude of rotation. If, however, the telephones are connected differentially, the mirror *mm'*, if properly adjusted, merely moves parallel to itself, fore and aft, and the fringes remain stationary. More usually, however, there is a difference in the size of ellipses (*cæt. par.*) in the two cases and at other times there is scarcely any difference at all appreciable. In such cases it seems probable that the periods of the two filaments *ee* and *e'e'* on opposite sides of *mm'* are not the same, or differ in amplitude (attracting forces of different strengths), so that the fore-and-aft motion is accompanied by more or less rotation, residually. It is in fact difficult to make the two telephones, etc., quite identical in action.

It is for the investigation of this question that the adjustment pushing-screws *l*, *l'* and springs *s*, *s'* (pulling toward the rear), the telephones being on a vertical axes, were provided. If one of these, *t* for instance, is placed in a definite effective position, while *t'* is relatively far from its armature *h'*, the screw *l'* may be gradually pushed forward, diminishing the distance to the minimum. The effect of this is further to rotate *mm'*, and if the turning of *l'* is cautiously done the fringes may be passed from top to bottom of the telescopic field by *l'* and restored to position by the micrometer. After each step of the experiment the fringes are observed, when actuated by the alternator, in the two positions of the commutator of one telephone. In this way it is possible to find the adjustment in which for one position of the commutator there is excessive motion of fringes, whereas for the other there is practically no motion, as in the example above. If the distance (*t'*,*h'*) is markedly larger or smaller, the distinction of the two positions of the commutator is lessened and may even vanish.

There is usually some vibration figure (1/1, 3/4, etc.) best adapted for the given adjustment, even if the other figures appear. When this is chosen, the distance from magnet to armature (*t'* to *h'*) makes little difference within reasonable distances. Such a result is not unexpected, for the whole phenomenon is relative, larger differences corresponding to larger total forces. The distance between magnet and armature (here on one side) does, however, affect the tension of the string, since the forces and the stretch are greater for smaller distances and the period therefore smaller. This is the simplest method of obtaining unison on the two sides of the bifilar, and as the magnet *t'* is set by its adjustment screw *l'*, the motion of the fringes while oscillating is seen in the field of the telescope and they are thus never lost. Regulation at *a*, *b*, figure 70, is more difficult.

Adjusting the bifilar as to tension in this way, there is one position or distance between *h'* and *t'* pretty sharply determinable for which the fringe bands change to stationary ellipses in the *absence* of all current. This peculiar result is at first puzzling, but since it is quite synchronous with the period of the telescope (stationary ellipse), it is obvious that the motion of the objective is the cause of the phenomenon and that the fibers are now in unison with its period. For distances *h'*, *t'*, greater or smaller, the ellipses soon return to bands. The effect of the alternating current on the stationary ellipse is very beautiful. It now oscillates very much like a smoke-ring for one commutator position, whereas it passes in an accentuated way through all phases for the other. Naturally, very complicated displays are also obtained in this double superposition; but practically the vibration of the telescope objective does not disturb the bifilar of the interferometer, unless under the exceptional condition of complete unison, even when both instruments are on the same (insulated) table, a convenience not at all necessary.

Utilizing the preceding adjustment giving ellipses and bands, respectively, in the two positions of the commutator, many experiments were made to detect a change of phase when a large inductance is placed on both sides of one of the telephones. But in none of the experiments thus far was any difference discernible to be attributed to the presence of the inductance. The endeavor to produce in part, by the mere insertion of inductance, an effect similar to commutation has not, therefore, been realized.

Among other promiscuous experiments I may refer to the use of a variety of telephones, single and bipolar; to changes in their position, sometimes with their poles (as at *mm'*, fig. 70) between, and sometimes with the poles on the outside of the wire filaments *ee*, *ee'* (as drawn in the figure); to bifilars of thread instead of wire; to bifilars of watch-spring; to different sizes and kinds of armatures, etc. Coils, moreover, of different resistances and size of wire were tried, for instance, in figure 73 (which preserves the notation of fig. 70), where *C* is the coil, *h* the soft-iron (screw) armature, and *M* a strong inducing magnet. In the absence of *M*, no effect was obtained, even when *C* was provided with a core of soft iron reacting on *h*. In other words, a magnetizing system is inefficient. In the presence of the magnet *M*, however, the results were marked, but not better than the above, while the system itself is more complicated. The replacing of *h* by a hard-steel permanent magnet gave good results, but of inferior sensitiveness. To obviate the annoyances of contact between armature *h* and *M*, the latter might advantageously be replaced by a small magnetizing coil surrounding the free end of *h* and supplied separately with current. Endeavors were also made to utilize the repulsion between a permanent magnet at *h* and a similar pole at *M*. But *M* in such a case reversed the polarity of *h*.

As a tuned system responding to definite periods only, the vibration interferometer is quite sensitive, provided the average currents are of the order of several microamperes. Between the types of compound vibration-curves corresponding to frequency ratios of 4/3, 3/2, 1/1, 2/3, 3/4, there is usually an unbroken band of fringes. If the ratio of periods remains fixed, the vibration curve of course remains fixed, which is the usual sharp acoustic criterion. When the ellipses (out of tune) change continuously between lines of different inclination, the passage in one direction is often gradual, whereas in the reverse or return direction it is almost sudden. Linear forms flop into linear forms, as it were. No doubt this is related to the vibration of a bifilar system like the above, where the two ends are liable to vibrate alternately. When the average currents approach the order of 10^{-4} cm. the bands become sinuous for all periods not excessively high or low, as already stated.

DISPLACEMENT INTERFEROMETRY BY THE AID OF THE ACHROMATIC FRINGES

PART IV

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PREFACE.

The anomalous behavior observed in the last report, in treating the elastic deformations of small bodies on the interferometer, induced me to endeavor to devise a different method for the same purpose. This led to the construction of the contact lever, using achromatic fringes described in the first chapter. The instrument at once functioned admirably, when employed either as a surface tester or as a spherometer.

The contact lever is then modified (Chapter II) for the interpretation of the elastic discrepancy specified, and it is shown that both the new and the old methods lead to trustworthy results, even for material as rigid as brass, if the rods examined are sufficiently slender.

A different kind of application of the contact lever is made in Chapter III. The very small elongations with subsequent contractions experienced by iron in magnetic fields are peculiarly interesting, because these phenomena are at their maximum variation after the metal has become magnetically saturated; so that something persists here, of which the magnetic moment gives but an inadequate account. Hence particular attention is given to the occurrences in strong magnetic fields, though the behavior in very weak fields is also explored. With metals other than iron no effect was observed.

An instrument which lends itself with equal facility to the measurement of thermal expansion and to the determination of elastic moduli is in a measure self-contained for the solution of many thermodynamic problems. A project of this kind, bearing on the specific heat of liquids under pressure and temperature, is discussed, with the requisite experimental data, in Chapter IV.

Chapters V and VI contain contributions to the electro-dynamometry of very weak (telephonic) alternating currents. No available effect is obtained unless the vibrator of the measuring-instrument is sharply in resonance with the alternation of current. When it is so, the response is astonishingly large and very definite in amount. In Chapter V the measurement is made by means of the vibrating telescope, the vibrator of the telephonic system carrying the objective. This chapter is merely introductory to the next, and the sensitiveness is not beyond a few micro-amperes per ocular scale-part of reasonable value (0.01 cm.). Within these limits, however, it may be very serviceable —for instance, in determining the number of turns in each of a variety of secondary coils successively slid over the same long solenoidal primary.

The sensitiveness may be increased upwards a hundred-fold, however (so that 10^{-8} ampere per fringe is measurable), by placing an instrument similar to the last on the displacement interferometer adjusted for achromatic fringes. The reading in such a case must be made with a vibration telescope, synchronized with the alternating current in the primary and with the objective vibrating normally to the displacement of fringes. The measurement is thus somewhat awkward, and consists in determining the range of fringe ellipses parallel to the direction of the vibration of fringes. To make amends for this, however, both the amplitude and the phase of the induced current are given by the form of the vibration ellipses obtained, whether modified by resistance, inductance, or capacity. So sensitive an apparatus naturally catches all the quivering stray magnetic fields in the room; but here again any such effect, which might at first sight seem to be fatal, may be compensated by the primary solenoid (for instance) almost as easily as the needle of an astatic galvanometer. Indeed, in the absence of current (secondary), the needle may be given any reasonable amplitude or phase. It is shown, furthermore, that the persistence of the symmetrical ellipse, with its axes respectively parallel to the directions of vibration, is a strikingly accurate criterion of resonance.

Chapter VII shows that a slight but essential modification of a form of interferometer used by Michelson and Morley, makes this apparatus virtually self-adjusting, while satisfying many of the requirements of displacement interferometry. This is a very great convenience when many separate adaptations of apparatus to the interferometer have to be made successively; for the wearisome search for fringes is thus reduced to a minimum. It is even possible to put a part of one of the mirrors of the interferometer on a micrometer screw for direct measurement, though the instrument is then no longer quite self-adjusting. The endeavor to use this device for finding the refraction of solid media apart from form did not, however, furnish results of practical value. On the other hand, a possible design of this kind for measuring the Fresnel coefficient is tested with a promising outcome in Chapter VIII.

An interesting class of interferences obtained by superposing the fringes resulting from dispersion on identical fringes resulting from the inclination of rays, is discussed in Chapter IX. It is possible in this way to obtain sharp spectrum fringes in the very luminous spectrum of an indefinitely wide slit and to specify the angular orientation of the spectro-telescope on its axis; for the fringes, if small, jump out of an unbroken spectrum band suddenly, when a definite angle is reached. Both of these possibilities are of practical value.

A number of results incidental to the preceding work are collected in Chapter X. Evidences of continuous micrometric convection currents within liquids, obtained from the shadows of motes in a highly dispersed spectrum, the satellites of the achromatic fringes already referred to in the preceding report, peculiarly brilliant phenomena obtainable in connection with Herschel's fringes, and other subjects are here treated.

Finally, in Chapters XI and XII, I have returned to certain gravitational experiments begun in the last report. The former, in which the deviations of the horizontal pendulum are read off by the displacement of achromatic fringes, is very definite in its evidence of the effect of temperature distributions within the supporting pier. Chapter XII is a continuation of the endeavor to follow the actual motion of a gravitation needle, under periodic gravitational attraction, with a view to deducing conclusions from that motion. The apparatus ultimately met with serious accident in the endeavor to exhaust it; but though the experiments have not been concluded, the progress made is encouraging.

CARL BARUS.

BROWN UNIVERSITY, *July, 1919.*

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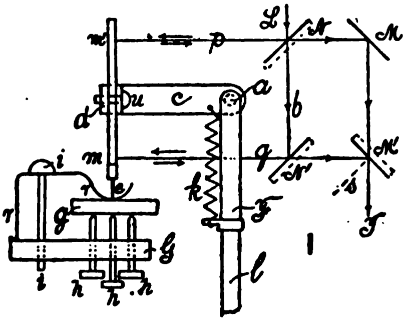
CHAPTER I.

AN INTERFERENTIAL CONTACT LEVER, WITH ACHROMATIC FRINGES.

1. Apparatus.—The method heretofore described for the measurement of small angles by the aid of the rectangular interferometer, lends itself conveniently for the construction of apparatus like the contact lever or the spherometer. Having in view work needing such instruments, I designed the following simple apparatus for the purpose:

Figure 1 is a plan of the design; figure 2 an elevation of the fork and appurtenances; figure 3 finally shows the same apparatus adapted for use as a spherometer. The interferometer receives the white light from a collimator at L. After the reflections and transmissions controlled by the mirrors M, M', N, N', and the auxiliary mirror mm', as indicated in the figure, the light is conveyed into the telescope at T for observation of the interferences. The mirror M' is on a micrometer with the screw s normal to its face.



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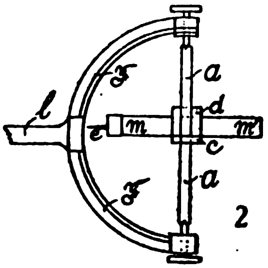

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It is through the mirror mm' that the small angles are to be measured, and this is therefore mounted at one end of the lever dc, capable of rotating around the long vertical axle aa, in the circular fork FF. The latter is rigidly mounted on the bed of the apparatus by aid of the stem l in the rear. The lever c is bent upward at right angles at d, and it is here that the mirror mm' is firmly secured by bolts, etc., as at u. The spring k draws the lever toward the front of the diagram, so that the blunt metal pin e suitably attached to the end of mm' may be kept in contact with the glass plate g to be tested.

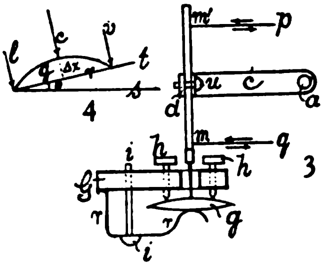
The plate g, in order to be examined as to its degree of plane parallelism, must be capable of sliding up and down, or right and left, under standard conditions. To obtain these the stout bar G (rigidly attached like l to the base of the apparatus) has been provided, carrying three set-screws h, h, h, the points of which lie in the same circumference about 120° apart. They therefore constitute a kind of tripod against which the plate g is firmly pressed by the flat spring or clip rr and screw i. This method of mounting may be appropriately varied in accordance with the tests to be made on the plate *g*, its shape, etc. Similarly, the set-screws *h, h, h* may be placed nearer together or further apart in appropriate screw-sockets, and finally, the lever *c* may be lengthened or shortened at pleasure. The pin *e* remains in permanent contact with the plate *g* in consequence of a wide circular hole in the clip *rr;* or *e* may clear *rr*, above or below it.

If but one face of the plate *g* is to be tested, the system *Ghrg* must slide as a whole, right and left, nearly parallel to the rays *p, q*. In such a case everything will depend on the excellence of the slide carrying the system. I did not attempt to make such arrangements, as I had no need of data of this kind; but the parts *MM'*, *NN'*, *Fcmm'*, and *Grg* were nevertheless mounted on heavy slides (lathe-bed fashion) for convenience in securing a variety of adjustments.

In figure 3 the bar *G* has been reversed in position and the contact pin *e* now passes through a circular hole in *G*, to be in contact with a lens *g*, for instance, kept pressed to the tripod screws *h, h, h* in the same way as before. The latter should in general be much closer together than the figure shows. The instrument is now a spherometer.

The experiments indicated that the mounting of the contact-pin *e* to the extremity of the mirror *mm'* may be the occasion of annoyances; for on sliding *g* right and left, or even up and down, the mirror *mm'* is liable to be flexed. In such a case the achromatic fringes rapidly lose sharpness, not to speak of the errors involved. I endeavored to avoid this by keeping the pin *e* out of contact with the plate *g* by a special lever (not shown) while *g* was being displaced and to test a number of successive contacts thereafter; but it is best (and I eventually did this) to mount *e* on a separate rigid cross-piece parallel to *mm'* and firmly attached to *c*. In such a case no flexure of *mm'* can occur and the contacts may also be repeated at pleasure. Before each reading the bar *G* should be gently tapped.

The achromatic fringes can be found only through the spectrum fringes. This is not usually difficult, remembering that not only must the slit-images in the spectrum be in contact throughout, but the two beams must be locally in contact on the mirror *M'*. Moreover, the mirrors *M'* and *N'* must be equally thick and the silvered faces all turned towards the auxiliary mirror *mm'*.



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2. Equations.—If the mirrors *M*, *M'*, etc., are set at an angle *i*, if the deflection of the auxiliary mirror is θ, and if the breadth of the ray parallelogram *MM'* or *NN'* is *b*, we may write

$$b\Delta\theta = \Delta N \cos i \tag{1}$$

where ΔN is the displacement at the micrometer at *M'*.

If r is the length of the lever c, figure 1, and Δx the displacement of the pin e

$$r\Delta\theta = \Delta x \tag{2}$$

Hence

$$\Delta x = (r \cos i/b)\Delta N \tag{3}$$

The apparatus is more sensitive as r is smaller and b is larger. In the instrument used (adapted from an earlier apparatus),

$$r = 11 \text{ cm.} \qquad b = 10 \text{ cm.} \qquad i = 45^\circ$$

so that

$$\Delta x = 0.778\Delta N \tag{4}$$

But the main condition of sensitiveness is contained in the size of the fringes, and these may be made indefinitely large by suitable rotation of the mirrors M and M', for instance, in like direction on a horizontal axis (local coincidence of rays on M'). Since

$$2\Delta X \cos i = n\lambda$$

in case of the passage of n fringes, equation (3) becomes

$$\Delta x = nr\lambda/2b \tag{5}$$

so that the limiting sensitiveness ($n = 1$) would be (with the above data)

$$\Delta x = 11 \times 60 \times 10^{-6}/20 = 33 \times 10^{-6} \text{ cm.} \tag{6}$$

for a single fringe, a few tenths of which may be registered with certainty. When the achromatic fringes are used it is, however, usually more convenient to standardize the ocular plate micrometer in the telescope directly by aid of the screw micrometer s, at M', figure 1. If the ocular plate is divided in tenth millimeters along a centimeter of length and the fringes are of moderate size, one may estimate that about 40 scale-parts correspond to $\Delta N = 10^{-3}$ cm., so that a single scale-part of displacement of the achromatics is equivalent to $\Delta N = 25 \times 10^{-6}$ cm., while a few tenths of a scale-part may here also be estimated.

If the apparatus (fig. 3) is to be used as a spherometer, the ordinary method of measuring from a plate of glass is at once available. If r is the radius of the circle of the tripod and ΔX the height of the central foot, we obtain, as usual, for the radius R required

$$R = r^2/2\Delta x \tag{7}$$

This method gives good results for lenses of all curvatures, however strong, as the tests below indicate. But it is not necessary to use the plate to obtain a fiducial reading, provided the system Gr carrying the lens g is on good right and left slides. For in figure 4, let θ be the angle between the plane of the tripod and the slides, and let three readings of ΔN be taken for three preferably equidistant points, l, c, v, of the lens, by sliding Gg over equal distances, r. Let the reading be

$$y = N \qquad y' = N + r \tan\theta + \Delta N \qquad y'' = N + 2r \tan\theta \tag{8}$$

where ΔN corresponds to Δx in figure 4. Hence

$$2\Delta N = 2y' - (y + y'')$$

and equations (4) and (7) apply as before. This method also gives good results even for short distances, r.

3. Observations.—The use of the apparatus, figure 1, with the strip of glass g to be tested sliding up or down, did not at first give satisfactory results, because the mirror mm' was too thin (2 mm. thick). It was found however, that on breaking contact at e during the sliding of g between successive positions, or by gently tapping the bar or standard G, very fair results were obtainable. There would have been no difficulty in using a thick glass mirror mm' (0.25 inch or more), in which case the annoyance of flexure would have been negligible. The following is an example of results obtained, the position of the glass strip g being read off on a parallel vertical millimeter scale:

Position	41.1	37.5	34.3	31.6	30.4	32.7	34.7	38.7	40.8 cm.
$10^3\Delta N$	12.9	10.6	8.6	7.2	6.8	7.9	9.0	11.2	11.8 cm.

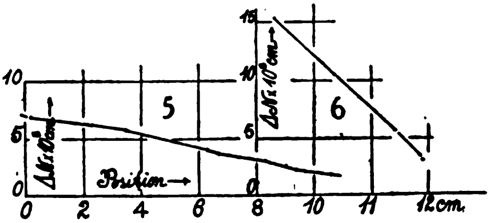
These data are shown in figure 5 and the direct and return series are consistent. The average slope of the strip, which is not quite uniform, may be estimated at

$$\Delta N = 5 \times 10^{-4} \text{ cm.}$$

per centimeter of length, so that

$$\Delta x = 3.4 \times 10^{-4}$$

per centimeter of length.



Using the ocular micrometer, it was found that one scale-part corresponded to one-fortieth of $\Delta N = 10^{-3}$ cm. Tests along a single centimeter of the glass strip gave the results

Position	34.55	35.5	34.5 cm.
Ocular scale-parts	44.3	64.0	45.3

i.e., a difference of 19.2 ocular scale parts per centimeter of length, so that

$$\Delta N = 19.2(10^{-3}/40) = 4.8 \times 10^{-4} \text{ cm.}$$

a result virtually identical with the preceding, as no refinement was attempted.

Tests made with another piece of plate glass (the auxiliary mirror provided with an independent cross-arm and the apparatus gently tapped before observation), gave the results following:

Position	40.8	35.6	37.7.	39.8	40.7 cm.
$10^3\Delta N$	15.8	40.6	30.7	21.1	16.7 cm.

They are constructed in figure 6. Hence per unit of length of plate

$$\Delta N = 0.00239 \text{ cm.} \qquad \Delta x = 0.00186 \text{ cm.}$$

As the micrometer read to about 10^{-4} cm., a corresponding error in the individual data is inevitable.

Tested within 5 mm. by the ocular micrometer (scale-part equivalent to $\Delta N = 46 \times 10^{-6}$ cm.), the mean displacement was about 50 scale parts, so that per centimeter of length $\Delta N = 0.0023$ cm. The results for so small a length are complicated by the difficulty of securing the same lateral position as well as the same longitudinal position, since the thickness changes in both directions. Lateral sliding, which is equivalent to lateral flexure of the contact lever is particularly to be guarded against, but it vanishes on tapping.

Experiments were now made with the design figure 3, except that the mirror mm' was left free, while a special arm (conical tube) parallel to mm' carried the stylus e. Although small fringes were used, the behavior of the apparatus as a spherometer was quite satisfactory. The three set-screws h, h, h were equidistant on a circle of radius $r = 1.08$ cm. Sliding the lens g (about 5 cm. in diameter) so that its center and the outer parts of four quadrants lay successively under the pin e, the readings obtained after gentle tapping were

	Top.	Bottom.	Right.	Left.	Center.
$10^3\Delta N =$	2.00	2.00	2.05	2.05	2.00 cm.

The screw micrometer was thus not sufficiently sensitive to register inaccuracies.

The apparatus used as a spherometer gave for:

	Plate.	Lens.	Plate.
$\Delta N \times 10^3 =$	28.5	23.2	28.4 cm.

so that the height above the plate corresponded to

$$10^3\Delta N = 5.2 \text{ cm., or } 10^3\Delta x = 4.04 \text{ cm.}$$

If R is the radius of curvature of the lens,

$$R = \frac{r^2}{2\Delta x} = \frac{(1.08)^2}{2 \times 4.04 \times 10^{-3}} = 144.3 \text{ cm.}$$

The fringes, however, were here too fine to admit of greater precision. In another experiment with somewhat larger fringes, the data were

	Lens.	Plate.	Lens.
$\Delta N \times 10^3 =$	10.48	15.52	10.53

Hence

$$10^3\Delta N = 5.02 \text{ cm., or } 10^3\Delta x = 3.91 \text{ cm.}$$

whence

$$R = \frac{(1.08)^2}{0.00781} = 149 \text{ cm.}$$

The fringes in the former case were not large enough to admit of better agreement.

These experiments were made merely for the purpose of showing that there is no difficulty, so far as the fringes are concerned, in the successive substitution of plates and lenses. The pin *e* should be removed from contact by an auxiliary lever during this exchange and the apparatus gently tapped before a reading is taken.

The lens tested by the ocular micrometer (here $10^3\Delta N = 0.037\Delta e$) showed at the left center and right side readings of 1.0, 1.0, 0.8 scale-parts, respectively. Thus the three values of $10^6\Delta x$ are 28.9, 28.9, 28.4 cm., respectively. The difference is but 5×10^{-6} cm., the positions taken being about a centimeter apart.

A concave lens was next tested, giving the readings

Lens	0.0277	0.02895 cm.
Plate	.0216	.02280 cm.
ΔN	.0061	.00615 cm.

Thus $\Delta x = 0.00478$ cm., $r = 1.08$ cm., as before, and therefore

$$R = 122 \text{ cm.}$$

The front and reversed sides of the lens calipered did not differ by more than $\Delta x = 3 \times 10^{-6}$ cm.

In the case of two magnifying lenses the readings obtained on different days were (all data in centimeters).

Lens.	Reading.		$\Delta N \times 10^3$		Mean $\Delta x \times 10^3$	R
No. 1....	0.06275	0.06237	35.5	35.7	27.70	21.05 cm.
No. 2....	.02370	....	74.55	....	58.00	10.05
Plate....	.09825	.09807	....	....	.....	∞

In case of such large displacement (nearly a centimeter) the fringes are liable to rotate considerably unless they are vertical. The latter should therefore be selected; otherwise the allowance for rotation may be troublesome.

Though the slides were quite inadequate for precise measurements of this kind, I was nevertheless able to test the method of figure 4 and equations (8). Three readings $r = 0.325$ cm. apart gave, respectively

$$y = 0.0148 \qquad y' = 0.01975 \qquad y'' = 0.0260 \text{ cm.}$$

Hence

$$2\Delta N = 2y' - (y + y'') = 0.0013 \text{ cm.} \qquad 2\Delta x = 0.00101 \text{ cm.}$$

Thus $R = 106$ cm. The observations, but for the defective slides, were very satisfactory.

CHAPTER II.

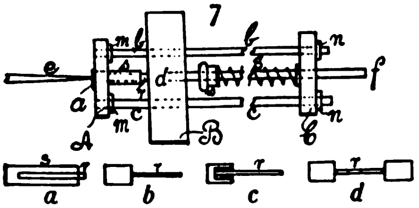
ELASTIC DEFORMATION IN SMALL BODIES, MEASURED BY THE PRECEDING CONTACT LEVER.

4. Introductory.—In the preceding report I communicated a series of experiments on the traction modulus of small bodies, using an interferometer design which worked admirably so far as the optic measurements were concerned. The mechanical part of the contrivance showed an apparent yield, the nature of which I was unable to detect, but which seemed to be in some way associated with the flexure of parts of this massive apparatus. In fact, pulleys and weights were used for imparting stress. It may be argued that any contrivance of this kind, however convenient in other respects, is dangerous because of the force couples introduced, even when the rigid parts of the apparatus are nearly 2 inches thick, as in the case in question.

In the present apparatus all this is completely avoided by the use of springs to impart stress, and the above contact lever to measure strain. True, friction enters into functioning of such an apparatus to a menacing degree. It thus becomes an experimental question to determine how far it can also be eliminated by judicious tapping, etc.

5. Apparatus.—The simplest of the apparatus designed is shown in figure 7. The rod to be tested, 2 to 3 cm. long, is at *r* held in a brass sheath *s*, loosely fitting it. This is screwed into the middle of the massive brass cross-piece *A*. A little disk of glass has been attached at *a*, and the end *e* of the contact lever touches it, to indicate the small displacements. The longitudinal displacements Δx of the pin *e* are observed by the interferometer, as explained in the preceding paper.

7



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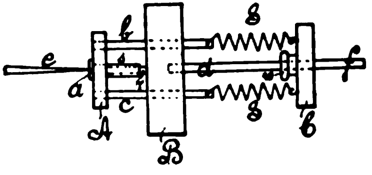


B is a cast-iron brick, about 10 inches high, 2 inches thick, and 3.5 inches broad, provided with two horizontal (0.25 inch) perforations, parallel to each other and normal to the large face. Through these pass the 0.25-inch brass rods *bb* and *cc* loosely, rigidly connecting the cross-piece *A* with the similarly massive cross-piece *C* (screws and locknuts *m*, *n*). The rectangle *AC* is thus free to slide in *B*, except so far as it is limited by the contact of the rod *r* with the smooth face of the brick *B*.

To apply stress, the system *d*, *w*, *S*, *f* has been provided, consisting of the stiff open spring *S* encircling the brass rod *df*, firmly screwed into the brick

B at d, but passing loosely through a perforation in the middle of C. The end near d of the rod df is threaded (20 threads to the inch) so as to admit of the compression of the spring S by aid of the thumb-nut w. S was a precision spring taken from an indicator apparatus and provided (as usual)with two end brass collars. It is essential that the sliding parts of the apparatus work smoothly and with a minimum of friction. Such as exists may be eliminated by tapping b and c before each observation. Thrusts up to 15 to 20 kg. may be easily applied by the thumb-nut w. These stresses act in the direction ar and fd collinearly, and there are no couples endangering the accuracy of the elastic displacements of r. The stress is standardized in terms of the observed rotation of the thumb-nut w. Figure 7 (a) to (d) are details, showing different methods of clutching the rod r.

Figure 8 shows a similar apparatus in the same notation. Here the spring SS acts by tension and more and more strongly as the thumb-nut w advances f. The apparatus is slightly more complicated but offers certain differences in relation to the friction of parts.

6. Observations. Hard rubber.—As in the preceding paper, if Δx is the longitudinal compression of the rod r is the sheath s,

$$\Delta x = (r \cos i/b)\Delta N \tag{1}$$

where ΔN is the displacement of the micrometer at $i = 45°$ to the rays, b the breadth of the ray parallelogram, and r the effective length of the contact lever. Furthermore, since the modulus E for the length of rod L and section A is

$$E = (F/A)/(\Delta x/L)$$

F being the thrust,

$$E = F\frac{L}{A}\frac{b}{r \cos i \Delta N} \tag{2}$$

The ocular micrometer, if used, is to be standardized in terms of ΔN by direct comparison.

To graduate the spring S, the apparatus ABC, figure 7, was detached from the interferometer and the brick B fastened to the edge of a strong flat table with its large face toward A lowermost and horizontal. The rectangle AC was thus vertical, A below C, just clearing the edge of the table. Weights from 1 to 9 kg. were now hung from A, compressing the spring S by measurable amounts. In this way it was found that the stretch 0.7 mm. corresponded to 1 kg. Since the threads of w were 1.275 mm. apart, it follows that 1 rotation of the thumb-screw w corresponds to 1.82 kg. or 1.78×10^6 dynes.

In the interferometer, $b = 9.3$ cm., $r = 11.0$ cm., were directly measured.

The test-rod r was here of hard rubber, of length $L = 2.47$ cm., diameter $= 0.377$ cm., and area $A = 0.112$ cm^2. Hence for n turns of the screw w, equation (2)

$$E = n \times 1.78 \times 10^6 \frac{2.47}{0.112} \frac{9.3}{11 \times 0.707 \times \Delta N} = 4.69 \times 10^7 \frac{n}{\Delta N} \tag{3}$$

or if we express ΔN in 10^{-3} cm., $E = 10^{10} \times 4.69 \dfrac{n}{\Delta N}$

The fringes found without difficulty were small, though adequate for the purpose. Measurements were made in cycles, care being taken to repeatedly tap the movable parts of the apparatus before each reading, and these came out remarkably smoothly at once. The first results were (initial load about 2 kg.):

No. of turns of screw, n 1 2 3 4 5 4 3 2 1
$10^3 \times \Delta N$ 24.7 26.9 29.0 30.9 32.8 31.1 29.1 27.3 25.1 22.6 cm.

These data are also given at *a*, figure 9. The outgoing branch is nearly straight and regular, though the first observation was lost. The return branch is less regular, as would be expected, but the departures lie within 5×10^{-4} cm. If we take the differences between observations $n=3$ turns apart, in the outgoing series the results would be

$10^3\Delta N$	6.45	6.15	5.85 cm.
$10^{-10}E$	2.18	2.29	2.41
Loads about	2–5	4–6	5–8 kg.

The rod therefore grows steadily more rigid as the load increases, as shown at *b*, figure 9, a result also to be anticipated. Of hysteresis there is no certain indication, since the higher results in the return may be otherwise explained. If the means of advance and return series had been taken, the values of $10^3\Delta N$ would have been 6.47, 6.10, 5.67, and their mean difference from the above is within the errors of the screw micrometer reading to 10^{-4} cm.

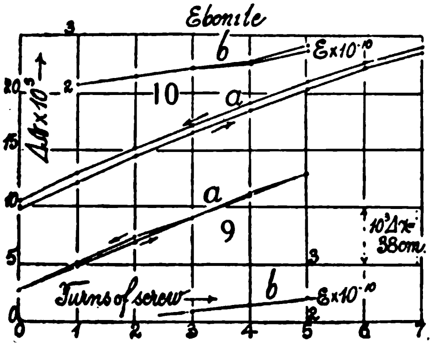


TABLE 1.—Hard rubber rod. $L=2.47$ cm. $A=0.112$ cm². $E=10^{10}\times 4.69n/\Delta N$. Initial load about 2 kg. Additional load about 1.8 kg. per turn ($n=1$).

n	10^3N	$10^2\Delta e$ ($n=1$)	$10^3\Delta N$ $n=3$	$10^{-10}E$	n	10^3N	$10^2\Delta e$ ($n=1$)	$10^3\Delta N$ $n=3$	$10^{-10}E$
0	4.8	30	6.65	2.12	*7	19.0	18	5.00	2.81
1	7.1	28	6.20	2.27	6	17.6	22	5.55	2.54
2	9.4	26	5.85	2.41	5	15.9	24	5.85	2.41
3	11.4	26	5.70	2.47	4	14.0	24	6.10	2.31
4	13.3	26	5.15	2.73	3	12.0	24	6.55	2.15
5	15.2	26	....	....	2	10.0	24	....	....
6	17.1		....	....	1	7.9		....	....
7	18.5	..	....	....	0	5.5	..	....	....

* Later.

The next experiments were made after some improvements of apparatus and carried out with the same rod through a range of about 15 kg. The rod stood the stress well, except at the end, when it showed slow viscous shrinkage. The data are given in figure 10, *a* and *b*, and in table 1 and contain the running displacement ΔN of the micrometer screw, as well as the successive displacements Δe measured on the ocular micrometer.

It has been found convenient for the sake of comparison with the preceding set to compute E from three turns (5.46 kg.) of the screw, and ΔN is so specified. For loads up to five additional turns (total 11 kg.) the data are practically identical, both in the outgoing and return series, and identical with the preceding series. At six turns (total 13 kg.) the rod yields, but at seven turns it again stiffens in both cases. As a whole the data are quite as good as the reading of the micrometer screw admits.

The values of Δe are rough, for the fiducial mark of the screw was necessarily taken near the middle of the field of the telescope, as a result of which the large displacements Δe were thrown off to the edge of it, particularly in the return series. Moreover, very small fringes were used for convenience, as these were adequately mobile. Under these circumstances about $\Delta e = 13$ ocular scale-parts correspond to $10^3 \Delta N = 1$ cm. To use Δe to advantage, a special adjustment insuring the illumination of 100 scale-parts is desirable and the fringes should be large. Only in case of more rigid rods where ΔN fails is this necessary.

7. The same. Continued.—To determine the limits toward which E ultimately approaches, it is advisable to examine a thinner rod; for beyond seven or eight turns of the compressing screw, the stress put on the mechanism in further twisting is liable to dislocate certain of its parts or to alter its position. The convenience of the thumb-screw would be unavailable. A hard-rubber rod of the same material was therefore made more slender, the dimensions being, length $L = 2.55$ cm., diameter 0.215 cm., $A = 0.0363$ cm.^{2} The ratio of length to diameter thus exceeds 10/1. Large fringes were installed unnecessarily, for the sensitiveness of the interferometer is here excessive. The dimensions given lead to

$$E \times 10^{-10} = 14.95/\Delta N$$

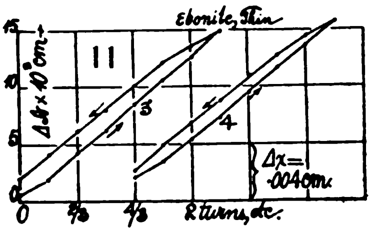
if ΔN is in 10^{-4} cm. Omitting the earlier series, the results of the third and fourth are given in table 2 and figure 11, the twists of the compressing screw being successively increased by one-third of a turn, or about 0.6 kg. There was a small permanent initial load, as usual.

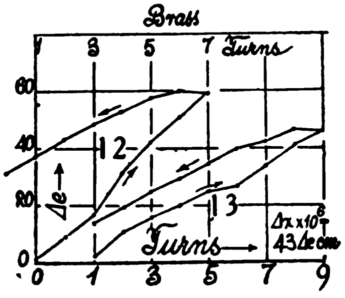
The results as a whole are good. The ascending and descending branches of the graphs are straight, except at the beginning or the end. The latter is

TABLE 2.—Thin hard rubber rod. $L=2.55$ cm.; diameter 0.215 cm., $A=0.0363$ cm.^{2}; $E\times10^{10}=14.95/10^3\Delta N$. Initial load about 0.5 kg.; additional load 1.8 kg per turn ($n=1$).

n	Series 1.			Series 4.		
	10^3N	$10^3\Delta N$ $n=1$.	$10^{10}E$	10^3N	$10^3\Delta N$ $n=1$.	$10^{10}E$
	cm.	cm.		cm.	cm.	
0	0.4	...	....	2.0	...	....
⅓	1.9	6.5	2.30	3.5	6.0	2.49
⅔	4.2	6.4	2.34	5.4	6.2	2.41
1	6.1	6.3	2.37	7.2	6.3	2.37
1⅓	8.4	6.3	2.37	9.4	6.2	2.39
1⅔	10.5	...	....	11.6	...	....
2	12.5	...	....	13.5	...	....
2⅓	14.7	...	....	15.7	...	....
2	13.5	...	....	14.3	...	....
1⅔	12.1	...	....	12.7	...	....
1⅓	10.1	6.0	2.49	10.9	5.9	2.53
1	7.9	6.0	2.49	8.7	6.1	2.45
⅔	6.1	5.9	2.53	6.8	6.1	2.45
⅓	4.1	...	....	4.8	...	....
0	2.0	...	....	2.6	...	....

difficult to interpret; for the reversal of the twist may imply some dislocation of the spring, like a twist around its axis, for instance. In table 2, E has therefore been computed from the data of ΔN for $n=1$, leaving out the irregularities specified. The values of E in a given branch (ascending or descending) of the cycles is virtually constant. The mean values in series 3 are (ascending) 2.34, (descending) 2.50; in series 4, (ascending) 2.41, (descending) 2.48. The rod is always apparently more rigid in the case when stress is removed than when it is added. But this difference decreases and would probably vanish if the experiments were indefinitely continued. It eventually vanishes from triplets. The mean of the whole of the third series is $E\times10^{-10}=2.42$; of the fourth, $E\times10^{-10}=2.44$, which may be regarded as satisfactory for the body in question. We may note that a dimension-ratio of 10/1 has been required to obtain this result.





8. The same. Brass.—A brass rod was now inserted to test the rigidity of the apparatus. The dimensions of the rod were $L=2.50$ cm.; diameter =

0.205 cm. at one end and 0.215 at the other; part of a hard-brass nail. The section was therefore $A=0.035$ cm². Hence equation (3) becomes

$$E=n\times 1.78\times 10^{6}\frac{2.50}{0.35}\;\frac{9.3}{11\times 0.707\times \Delta N}$$

If ΔN is given in 10^{-3} cm.,

$$E=10^{11}\times 1.52n/\Delta N \tag{5}$$

If the ocular micrometer shows a displacement Δe (in case of brass the micrometer screw is insufficiently sensitive) equation (5) is to be modified by first finding $\Delta e/\Delta N$ by direct comparison.

The tests gave an average value of $\Delta e/\Delta N\times 10^{3}=17.1$. Consequently

$$10^{-11}E=1.52\times 17.1\frac{n}{\Delta e}=26\frac{n}{\Delta e}$$

for n turns of the compressing-screw.

The behavior of the brass rod was peculiar and the first two series for increasing and decreasing pressures are shown in figures 12 and 13. The mean data are

Δe per turn	9.6	5.0	5.3	5.2
E 10^{-11}	2.7	5.2	4.9	5.0
Series	(1) F increasing	(1) F decreasing	(2) F increasing	(2) F decreasing

The first compression seems merely to have established firmness at the seats at the end of the rod. Thereafter the value of E comes out fairly uniformly, but it is only one-half the normal value; *i.e.*, Δe is twice the reasonable value, so that something else is simultaneously yielding. On the removal of pressure there is at first no apparent elongation of the rod, but rather a contraction. The maximum load was about 16 kg., beginning with 2 kg. There is no evidence of viscosity, but rather of permanent contraction or set. The apparatus was vigorously tapped before each reading, which was thereafter constant. The optical system functioned admirably. A piece of thin mica placed under the contact lever showed $\Delta N=0.0229$ cm. and therefore $\Delta x=0.01786$ cm. The same piece calipered with a micrometer screw proved to be 0.0180 cm. thick, practically a coincident result.

The next four series were made with a higher initial load (4 kg.), but showed the same character. In No. 3 figure 14, there was an accidental dislocation owing to the endeavor to use a wrench at the thumb-screw. If we take mean results for the outgoing and return series the data are

Series	3	4	5	6
Δe per turn	4.7	4.4	4.7	4.0
$10^{-11}E$	5.5	5.9	5.5	6.5

The normal value of the modulus is being slowly approached. As the displacement is too large, errors can not be referred to friction. Probably the two ends of the rod are fitting themselves to a more uniform seat.

In the next series (8) figure 14, the projecting head of the rod was ground flat with a flat carborundum stone, parallel to the abutment B, figure 7. It was also kept under eight turns (about 14 kg.) for 2 hours. The data are given in figure 14, series (8) and the mean results of the straight lines are ($\Delta e = 17 \Delta N \times 10^3$)

	Returning	Outgoing
Δe per turn,	3.6	4.2
$E \times 10^{-11}$	7.2	6.2

The return series has as usual the larger modulus; but the outgoing series is here above the return series in Δe. One should notice that the mean displacement Δe per turn is only about $\Delta e = 3.9$ equivalent to $\Delta N = 23 \times 10^{-5}$ cm. or $\Delta x = 18 \times 10^{-5}$ cm. Since the true value or Δe should be 2.6 instead of 3.9, $\Delta e' = 1.3$, equivalent to $\Delta N = 0.00008$ cm. or $\Delta x = 0.000{,}06$ cm. (*i.e.*, about the wave-length of light) is lost in the apparent yield of the apparatus per turn of compressing-screw.

The rod was now left under stress (14 kg.) for about an hour and then showed the displacements Δe given in figure 14, No. 9. The slope of the straight part of the curve, *i.e.*, the modulus, has remained about the same. One may note also that moderate loads (below 7 kg. on 0.035 cm^2.) are best adapted to bring out linear graphs, quite up to the absence of all load.

On removing the rod from its sheath there was no evidence of any change of shape and it still fitted the sheath loosely; only at the neck of the sheath the rod showed ringlike chafing.

9. The same, continued.—By way of contrast a thick solid brass rod, $L = 2.34$ cm. and 0.376 cm. in diameter, $A = 0.111$ cm^2., was now put into the sheath and tested, the aim being to determine the limit of measurement.

We should thus have ($\Delta e / \Delta N \times 10^3 = 16$ here)

$$10^{-11} E = n 1.78 \frac{2.34}{0.111} \frac{93 \times 16}{11 \times .707 \Delta e} = 7.18 n / \Delta e$$

The first and second series are given in figure 15, (1) and (2), and the mean data are:

Δe per turn	2.7	4.9	3.7	4.6 scale-parts.
$E \times 10^{-11}$	2.6	1.5	1.9	1.5

These values of E are much too small, as was to be expected. The apparatus yields much more than in the preceding case. Δe per turn should here be but 0.7 scale-parts.

The foot of the rod r, figure 7, was now ground down with a small flat carborundum stone placed between r and B and slid up and down under spring pressure. This is the best way to obtain parallel surfaces. Results so found are given in figure 15, (1) to (4), beginning with a decreasing thrust. If we take the mean rate for the higher pressures (the rod loosens below four turns), the data came out ($\Delta e / \Delta N \times 10^3 = 18$) as follows:

Series	(1)	(2)	(3)	(4)
Δe per turn	...	2.7	1.2	1.7
$E \times 10^{-11}$	...	3.0	6.8	4.5

The first series is irregular. The present data are a considerable improvement on the preceding, but they are far from indicating the rigidity of brass. A concluding series, made with but slight tapping, showed the following two rates:

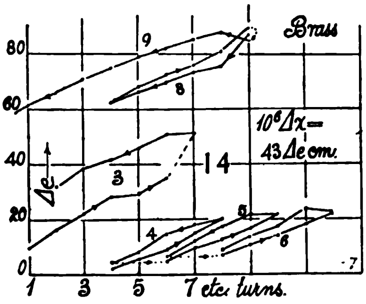
Turns	8	7	6	5	4	3	2	1	0
Δe	69	68	67	65	64	62	59	54	49
$E \times 10^{-11}$			6.2				1.9		
Δe per turn			1.4				4.3		

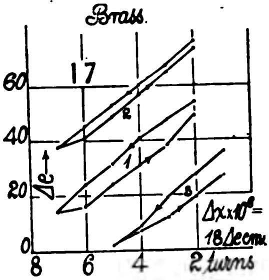
Finally, in a series of measurements in triplets, between three and four turn of the forcing-screw, values of $E \times 10^{-11}$ between 1.3 and 2.4 were obtained, showing therefore no improvement on the preceding work.

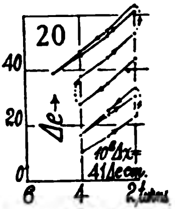
The interferometer was now modified to guard against displacements on tapping, large fringes were installed, and readings were made several times before and after tapping. There was but little difference. Examples of the results are given in figure 17, (1) and (2), where $\Delta e/\Delta N \times 10^3 = 43$, and therefore

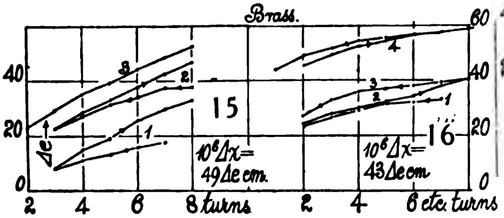
$$10^{-11}E = 19.3/\Delta e$$

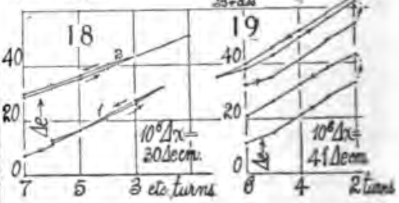
In the first series Δe per turn was 8.0, and hence $E \times 10^{-11} = 2.4$; in the second $\Delta e = 7.8$ and $E \times 10^{-11} = 2.5$. Seeing that a scale-part in figure 17 is but 23×10^{-6} cm., these results are experimentally very good. But their absolute value, as given by E, is nevertheless very low. The rates for the outgoing and return series are identical. The back-lash, as it were, on passing from one to the other is probably in the apparatus.











In the triplets naturally higher values of E appeared; for instance, between three and four turns of the screw, per turn,

$$\Delta e = 6,\ 5,\ 5,\ 6,\ 5,$$

in successive separate experiments. Thus, mean $\Delta e = 5.5$ and $E \times 10^{-11} = 3.5$.

10. The same. Continued.—The last experiments were made with a brass rod, shouldered as indicated in figure 7*b*, the large end (0.25 inch in diameter) being threaded and screwed into the cross-piece A. The dimensions of the thin part were: length $L = 1.8$ cm., diameter $= 0.22$ cm., $A = 0.038$ cm.^{2} The results are shown in figure 18 (1), where $\Delta e/\Delta N \times 10^3 = 29$. The mean rate per turn is $\Delta e = 4.9$ and $10^{-11}E = 29.2/4.9 = 6.0$.

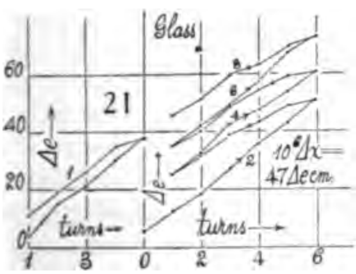
The end of the same rod was now ground flat again with the carborundum plane. The results in figure 18 (2) are often coincident in outgoing and return series, but show slight hysteresis. The mean rate ($\Delta e/\Delta N \times 10^3 = 26$ here) is $\Delta e = 3.6$ per turn and $10^{-11}E = 26.2/3.6 = 7.3$ the highest value yet found. Either the regrinding or the gradual hardening of the rod has been effective.

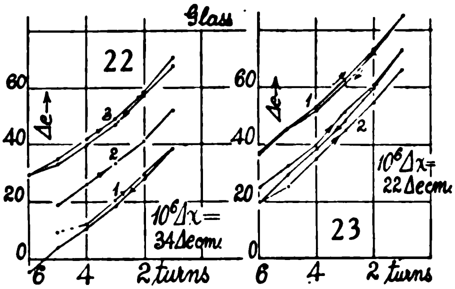
It seemed therefore worth while to further decrease the section. This was done, the dimensions being, $L = 1.8$ cm., diameter 0.175 cm., $A = 0.0199$ cm.^{2} The results are given in figure 19, where $\Delta e/\Delta N \times 10^3 = 26.0$ and therefore $E = 5 \times 10^{11}/\Delta e$. The graphs have been separated for clearness. The rates per turn lie between $\Delta e = 5.6$ (returning) and $\Delta e = 6.1$ (outgoing), so that $E \times 10^{-11}$ is between 8.9 and 8.2, respectively. This is so near the normal value for brass that a further decrease of section of the rod figure 7*b* was undertaken. The final dimensions were $L = 1.8$ cm. diameter $= 0.138$, $A = 0.015$ cm.^{2} The results are given in figure 20, care having been taken not to overstrain the thin rod. Here $\Delta e/\Delta N \times 10^3 = 25.8$, $E \times 10^{-11} = 66.4/\Delta e$, and Δe per turn lies between 6.7 (outgoing) and 8.1 (returning). Hence $E \times 10^{-11} = 9.9$ and 8.2, respectively, so that the normal modulus of brass has actually been reached.

11. Glass.—The glass rod tested was $L = 2.33$ cm. long, 0.37 cm. in diameter, so that $A = 0.107$ cm.^{2} Hence, since $\Delta e/\Delta N \times 10^3 = 16.7$ here,

$$E = n \times 1.78 \times 10^6 \times (2.33/0.107) \times (9.3/11 \times 0.707) \times 16.7/\Delta e = 10^{11} \times 7.7n/\Delta e$$

The first series of results is shown in figure 21 (1). Both branches are practically parallel, whence per turn $\Delta e = 7.6$ and $E \times 10^{-11} = 1.02$. Notwithstanding the smooth results, this datum is about five or six times too small. Seven





other series were now worked out, of which figure 21 (2) shows an example. The mean rates per turn were from $\Delta e = 7.7$ to $\Delta e = 6.2$, corresponding to module from $E \times 10^{-11} = 1.0$ to 1.25. I also resorted to the method of observing in triplets between definite steps of pressure. The results were all of the same low order.

With a more robust interferometer the series of results in figure 22 were obtained. The outgoing and return data are here more nearly coincident, but the graphs are not as a rule straight. The mean rate may be taken from figure 22 (1) and is per turn $\Delta e = 8.6$. The fringes were of moderate size, so that $(\Delta e/\Delta N \times 10^3 = 27.5)$

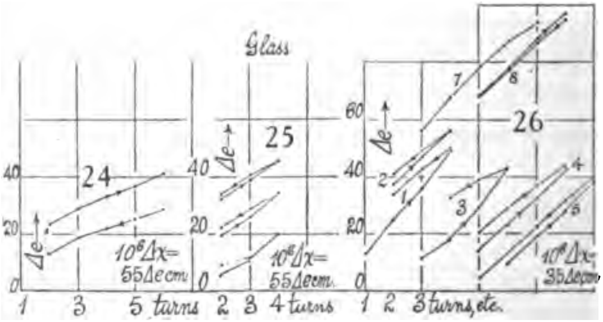
$$10^{-11}\mathrm{E} = 12.75/\Delta e = 12.75/8.6 = 1.5$$

Larger fringes were now installed, giving $\Delta e/\Delta N \times 10^3 = 34.8$. The results are shown in figure 23 and are again nearly coincident, but lie on curved loci, often with two definite rates. In the first series the larger is $\Delta e = 8.4$ per turn, in the second series $\Delta e = 10.0$ per turn. Hence

$$10^{-11}E = 16.1/\Delta e = 1.9 \text{ and } 1.6, \text{ respectively,}$$

larger than the preceding, probably because the face had been reground. But this order is still only about one-third of the normal modulus of glass.

The endeavor was now made to proceed as in the case of brass above, with a shouldered rod and thinner sections. With this in view, the glass rod was fixed in a small hollow cup (fig. 7*c*) with fusible metal. The cup being threaded, was thereupon screwed into the cross-piece A, figure 7. The first glass rod was $L = 1.9$ cm. long, 0.32 cm. in diameter, $A = 0.083$ cm.^{2} Very small fringes were used, $\Delta e/\Delta N \times 10^3 = 13.3$, so that $E \times 10^{-11} = 6.5/\Delta e$. The results in figure 24 conform to $\Delta e = 3.4$ to 4.2 per turn of screw, which correspond to $E \times 10^{-11} = 1.9$ to 1.5. This is therefore no improvement.

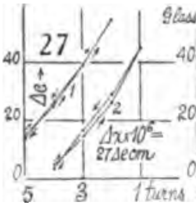


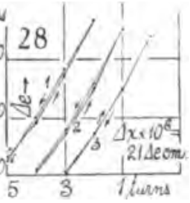
The rod was then drawn thinner, viz, to $L = 1.9$ cm., diameter 1.9 cm., $A = 0.284$ cm.^{2} The results in figure 25 are necessarily confined to small stresses and show a gradual improvement. As $\Delta e/\Delta N \times 10^3$ is here 14.2, $E \times 10^{-11} = 19.1/\Delta e$. Δe per turn lies between 6.2 and 6.8, so that $E \times 10^{-11}$ is 3.1 to 2.8, respectively. Experiments made in triplets gave $\Delta e = 6$ or $E \times 10^{-11} = 3.2$. The discrepancy is undoubtedly connected with the difficulty of securing an adequate seat for this brittle and slender glass rod. There was also evidence of apparent viscosity, to be attributed to the relaxing clutch of the fusible metal.

After grinding the outer face of the rod again, experiments were made first between two and four turns and thereafter between two and five turns of the screw. The results are given in figure 26. One notices that the successive cycles gradually close up. After the sixth series there is practical coincidence and the rates per turn of the screw are $\Delta e=8.9$ (ingoing) and $\Delta e=9.8$ (outgoing). The fringes showed $\Delta e/\Delta N \times 10^3=22.2$. Hence $E\times 10^{-11}=29.9/\Delta e=$ 3.4 and 3.0. This rod showed marked (apparently) viscous contraction, quite apart from the hysteresis of the cycles. Since $\Delta e=10^{-3}/22.2=10^{-6}\times 45$ cm., and $\Delta x=0.78\Delta N$, Δx being the contraction of the rod, it follows that $\Delta x=10^{-6}\times 35\Delta e$ cm. The rod left for 45 minutes under the compression of five turns of the screw showed an apparent contraction of $\Delta e=4$ scale-parts or $10^{-6}\times 140$ cm., or $10^{-6}\times 74$ cm. per centimeter of length. More systematically in the following tests (reduced to a centimeter of length).

Time	50	80	120	195 minutes
$10^6\Delta x$	0	140	350	787 cm.

On taking the rod out, however, it became clear that the real reason for this was the gradual yielding of the fusible metal clutch. It is thus probable that the whole length of the glass rod, instead of the part projecting from the fusible metal plug (fig. 7, *c*), should have been inserted into the equation, making E larger than above given. Hence I returned finally to the sheath method (fig. 7, *a*) using a thin glass rod, $L=2.54$ cm. long, 0.185 cm. in diameter, $A=0.0269$ cm.^{2}. The results are given in figure 27. The graphs are nearly coincident, but curved. The mean rates for the higher loads are (per turn of the screw) $\Delta e=10.4$ (incoming) and $\Delta e=8.6$ (outgoing). $\Delta e/\Delta N\times 10^3=28.8$ being the fringe factor, $E\times 10^{-11}=57.86/\Delta e=5.5$ and 6.8, respectively. Hence here, also, as in the case of the brass rod above, the normal value of the modulus has been reached; *i.e.*, one may expect the data for E to be correct in their absolute values, if the ratio of length of rod to diameter is of the order of 10 to 1.





Another series with a different adjustment of the same rod is shown in figure 28, the fringes being larger, $\Delta e/\Delta N\times 10^3=37.3$. Triplets gave the following mean results:

Turns of screw	4–5	3–4	2–3
$\Delta e/n$	13	15	17.5
$E\times 10^{-11}=74.9/\Delta e$	5.8	5.0	4.2

12. Conclusion.—The present experiments, made with a totally dissimilar apparatus and in a different manner, are nevertheless (notwithstanding the relative simplicity of the present design) not markedly superior to the earlier experiments, as a whole. The misgiving felt regarding the force couples entering into the earlier method was not therefore justified. Both apparatus function admirably so far as the optics of the method are concerned; particularly is this so when one considers the admissibility of the rather rough treatment needed in work of the present kind. Both apparatus are liable to give misleading results from the same cause, *i.e.*, from an insufficiently uniform and continuous contact of the two ends of the rod with the abutments. From this results appreciably unequal distribution of stress in the sections of the rod and possibly flexure. There seems to have been no serious yield in the abutments, etc., of either apparatus.

The values of the modules E as a consequence come out too small. There can therefore (tapping admitted) have been no serious discrepancy from friction in the application of stress; for this would have made E too large. Moreover, all slight dislocations within the interferometer, as the result of the tapping or jar, were finally eliminated, so that the cycles are practically closed, or merely give evidence of a difference of slope in the outgoing and return series. Such an effect would be expected from viscosity and hysteresis.

I was at first inclined to regard the small values of the modulus E as an actual or trustworthy result, in keeping with the peculiar crushing stress applied. But inasmuch as E may be increased to the normal value by successively decreasing the diameter of the rod in the case of glass and even of brass, the small values of E must be associated with the lack of contact at the abutments of the rod. Rods about 1 to 2 cm. in length should not be thicker than 1 or 2 mm. (ratio about 10 to 1) if the results are to be correct in their absolute values. And here again a thin rod, *r*, with two thick ends, as in figure *7d*, if both ends are firmly clutched (without strain), is the ultimate desideratum. Figures *7b*, *7c*, *7a* (sheath, *s*), are admissible expedients, the latter being particularly convenient. The relative results are almost always smooth and admirable to a fraction of a wave-length; but for relatively large sections they can not be interpreted, owing to the sectional discrepancy in question. This also is relative in its character; at least for moduli markedly above 10^{10}. Thus it is as difficult to obtain the true modulus for a glass rod as for a brass rod, although the latter body is far more rigid.

It is not easy to interpret the apparent hysteresis in many of the above graphs; for this is always associated with possible changes in a complicated train of apparatus. Similarly the different rates in the outgoing and the return series may be variously explained. If the measurements are made in triplets between definite steps of pressure, this difference soon vanishes. Hence such a procedure is to be preferred.

CHAPTER III.

THE ELONGATION DUE TO MAGNETIZATION, MEASURED WITH THE INTERFERENTIAL CONTACT LEVER

13. Introductory.—The small longitudinal displacements due to magnetostriction have been frequently subjected to investigation and an excellent summary is given in Winkelmann's Handbuch, vol. 5, p. 307, et seq., 1908. The measurements of Professor Knott and his students, notably Nagaoka and Honda (Phil. Mag., vol. 37, p. 131, 1894), are particularly complete. In 1911,* my son, Mr. Maxwell Barus, and I used these phenomena for the purpose of testing a peculiar type of interferences then under discussion.

The present purpose is similar, being a test of the contact lever described in Chapter I; *i.e.*, to find how small a field can be detected from the elongations of an iron rod within it, and vice versa. Furthermore, to find to what degree the magnetic field may increase, before the subsequent contractions appreciably cease.

The elongation phenomena are necessarily complicated by the occurrence of hysteresis loops, to which the present paper (in which the measurements are not made by the continuous variation of currents and field, but by successively making and breaking the circuit) will give no attention. This subject has been adequately explored by Professor Knott† and the authors cited. The chief interest in this paper is rather the continued increase of the contractions due to magnetization, not only after the latter has practically reached a maximum, but in a marked degree, and at an unexpectedly persistent rate relative to the magnetizing field, so far as I have gone (fields up to 800), indefinitely. There is no sure indication of a cessation of the contraction. Hence the magnetic contribution of the present paper is to lie in the treatment in strong fields.

14. Apparatus.—The contact lever shown in figures 1 and 2 of Chapter I was modified as indicated in figure 29 (plan), where F is the semi-circular fork in a vertical plane, rigidly attached to the bed-plate of the interferometer by a strong clutch (not shown) holding the cylinder g or handle of the fork. The vertical axis a of the contact lever is secured between the screw-pivot b of the fork. The horizontal strip of brass d, rigidly fastened to the middle of the axle a, carried at its end the auxiliary mirror mm' of the quadratic interferometer. For this purpose, a short length, f, at the end of d has been bent upward at right angles to d, so that mm' may be held between plates of brass by the yoke-shaped steel slip c. At the side of the lever is a vertical brass plate inset c, to which a small glass plate n has been fastened with cement. It is against this that the conical end of the iron rod *rr* to be examined pushes. The spring *S* attached to the blade *d* and a lateral projection from the fork *F* (or other appurtenance) insures continuous contact at a constant pressure.

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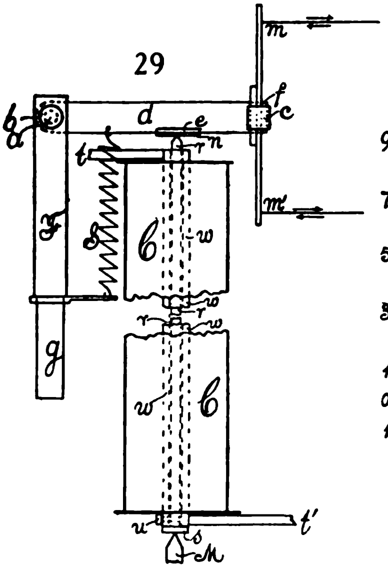
* Carnegie Inst. Wash., Pub. No. 149.

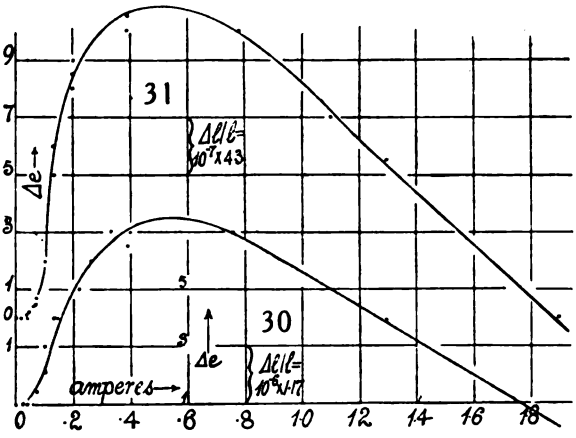
†Knott; Phil. Mag. (5), XXXVII, p. 141, 1894.

The iron rod *rr*, about 43 cm. long and 0.67 cm. in diameter in the first experiment, is enveloped by a tubular water-jacket *ww*, 1.40 cm. in external diameter. Through this a current of water entering at *t* and leaving at *t'* is kept flowing from a large copper Mariotte flask about 50 cm. high and 30 cm. in diameter. The water contained is at the temperature of the room. Two such flasks were at hand to be used alternately. The magnetizing coil *CC*, 26 cm. long and 3.7 cm. in external diameter, is wound immediately on the tubular water-jacket.

The rod *rr* fits the tube *ww* loosely and is centrally attached at the remote end by aid of a bushing *s* and a small bolt *u*. The conical front end is free.

29





The coil *CC* is held in position by a large clutch (not shown) encircling it at the middle and attached to the bed-plate of the interferometer. It is additionally attached at the tubulures *t* and *t'*. Finally the conical end *M* of a micrometer-screw (also rigidly attached to the bed plate) gives the remote end of the rod *rr* any desirable fiducial position. This micrometer *M* has the further advantage of permitting an independent corroborative standardization of the contact lever, as there would obviously always be sufficient elastic yielding in the apparatus to considerably shift the interference fringes; but it is better to free the rod from the coil at the rear end, also.

In the experiments made, the breadth of the ray rectangle *mm'* of the interferometer was $b = 9.7$ cm.; the normal distance between the rod *rr* and the axis *a*, 7.0 cm.; the length of the contact lever *a* to *mm'*, 10.6 cm.; and the length of the axle *a*, 10 cm.

The rod *rr* may be easily withdrawn and others inserted.

15. Observations.—The helix C in figure 29 was slender in shape, the length being 37 cm. and the diameter within being about 1.5 cm. There were about 11.2 turns per centimeter per layer and 8 layers of wire, so that the field within may be estimated at $H=110i$ gauss, i being the current in amperes. The current 0.01 to over 2 amperes thus corresponded to field from 1 to over 200 gauss.

The rod first selected was of low-carbon shop steel, 43 cm. long. Thus it projected a few centimeters beyond either end of the helix.

The displacement of fringes observed was characteristic, being (in the smaller fields) slow and deliberate on closing the circuit (so that their motion could almost be followed by the eye), but very rapid on breaking the circuit. In the higher fields (200 gauss) there is always much jarring on closing the circuit, as though the rod passed through the whole antecedent cycle of elongations. The fringes are turbulently displaced and only gradually subside. Reading is more difficult.

The experiments were begun with small fringes (about 0.1 mm. in the ocular), and the readings Δe were made in terms of an ocular micrometer scale, which was a centimeter divided into 0.1 mm. Comparing this with the datum ΔN of the displacement micrometer normal to one of the mirrors of the interferometer, the relation was found to be

$$\Delta e = 22\times 10^3 \Delta N \text{ cm., or } \frac{\Delta N}{\Delta e} = 4.5\times 10^{-5}$$

If ΔN corresponds to the angular displacement, $\Delta\theta$, of the contact lever and to Δl of elongation of the iron rod r in the helix C (fig. 29) we may write as above,

$$(1) \qquad 2b\Delta\theta = 2\Delta N \cos i$$

if i is the angle of incidence (45°) at the mirrors of the interferometer and b the breadth of the ray parallelogram.

But

$$(2) \qquad \Delta l = r\Delta\theta$$

if r is the normal distance of the line of thrust of the rod rr from the axis a of the contact lever. Thus

$$(3) \qquad \Delta l = \frac{r\cos i}{b} \qquad \Delta N = \frac{r\cos i}{b}\left(\frac{\Delta N}{\Delta e}\right)\Delta e$$

If l is the length of the iron rod $\Delta l/l$ will be the datum required.

If the above data are inserted, the coefficient becomes ($l=43$ cm.)

$$\frac{\Delta l}{l} = \frac{7.0\times 0.707}{9.7}\, 10^{-5}\times 4.5\times\frac{1}{43}\Delta e = 10^{-7}\times 5.34\Delta e$$

so that

$$\Delta l = 10^{-5}\times 2.3\Delta e \text{ cm.}$$

The results obtained with these small fringes are given in figure 30, where the abscissas show the current i passing through the helix and the ordinates the corresponding fringe displacements, Δe, in the ocular.

Owing to tremors of the laboratory and possibly also to actual instabilities connected with the magnetization, the results are not smooth. But such disturbances are secondary. As a test of the functioning of the contact lever the data are very satisfactory. Thus it was quite possible to ascertain $\Delta e = 0.1$, *i.e.*, elongations of but $\Delta l/l = 5 \times 10^{-8}$, equivalent to $\Delta l = 2.3 \times 10^{-6}$ cm. It is interesting, therefore, to note that the current must exceed 0.02 ampere before any elongation can be detected. After this, however, the elongations abruptly begin and increase rapidly to a maximum, which is reached before saturation. They then decrease somewhat more slowly and eventually become negative. In the strong fields the contact lever is thrown into violent vibration on closing the circuit, and the reading is less certain.

The next experiments, figure 31, were made with somewhat greater care and with larger fringes. The standardization of the ocular micrometer showed

$$\Delta e = 5.5 \times 10^4 \Delta N \text{ cm. or} \left(\frac{\Delta N}{\Delta e}\right) = 10^{-5} \times 1.82$$

Hence

$$\Delta l/l = 10^{-7} \times 2.16 \Delta e$$

the data being otherwise the same as above. Here

$$\Delta l = 10^{-6} \times 9.3 \Delta e \text{ cm.}$$

The results in figure 31 are smoother than in figure 30, and but for the incidental difficulties mentioned they would probably be quite smooth. The character of both curves is about the same as to maxima and neutral points. The latter maximum corresponds to about

$$\frac{\Delta l}{l} = 10^{-6} \times 2.4$$

(the former being somewhat too large), which is smaller that the values formerly found for pure Swedish iron, as was to be expected.

A number of supplementary experiments were made to see whether the observed $\Delta l = 0$ for currents below 0.02 ampere might not be equivalent to an initial small minimum. But Δl remained *persistently* zero, while currents decreased from 0.02 to 0.001 ampere. At 0.004 ampere the field was reversed, but no significant Δl could be detected. The fringes just moved ($\Delta e = 0.1$) when i was about 0.035 ampere, indicating a field of 3 or 4 gauss.

A rough test made of the equation by pulling the rod rr forward by the backstop-screw M, figure 29, gave corroborative results. We have

$$\Delta l = \frac{r \cos i}{b} \left(\frac{\Delta N}{\Delta e}\right) \Delta e = 0.51 \left(\frac{\Delta N}{\Delta e}\right) \Delta e$$

and if $\Delta\varphi$ refers to the turns of the screw,

$$\Delta l = \frac{r \cos i}{b} \left(\frac{\Delta N}{\Delta e}\right) \left(\frac{\Delta e}{\Delta \varphi}\right) \Delta\varphi$$

$(\Delta N/\Delta e)$ was found to be $10^{-5}\times 3.3$ cm. per scale-part and $(\Delta e/\Delta\varphi)=11$ scale-parts per degree of turn. Hence, with the above data,

$$\Delta l = 0.51\times 10^{-5}\times 3.3\times 11 = 10^{-4}\times 1.8 \text{ cm.}$$

The back-screw having 40 turns to the inch, *i.e.*, a pitch of 0.0635 cm., gives us $10^{-4}\times 1.76$ cm. per degree of turn. This is as close as the observations warranted. The rod must of course be free at both ends, except for the stop-screw and the contact lever. Even in such a case the intermediary rod *rr*, whose end-face is not rigorously true, is not favorable to sharp results.

Another feature may be mentioned here. The expansion of the coil when carrying very large currents is a thrust on the back-stop *M*, which is quite appreciable and appears as an apparent contraction of the rod. The effect is eliminated in the triplets. It was not quite eliminated by the water-jacket, which protects the rod only or chiefly.

16. Vibration telescope.—To test the surmise just stated, the vibration telescope, heretofore described, was installed. It was then found that the even band of fringes, drawn out by the vibrating objective, broke up into strongly sinuous lines on making and particularly on breaking the circuit through the helix. When the circuit was made and broken alternately, the waves separated further into a succession of discontinuous pulses of more than double the amplitudes of the waves. With the field properly adjusted by passing 1.8 to 2.0 amperes through the coil, there was no further observable displacement after the strong wave-lines produced immediately after closing the circuit had subsided.

The question is therefore pertinent whether in continually stronger fields the contraction of length (which follows the expansion) continually increases long after the rod is saturated. I therefore made a large number of measurements in stronger fields with the coil suitably water-cooled. The flow of water produced some slight sharp sinuosities in the otherwise even fringe-band of the vibration telescope, but this was not seriously objectionable. The main results obtained were

$$(\Delta N/\Delta e) = 10^{-5}\times 4.7 \left\{ i = \begin{array}{ll} 4.1 \text{ ampere,} & \Delta e = 10 \text{ scale-parts} \\ 7.4 \text{ ampere,} & 18 \text{ scale-parts} \end{array} \right.$$

The other data for the rod correspond to

$$\Delta l/l = 10^{-7}\times 5.64\Delta e \qquad \text{and} \qquad \Delta l = 10^{-5}\times 2.42\Delta e \text{ cm.}$$

Thus the contraction continually increases long after practical saturation, even in fields approaching 800 gauss. It is therefore desirable to reduce the mean data of figure 31 and the present data to the same scale, and to plot the whole curve.

The results as given in the curve figure 32 are not as smooth as was expected; but observation is difficult, because the heat produced by strong electric currents strikes through the water-jacket when the water-current is slow, and a strong water circulation is apt to shake the fringes abnormally, and therefore even the vibration telescope is in this respect unavailing. One would not be justified in asserting that the observations fall very far off of the mean rate indicated in figure 32, disregarding altogether the possibility of hysteresis. The interesting feature of the diagram is thus the continuance of the contractions even in very strong fields.

17. Theoretical observations.—To account for such a graph as figure 32, as a whole, one may argue that the initial elongations are to be referred to the rotations of the molecular magnets. For these elongations are coextensive in field variations with the marked increase in magnetization. It would then seem further plausible that thereafter the attractions between the oriented molecular magnets may be instanced to account for the persistent contractions in continually increasing fields. Thus it seems worth while to endeavor to ascertain whether such a supposition would conform with any reasonable value of the susceptibility k of the iron, which one may estimate as decreasing from over 100 to less than 10, as the rod approaches saturation ($H=150$ gauss) and to decrease thereafter asymptotically to zero.

If p is the force per square centimeter of section of the rod and E Young's modulus,

$$p=E\frac{\Delta l}{l} \tag{1}$$

regarding the magnetic stress as traction.

Using the expression for the potential of a disk, the magnetic force F, per unit pole in a narrow crevasse normal to F, between molecular layers of magnetic surface density σ is

$$F=4\pi kH+H$$

where k is the susceptibility of the metal and H the exciting field without, so that $\sigma=kH$. Hence the force per cm.^{2} should be $p'=F\sigma$

$$p'=(4\pi k^2+k)H^2 \tag{2}$$

Equating $p=p'$ in equations (1) and (2) modified,

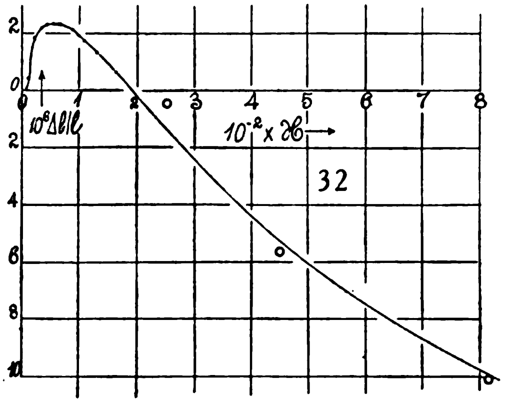
$$\frac{\Delta l}{l}=\frac{4\pi k^2+k}{E}H^2 \tag{3}$$

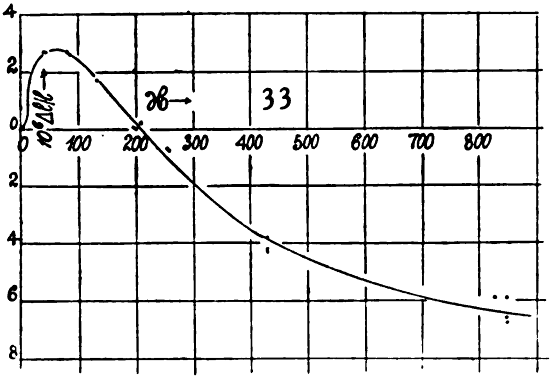
If the data in figure 32 are taken above 200 gauss, supposing that these are far enough removed from the initial complications, the estimates would be ($E=2\times10^{12}$)

H	$10^6\Delta l/l$, (observed)	k
250 gauss	1.0	1.6
400	3.5	1.9
800	11.0	1.6

Similarly, the data for Swedish iron in the next paragraph if treated in the same way, give $k = 1.9$, 1.5, 1.4, in fields of 500, 700, 800 gauss, respectively. The assumption of a mean susceptibility of this order, which seems more than given by the behavior of iron, would suffice to account for the observed contractions.

Hence the estimate of susceptibility obtainable in this way, though treated as a constant, is not of a wholly unreasonable order of mean value. The curve of equation (3) can not, of course, conform with the data of figure 32;





for k is essentially variable and an initial length free from the initial complications must be assumed. Further inquiry will be made in the next paragraph.

18. Further observations.—As there is something objectionable in a result pieced together out of separate observations, the data of table 3 and figure 33 were investigated in a single series. To reduce the heat discrepancy a brisk current of water was passed through the tubular water-jacket. This seemed the safer plan, even though the fringes were shaken. The observations were made in triplets and largely confined to the higher fields.

TABLE 3.—Elongation of an iron rod, length 43 cm.; diameter 0.67 cm. $H = 110i$ gauss. Factor 4.5×10^{-7}.

i	H	Δe	$10^6 \Delta l/l$	i	H	Δe	$10^6 \Delta l/l$
ampere	*gauss*			*ampere*	*gauss*		
0.74	81	6.0	2.7	2.3	253	− 1.5	−0.7
1.90	209	.5	0.2	3.9	429	− 9.4	−4.2
1.20	132	3.7	1.7	7.7	847	−15.0	−6.8
.80	88	6.0	2.7	7.7	847	−13.0	−5.9
.40	44	6.0	2.7	7.7	847	−15.0	−6.8
				3.9	429	− 8.5	−3.8
7.5	825	−13.0	−5.9	1.8	198	− 0.0	−0.0
3.9	429	− 9.5	−4.3	2.3	253	− 1.5	−0.7
1.8	198	− 0.0	−0.0	....	....	....	....

The curve in figure 33 crosses the axis (original length restored) in about the same strength of field as in figure 32. The curve is quite as clearly indicated, as may be expected (owing to the difficulties cited); but the higher observations ($H = 200$) are decidedly smaller contractions than are seen in figure 32. The reason is to be found in the method of observation in triplets, where (curiously enough) the third reading (field zero) is an apparent contraction in relation to the first reading in the absence of the field. The probable cause of this has been suggested above.

Figure 33 seems to indicate that the curve is approaching an asymptote, or, in other words, that the susceptibility implied in equation (3) is vanishing.

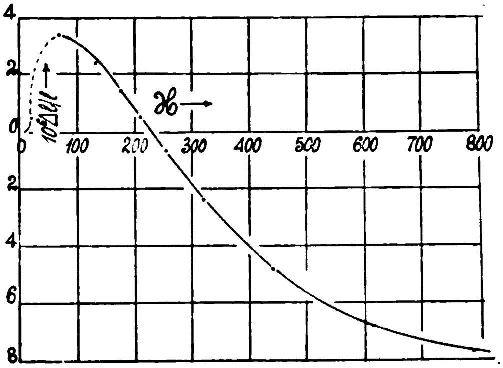


FIG. 34.

The apparatus in the preceding experiments was believed to be faulty in design, inasmuch as the clutch of the contact lever and of the coil were attached to the same rigid standard. This arrangement was therefore modified, so that the two mountings were quite independent, whereupon the anomalous results specified largely receded. The new data did not, however, differ essentially from the old and may therefore be omitted.

As a second test a rod 28 cm. long of Swedish iron was inserted, the extra length being pieced out by brass tubing soldered to each end, so that the iron lay quite within the coil. The data obtained are given in figure 34, chiefly in relation to higher fields and obtained with moderately large fringes. Data of the same kind were also investigated with smaller fringes, but as they were less uniform they, also, may here be omitted. The results of figure 34 for pure iron do not differ as much from the low carbon iron results of figure 33 as was anticipated. In the high fields, moreover, the former show definitely that the contraction has not subsided, though its rate is decreasing. First and third readings of the triplets were usually nearly the same, and but for the vibration of fringes due to the extraneous causes, the results would have been very satisfactory.

With the aid of the results for Swedish iron, equation (3) in paragraph 17 may be resumed. With this end in view, the curve, figure 34, so far as the higher fields ($H > 200$) are concerned, may be roughly reproduced by an equation of the form

$$\Delta l/l = A - B/H^2 \tag{4}$$

The data for the fields $H = 200$ and 600 then give us $A = 10^{-6} \times 7.3$; $B = 0.33$. The equation is not adequate, inasmuch as it either makes the final data too small or the initial data too large. If we accept it, however, in order to determine the order of value of k, we may use equation (3) above and, eliminating $\Delta l/l$ from (3) and (4), compute

$$k^2 = E(A - B/H^2)/4\pi H^2 \tag{5}$$

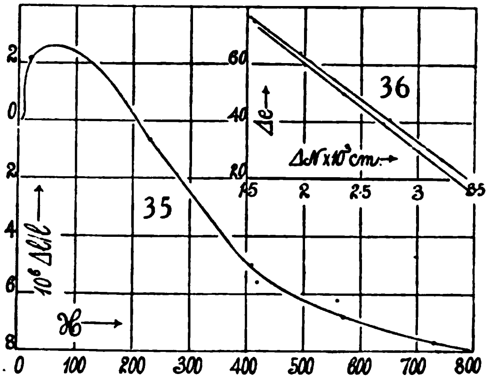
This expression can have a meaning in the higher fields only; but if we use it to find the order of values of k, the results are

$H = 400$	$k = 0.8$
600	.6
800	.4

Hence one may infer from this tentative result that small residual values of the susceptibility of iron, such as probably exist, are sufficient to account for the contractions observed in strong fields.

In addition to iron, a number of other metals, in particular bismuth, were examined by the same method; but no elongation was observable in fields up to 800 gauss. The bismuth rod was 39 cm. long. Large fringes were produced for which $(\Delta N/\Delta e) = 10^{-5} \times 1.86$. This makes $\Delta l/l = 10^{-7} \times 2.43 \Delta e$, the constants of the apparatus being as above. Assuming the $\Delta e = 0.5$ could still be recognized in a field of 800 gauss (the difficulty of observation being due to the rapid heat production by the current), it follows that the expansion must have been less than $\Delta l/l = 10^{-7}$. The same result applies to the other metals.

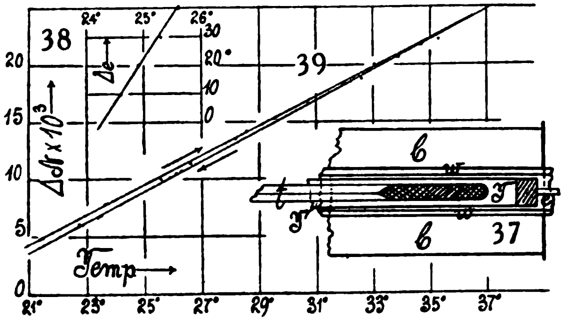
19. Magnetic elongations in a free-end coil.—Profiting by the experience of the preceding paragraph, I reinserted the original low-carbon iron rod and completed a series of data in the larger magnetic fields, the coil being free to expand at the end near the contact lever. These results are given in figure 35. They are not superior to the earlier results, however, as was in a measure to be expected, because all observations were made in triplets from which the thermal discrepancy largely vanishes. Figure 36 shows the relations of the fringe displacement Δe in the ocular to the micrometer displacement ΔN. The two curves were obtained successively and show slightly different rates $10^{-6}\times 3.45$ and $10^{-6}\times 3.33$. The reason of this is to be sought in the change of fringe-breadth if the rod appreciably expands. It also accounts for the difference of contractions in figure 35, found later from those for the same field found earlier in the series; for as the coil and rod inevitably increase in length during the measurements, the fringe-constant of the ocular micrometer changes, and this constant can be found only at the beginning or after the end of the measurement when all temperatures are again constant. The work with Δe should always be confined to the range of the ocular micrometer, which is here much exceeded.



20. Coefficient of expansion.—To arrive at a definite reason for the occurrence of the anomalous contraction mentioned above, it seemed desirable to modify the magnetic apparatus for the purpose of measuring the coefficient of expansion of a given metal tube. This could easily be done by using the coil merely as a heater.

Accordingly a thin-walled brass tube was prepared, effectively 40 cm. long, 7 mm. in diameter, with walls but 0.3 mm. thick. One end of this is shown at *T*, figure 37, inclosed by the water-jacket (now empty) *w*, of the coil *CC*. The tube *T* is closed at this end by a brass plug ending in the stylus *e* (to be made of invar), which actuates the plate of the contact lever, as above explained. The other end of the tube *T* was strengthened by a cylindrical sleeve soldered to it, the latter projecting beyond the coil *CC* and rigidly held by a clutch, the attachment being quite independent of the supports of the coil *CC*, for the reasons already indicated. To measure the temperature of the brass tube *T*, a long (60 cm.) normal thermometer (Baudin) *t*, graduated in tenth degrees, capable of sliding easily from end to end of the tube, was used. The stem projected considerably beyond the left end of *T*, figure 37, so that the whole thread could be left within the tube except during the reading.

The method of observation consisted in passing a current of a few amperes through the coil for a short time, breaking it, and making the observations at the interferometer and thermometer near the period of maximum temperature, after which the coil would begin to cool. These observations should have been made by two observers; but by confining the work to temperatures not more than 20° above atmospheric temperature, it was found possible to take both readings in succession without serious error.



A large number of experiments were made both in terms of the ocular micrometer, Δe (see inset, fig. 38; here the temperature increments must lie within 2°) and by the Fraunhofer micrometer ΔN at the mirror of the interferometer. These very soon showed the reason of the anomalous contraction referred to; for it was found that all the expansions were apt to begin with a contraction immediately after the heating current had been closed. Hence it is due to the initial expansion of the *coil* itself. As this was fastened at its two ends to the heavy wooden table, it seemed clear that the consequent flexure of the table was the ultimate cause of the interferometer discrepancy. The end C of the coil (fig. 37) must therefore be left free to expand, as well as the tube T.

Hence a modification of the apparatus was made by allowing the end C of the coil to recline on a large grooved wheel, which by slight rotation would admit of any expansion of the kind in question. With this improvement the anomalous contractions vanished and the work thereafter proceeded smoothly.

Of the results obtained I shall give only the example in figure 39, which shows the readings ΔN of the micrometer in 10^{-3} cm., at the different temperatures (21° to 35°) given by the abscissas. The locus is fairly linear, but the rate of expansion is slightly greater on cooling than on heating, as shown by the arrows.

If α is the coefficient of linear expansion we may write

$$\alpha\Delta t = \Delta l/l = (r \cos i/bl)\Delta N$$

so that

$$\alpha = \frac{r \cos i}{bl}\frac{\Delta N}{\Delta t}$$

where the increments Δl, Δt, ΔN, of length temperature and micrometer displacement (the latter brings the achromatic fringes back to a fiducial mark in the slit of the collimator) correspond to each other. In the given apparatus $r=7.2$ cm.; $b=9.7$ cm.; $i=45°$; so that the constant is

$$\frac{7.2\times 0.707}{9.7\times 40}=0.0131 \text{ or } \alpha=0.0131(\Delta N/\Delta t)$$

In the outgoing series $\Delta N/\Delta t=0.00125$; or $\alpha=10^{-5}\times 1.64$; in the ingoing or contracting series $\Delta N/\Delta t=0.00130$; or $\alpha=10^{-5}\times 1.70$.

These values are too small, doubtless for reasons connected with the distribution of temperature along the length of the expanding tube. That it was not sufficiently constant was tested by sliding the thermometer along the tube. Virtually a rod 38 cm. long, instead of 40 cm., has been exposed to the given temperature conditions. The two rates are to be similarly explained. The remedy would be a longer rod, or adequate jacketing. It would be well worth while to do this, since the curve figure 39 shows the method itself to be trustworthy. In the present place, however, these experiments are incidental and merely of interest in their bearing on the magnetic work, so that no further improvement was attempted.

CHAPTER IV.

ON THE PRESSURE VARIATION OF SPECIFIC HEAT IN LIQUIDS.

21. Introductory.—The measurement of the specific heat of a liquid in its relation to pressure is surrounded by so many difficulties that any method which gives a fair promise of success deserves to be carefully scrutinized. During the course of my recent work on interferometry I have had this in view, and the plan which the present paper proposes is particularly interesting, as it seems to be quite self-contained.

22. Equations.—If ϑ, p, ρ, c denote the absolute temperature, the pressure, the density, and the specific heat of constant pressure, respectively, of the liquid, and if $\alpha'=(dv/v)/d\vartheta$ is its coefficient of volume expansion, the relation of these quantities may be expressed by the well-known thermodynamic equation

$$d\vartheta/dp=\alpha'\vartheta/J\rho c \tag{1}$$

where J is the mechanical equivalent of heat, and the transformation is along an adiabatic. The main difficulty involved would therefore be the measurement of the temperature increment; for dp could be read off with facility on a Bourdon gage after a partial stroke of the lever of my screw compressor. It is my purpose to find $d\vartheta$ by the displacement interferometer. To fix the ideas, suppose the liquid in question is introduced into a long steel tube TT, figure 40, and that the tubulure p conveys the increments of pressure dp.

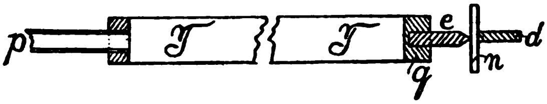


FIG. 40.

This end p is rigidly fixed. The other end q of the tube is free to move. By aid of the stylus e, the elongation is registered on the plate n of a contact lever d, read by interferometry, the lever being identical in construction with that in the apparatus (fig. 29) described in the preceding paper on magnetic elongation. Thus the interferometer will indicate the elongations due both to the pressure increment and to the corresponding temperature increment of the suddenly compressed liquid, and it becomes a question to what degree the two may be adequately separated. If Δl, Δp, $\Delta\vartheta$ are corresponding increments of the length l of the tube and the pressure and temperature of its liquid content, we may write successively, if $\Delta l=\Delta l'+\Delta l''$, etc.,

$$\Delta l'/l=(r_1^2/3k(r_2^2-r_1^2)) \qquad \Delta p=\beta\Delta p, \text{ (say)}, \tag{2}$$

$$\Delta l''/l=\alpha\Delta\vartheta \tag{3}$$

where α is the coefficient of expansion, k the bulk modulus, r_1 and r_2 the internal and external radius of the steel tube of length l. Hence

$$\alpha\Delta\vartheta = \Delta l/l - (r_1^2/3k(r_2^2 - r_1^2))\Delta p \tag{4}$$

and equation (1) becomes

$$\frac{d\vartheta}{dp} = \frac{\Delta l/(l\Delta p) - \beta}{\alpha} = \frac{\alpha'\vartheta}{J\rho c} \tag{5}$$

or

$$c = \frac{\alpha'\vartheta}{J\rho}\,\frac{\alpha'}{\Delta l/(l\Delta p) - \beta} \tag{6}$$

Hence c may be found from observations of Δl and Δp, provided α' and ρ, α, and β are sufficiently known.

23. Measurement of the pressure coefficient β.—For this purpose the tube TT, figure 40, is placed in a water-jacket of constant temperature, and β found directly. Experiments of this kind were contributed with some detail in an earlier paper.* The method then used consisted in finding β from the displacement of the spectrum ellipses under known conditions; but the present method of the contact lever and achromatic fringes may be considered preferable, particularly if the tube contains water (as a conducting medium for pressure), for which the thermal discrepancy is small.

Moreover since $\Delta\vartheta$ is primarily aimed at, β should be made as small as possible. This may be done by selecting relatively thick-walled tubes of small external diameter. A few data are here desirable. Using an ocular micrometer plate 1 cm. long with scale-parts of 0.01 cm. each and fringes of moderate size (1 or 2 scale-parts in width) we may write as in the preceding paper,

$$\Delta l/l = 3\times 10^{-7}\Delta e \tag{7}$$

where Δe is the displacement of the achromatic fringe on the ocular scale corresponding to the elongation Δl.

Hence for steel tubes ($k = 1.8\times 10^{12}$) the following data apply, β being reckoned per atmosphere:

	$2r_1$	$2r_2$	$\beta\times 10^7$	$\Delta e/\Delta p$
I	1.0 cm.	0.9 cm.	8.0	2.7
II	1.0	.8	3.3	1.1
III	.7	.6	5.2	1.7
IV	.7	.5	1.9	.63

24. Measurement of the thermal expansion, α.—For this purpose the tube TT (fig. 40) is to be clean and empty, the nozzle p removed, and a long-stemmed thermometer passed from end to end of the tube, through the end p, as in figure 37. Externally the tube is surrounded by a coil of wire for electric heating and appropriately jacketed. Measurements made in this way with a brass tube are given in figures 38 and 39 and are quite satisfactory, if the tube is properly protected from loss by radiation.

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* Carnegie Inst. Wash. Pub. No. 249, pp. 84–94, 1917.

If for the steel tube $\alpha=10^{-6}\times 12$, equations (3) and (6) then give us

$$\Delta\theta=\frac{3\times 10^{-7}}{12\times 10^{-6}}\Delta e=0.025\Delta e,$$

or about 40 scale-parts of the ocular micrometer per degree of temperature.

25. Available liquids.—It remains to select suitable liquids for experiment. For water at 27°, $\alpha=10^{-6}\times 27$, $c=1$, $\rho=1$, or $d\vartheta/dp=0.0019$; for ethylic alcohol at about 18°, $\alpha=10^{-6}\times 110$, $c=0.58$, $\rho=0.79$, whence $d\vartheta/dp=0.017$; for ether at 18°, $\alpha=10^{-6}\times 163$, $c=0.56$, $\rho=0.72$, whence $d\vartheta/dp=0.028$, pressures being measured in atmospheres. Thus for the four tubes specified in 23, the respective displacements in the ocular micrometer would be (per atmosphere):

I	$\Delta e'=2.7$	water	alcohol	ether
II	1.1	$\Delta e''=0.08$	0.68	1.14
III	1.7			
IV	.6			

The case of water may be dismissed, for here the thermal displacement per atmosphere, $\Delta e''$, is a small fraction of the elastic displacement in the ocular micrometer. But alcohol and ether show satisfactory conditions. Thus a sudden half-turn of the lever of the screw compressor producing 100 atmospheres would displace the fringes, in case of tube III and ether, 173 scale-parts elastically and 114 scale-parts thermally, together 287 scale-parts. Stops of 30 atmospheres would be advisable. Tube IV with 63 (elastic) and 114 (thermal) scale-parts, is even more advantageous. All this implies, however, the adequate absence of thermal conduction from liquid to tube, initially.

It remains to estimate the diminution of $\Delta\vartheta$ owing to the completed partition of heat between the liquid and the tube. If $\Delta\vartheta'$ is the increment of the combined system of liquid and tube, the ratio will be

$$\frac{\Delta\vartheta'}{\Delta\vartheta}=\frac{1}{+\frac{r_2^2-r_1^2}{r_1^2}\,\frac{c'\rho'}{c\rho}}$$

if c' and ρ' are the specific heat and density of the solid. This ratio for the tubes and liquids in §23 and the corrected $\Delta e''$ are as follows:

No.	$2r_1$	$2r_2$	$\Delta e'$	Water		Alcohol		Ether	
				$\Delta\theta'/\Delta\theta$	$\Delta e''$	$\Delta\theta'/\Delta\theta$	$\Delta e''$	$\Delta\theta'/\Delta\theta$	$\Delta e''$
I	0.9	1.0	2.67	0.83	0.07	0.69	0.47	0.66	0.75
II	.8	1.0	1.10	.68	.05	.49	.33	.45	.52
III	.6	.7	1.73	.76	.06	.60	.41	.56	.64
IV	.5	.7	0.63	.55	.04	.36	.24	.33	.38

Tubes of the type I are thus unsatisfactory. In case of tubes of the types II or III the thermal displacement would be but about 3 to 4 per cent of the elastic displacement in case of water; but in case of alcohol 30 and 24 per cent; of ether 47 and 38 per cent. In tubes of the type IV the advantages of thicker walls and small external diameter are further in evidence: alcohol and ether show ratios of 38 and 59 per cent. The problem of selecting the best tube admits of general solution, to which these data contribute.

If we combine equations (4) and (8) and put

$$A=\Delta l/l \qquad B=\Delta p/3k \qquad C=\rho'c'/\rho c \qquad x=(r_2^2-r_1^2)/r_1$$

the result is

$$\alpha\Delta\theta'=\frac{A-B/x}{1+Cx} \tag{9}$$

Here x is the ratio of solid and liquid sections and A the original total elongation. We inquire what value of x will make $\Delta\theta'$ a maximum provided A, B, C are constant. If the original thermal and elastic elongations are to be equal $A=2B$. Differentiating (9) and reducing:

$$\frac{1}{x}=C(\pm\sqrt{1+A/BC}-1) \tag{10}$$

and since x must be positive the radical is positive. Now if $A=2B$, for example

$$\frac{r_1^2}{r_1^2-r_1^2}=\frac{\rho'c'}{\rho c}(\sqrt{1+2\rho c/\rho'c^1}-1) \tag{11}$$

or the ratio of diameters $2r_1$ to $2r_2$ would in all cases have to exceed 0.65. If $A=3B$, the case of water remains nearly the same, but for ether and alcohol the diameter ratio approaches 0.9. Massive tubes implying complications owing to heat conduction are to be avoided so far as possible.

CHAPTER V.

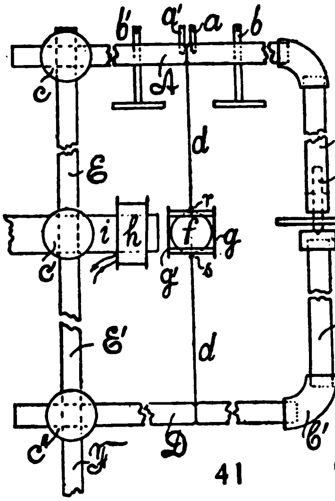
AN ELECTRODYNAMOMETER USING THE VIBRATION TELESCOPE.

26. Introductory.—The present paper is in the main contributory to the much more difficult experiments of the next chapter. The employment of a telescope with a vibrating objective did good service as an aid to the interferometry of vibrating systems. It seemed worth while, therefore, to see what could be got out of it, when used in connection with a telephone only, as a dynamometer. The experiments are of interest both because of the vibratory phenomena observable and in view of the peculiar method of optic observation developed. Its particular use in finding the magnetic field within a helix of any shape, but of unknown constants, deserves mention at the outset. The attempt made in §36 to detect the node in an organ pipe, bolometrically, may also be referred to.

27. Apparatus.—A front view (elevation) of the design is given in figure 41 and an enlarged detail (side view) in figure 42. The apparatus consists of a rigid rectangular framework of 0.25 inch gas-pipe *A*, *B*, *C*, *D*,*EE'*, *F* being the foot attached to a tripod. There may be a steadying foot at *C'*. *A* and *D* are attached to *EE'* by the stout clamps *c*, *c''*, so that *EE'*, lying behind the plane of *ABCD*, admits of the attachment of a suitable clamp *c'*, by which the telephone, *ih*, may be secured in the same plane. *B* and *c* may be forced apart slightly by the screw *n*, controlled by the broad thumb-nut *m*, the conical end of *n* rotating in a socket of the cap *p*.

The vibrating system consists of the bifilar wires of phosphor bronze or steel *dd'*, and the frame or carriage of the lens *f*, which is the movable objective of the telescope *T*, the latter part containing the ocular and a plate micrometer (centimeter divided in 100 parts). *T* may be at a considerable distance (50 cm. or more) from *f*, and supported by a convenient standard. The frame of the lens (which must hold it securely, cement being used if necessary) is of light sheet metal, the parts *gg'* being of sheet-iron (0.05 cm. thick), so as to be attracted by the magnet *i* of the telephone. The stiff cross-wires *r*, *s* of the frame are either soldered to the bifilar system *dd'* or otherwise attached to it (soft sealing-wax does very well for temporary experimental purposes).

The attachment and tension-control of the bifilar suspension is finally to be described, as its period must be synchronized with the alternating current. Results are obtainable only when the two periods are strictly in unison. In figure 41 the wires *dd'* are looped around a groove in the pipe *D* below, and the upper ends of *dd'*, after passing a similar groove in *A*, are bent around the posts *a*, *a'*, and wound respectively around the snugly fitting screws *b*, *b'*, the ends being secured against sliding by a fine hack-saw cut in the screws. To stretch a wire it is passed from the notch in *b* once or twice around it, thence around *a*, downward by the groove to *D* and then up in the corresponding way to *b′*. The lens carriage *gg′* is then attached with cement (after the wires are evenly stretched) with the cross-wires *r*, *s* on the opposite side of *dd′* to the pull of the magnet *i*. The magnet, in addition to the cement, thus guards against slipping. On turning the screws *b* and *b′*, any degree of tension may be imparted to the wires *dd′*, roughly. This simple device worked surprisingly well, and wires of different kinds may be easily inserted or replaced, the lens system being subsequently attached with cement; but it is better to loop the lower part of *dd′* around a special roller *G*, as indicated in figure 42 and used in my later tests, with the object of more easily reaching an equality of tension in the wires *d* and *d′*. The tensions are then roughly changed by the screw and nut *u*.



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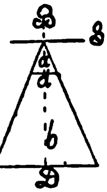

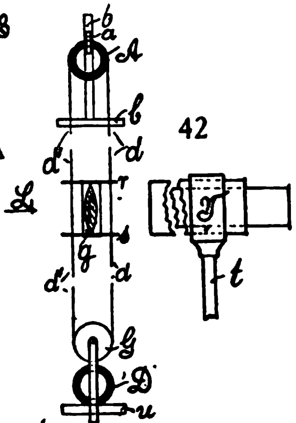

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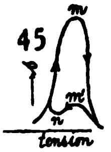

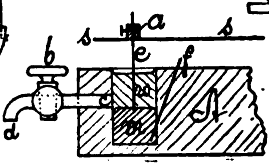

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The approximate tension having thus been obtained by the screws *b*, *b′*, or *u*, the finer variations are imparted by the screw *mn* which flexes the elastic rectangle *ABCDE* and thus gives to the bifilars *dd′* exactly the tension required. It is at the thumb-nut *m* that all adjustment is made during observation.

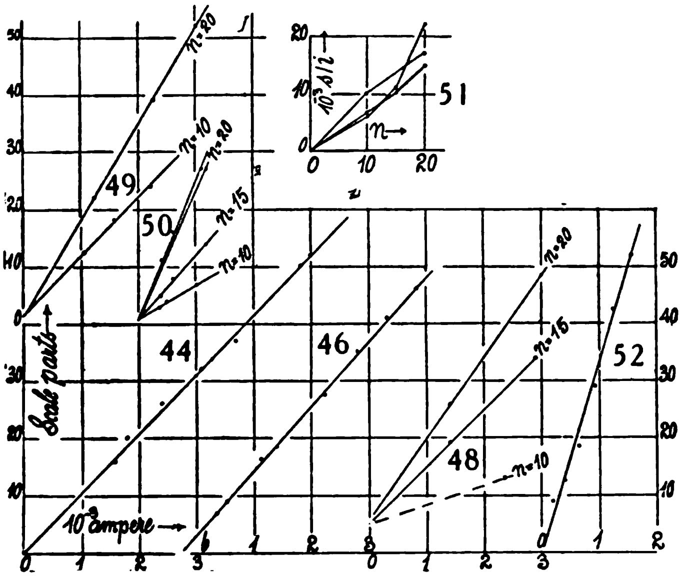
In my apparatus the rectangle was about 50 cm. long and 12 cm. wide. The wires *dd′* about 1.5 cm. apart and each about 0.025 mm. in diameter. Wires as thick as this require sharp adjustment as to tension, but they obviate, when tense, all objectionable quiver and the method given proved quite satisfactory. The tension is sufficient to admit of an air-gap of less than a millimeter between the plate *g′* and the magnet *i* of the telephone. Later, in the interest of sensitiveness, the telephone *hi* was also put on a stout spring micrometer-screw system so that the distance *ig′* could be nicely regulated.

As a source of light a distant vertical electric filament or a slit and collimator suggests itself, so that a fine line may be seen in the ocular, normal to the scale. In the case of parallel rays, however, the displacement of the image in the ocular would be no larger than the displacement of the objective, *f*, figure 41. To obtain increased displacement, the method of figure 43 is available, where *S* is the fine slit in front of a Welsbach burner at *B*. At *d* is the principal plane of the vibrating objective and at *D* the micrometer plate in the ocular. Again, if the length *d* represents the double amplitude of the objective and the sides of the triangle be drawn from *S*, the intercept *D* will represent the displacement in the ocular. If the distance *Sd* be *a* and *dD*, *b*, we may write

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f} \tag{1}$$

where *f* is the principal focal distance of the objective. Hence

$$\frac{D}{d}=\frac{b}{f} \tag{2}$$



Theoretically, therefore, any degree of magnification is possible by increasing *b* (the distance of *T* and *f*, fig. 42) and decreasing *f*. In the former case, some means of controlling the thumb-nut *m*, figure 41, from a distance would have to be provided. In the latter the lens *f* would have to be achromatic. In the present experiments I first used an ordinary spectacle lens at *f* with a slotted screen between it and the slit to diminish chromatic aberration; but there is no objection to the use of an achromatic lens at *f*, as I did later, particularly since a lens-breadth of a few millimeters will suffice; for there is an abundance of light.

To measure the width of the band of light produced by the vibration, the ocular *T* may be on an axle *t*, with slight friction. The zero of the micrometer scale may then be brought to coincide with one edge of the band by rotation on t and the width of the band read off at once on the micrometer scale, though it is probably as easy to read off both ends. In my earlier improvised apparatus, single scale-parts (0.1 mm.) only could be guaranteed; but with a perfected optical system there is no reason why tenths of scale-parts should not be equally trustworthy. This makes a scale of 1,000 parts in the ocular.

28. Observations.—There is sometimes difficulty in finding the coarse adjustment for resonance, unless the design of figure 42 is adopted. After this the adjustment at m is easy. As a source of alternating current, I selected a small induction coil with a mercury break (Kohlrausch's design) to facilitate the initial tests. This was put in series with a rheostat (to 30,000 ohms), a Siemens precision dynamometer reading to just within milliamperes, an ordinary telephone to indicate the continuous action of the coil, and the vibrator above described.

The Siemens dynamometer was first standardized with a Clarke cell. Accepting the effective current i as being

$$i=C\sqrt{\varphi}$$

where φ is the deflection on a scale at about 1 meter of distance, the constant $C=1.12\times 10^{-3}$ relative to amperes was found and the mean resistance of the coils included about 250 ohms. Virtual currents of 10^{-4} ampere would escape detection.

The coil was now started and measurements made simultaneously both at the Siemens dynamometer and at the vibrating telephone (slit distance $a=35$ cm., ocular distance $b=75$ cm.), with results an example of which is given in the diagram (fig. 44), the virtual currents of the dynamometer (abscissas, milliamperes) being compared with the width of the image band (in scale-parts, 0.01 cm.) seen in the vibration telescope. The relation of the two is as closely linear as the observations warrant, seeing that the optical system (spectacle lens) was imperfect and the currents of the induction coil essentially not quite uniform. In fact, it is an encouraging feature that the relation is so nearly linear, even within the limits of observation. To obtain these different virtual currents, resistances from 10,000 to 2,000 ohms were put into the circuit. The dynamometer showed deflections from 2 to 20 cm. on the scale. For larger deflections the coil current would have been too irregular for use.

Figure 44 shows that the vibrator indicates about 10 scale-parts per milliampere, that readings beyond 5 milliamperes would be possible, that the deflections are fairly proportional to the current, and that with a good optical system virtual currents as small as 10^{-5} ampere should have been perceptible. Judging from the inclosed telephone, I should estimate the currents of an ordinary voice at about 1 scale-part. The apparatus is thus very well worth further development and would be particularly useful where alternators with a definite period are in question.

Throughout these measurements a variety of interesting observations were made. It is obvious that current must not be increased until, with the approach of g to i, the vibration becomes unstable or shows a tendency to become torsional. The rather thick wires used are an advantage in this respect, as they prevent any sticking of the lens-carriage to the magnet and serious tremor during manipulation.

The method of observation consists in gradually increasing the tension of the wire from a slightly *low* value to beyond the maximum tension. In this case the band widens from a relatively small width to the maximum and then (as a rule) abruptly falls off to a small value. To repeat the observation the tension must often again be reduced to the low value and the whole operation repeated; but after some practice, maxima may be reached in the reverse direction unless the image band is too far spent and narrow.

When the current is broken and thereafter closed, a low width of band is obtained which will not usually widen unless the operation described is carried out from low tension. In other words, there are two cases of equilibrium for each current, corresponding to very different image-band widths. This is a curious result, for it means, virtually, that the magnetic forces and the stresses are in the relation of a doubly inflected curve to each other, so that there are three intersections, two for stable vibratory equilibrium; or else the two harmonic systems, the electrical and the mechanical, may vibrate in the same or in opposed phases, with the latter preferred. Figure 45 gives a diagrammatic comparison of the tensions and the band-widths, s; the maxima are at m and m'. The arrows indicate the cyclic result of change of tension.

Similarly each current requires its own particular maximum tension, which increases with the current. The difference is not large, but very operative, and for this reason the fine tension adjustment is essential.

29. Further observations.—The apparatus was then improved in a variety of ways, chiefly by the insertion of a small vibration objective about 1 cm. in diameter, achromatic and with a focal distance of but 5.8 cm. In this case the distance a and b could be decreased to 7 cm. and 35 cm. and the observer was thus conveniently near the adjusting-screw. The slit-image was white and about a scale-part in width. There would have been no difficulty in using much greater magnification.

TABLE 4.—Comparison of Siemens (φ) and telephonic dynamometer. $C = 10^{-3} \times 0.87$; $r = 710$ ohms; $i = C\sqrt{\varphi}$; $a = 7$ cm.; $b = 35$ cm.; $f = 5.8$ cm. Phosphor-bronze wires 50 cm. long; 0.025 cm. in diameter.

$R \times 10^{-3}$ ohms.	Siemens		Telephone s	$(R+r) \times i$	$10^{-3}L\omega$
	φ	$i \times 10^3$			
	cm.	*ampere*			
2	19.5	3.84	46	10.4	0.8
3	10.5	2.82	35	10.4	1.4
4	6.6	2.24	27	10.5	2.1
5	4.6	1.86	23	10.6	2.2
7	2.6	1.40	18	10.8	...
9	1.7	1.13	16	11.0	...
20	0.38	.54	9	11.2	...
30	0.17	.36	7	11.1	...

The results are given in table 4 and figure 46. The constant of the dynamometer was now $C = 10^{-3} \times 0.87$ relative to amperes and the total resistance in circuit 710 ohms. The frequency was the same as before. The same bifilar wires (phosphor bronze, 0.025 cm. in diameter and 50 cm. long) sustained the vibrations.

The results are a considerable improvement on the preceding and the discrepancies as a rule lie within 5×10^{-5} ampere. They are much more liable to be in the dynamometer than in the vibrator, as the former was not well adapted for these small currents. Curiously enough, the deflections begin with 3 scale-parts and not at zero. As the slit-image was about 1 scale-part broad, it is difficult to assign a reason for this. Like the slit-breadth, however, it appears merely as an initial constant and is thus not of much consequence.

If we compute the coefficient of induction as

$$L^2\omega^2 = \Delta(i^2(R+r)^2)/\Delta i^2$$

Δ being a differential symbol) from the first and fifth, second and sixth, etc., observations, the results for $L\omega$ are given in the corresponding column. L increases as if the resistances 2,000 to 30,000 ohms were not induction-free, implying a larger L at the higher resistances; but as the current is not harmonic, the reason may have to be sought elsewhere. After 10,000 ohms (or even below) the induction effect is practically negligible in comparison with the resistance, as appears from the column for $(R+r)i = E$. Hence the effective E is constant and about 11 volts, implying a maximum voltage of 15 volts.

30. Effect of frequency.—A special mercury interruptor was now made as shown in figure 47, having as its distinctive feature contrivances by which the mercury surface could always be kept clean and bright and furthermore adapted to give different frequencies. This consisted of a block of wood *A*, figure 47, impregnated with resinous cement, with a vertical hole to receive the mercury *m*, distilled water *w*, and the vibrator, *eas*. The spring *s* was actuated as usual by an electro-magnet (not shown) and the terminals *e*, *f*, for passing current through the mercury were of platinum, *e* being adjustable at the clamp *a*. In view of the glass stopcock *b* and the pipe *c d*, the water *w* could be withdrawn whenever desirable and the mercury surface washed by aid of a small wash-bottle. The apparatus functioned admirably for days, frequent washing presupposed. Different frequencies were obtained by sliding a weight along *ss*. These were estimated from the moments of inertia as $n = 10$, 15, and 20. The latter could just be counted in groups of 4 vibrations with a stop-watch. Higher frequencies were obtainable by using a stiffer spring *ss*.

The results obtained with this apparatus are shown in figures 48, 49, and 50, for the phosphor-bronze bifilar differently stretched. All give evidence (fig. 51) of the peculiar fact that the sensitiveness increases in marked degree with the frequency. In figures 48 and 49 the telephone used was the original one consisting of a file-blade and appropriate bobbin. The sensitiveness for the three frequencies is respectively in ocular scale-parts s_0 per milliampere.

	n	s_0
Fig. 48	20	14.8
	15	10.0
	10	6.5
Fig. 49	20	17
	15	....
	10	10

The apparatus was now modified by inserting a stronger (commercial) telephone. In this case the sensitiveness was so far increased that only currents not much exceeding milliamperes could be measured within the range of the ocular micrometer. The sensitiveness s_0 is

	n	s_0
Fig. 50	20	22
	15	11
	10	6

In endeavoring to reach still higher frequencies, the phosphor-bronze wires broke under the strain.

It is difficult to account for this effect of frequency, so peculiarly marked in the last instance, where the observations were very good. If different harmonics are in action in case of the separate frequencies, the overtones would have to respond in the case of the wires under less tension, and this seems to be anomalous. Nevertheless, it is often possible to detect two cases of resonance at different tensions, which I hold to be the fundamental, and the octave, the latter obtainable for very low tension. Some suggestions will be given in the two succeeding paragraphs, moreover. As the vibrators, interruptor, etc., are damped systems, the periods will vary with their amplitudes, so far as resonance is concerned.

31. Steel wires.—A number of trials were made with the bronze wires to secure higher tension, but without avail. A steel wire was therefore inserted, 0.036 cm. in diameter, and the spring of the interruptor stiffened. Curiously enough, the results with steel proved to be the reverse of the results with bronze wire. In case of frequencies estimated in the ratio of 20 and 40 per second, the sensitiveness was but 19 and 10 scale-parts per milliampere. With the tense wire but a single harmonic could be found. The less tense wire, however, responded once again on further decreasing the tension, with an increase of sensitiveness of about 26 scale-parts per milliampere.

In the work with these stiff wires, the tension is difficult to control to the nicety required. One should expect a greater amplitude for the less tense wire, as here found.

A thin steel wire 0.015 cm. in diameter was also tried in contrast. Though it showed considerable sensitiveness (up to 30 scale-parts per milliampere), the system was inconveniently subject to tremor during adjustment.

32. Adjustable telephone.—The question as to the most advantageous position of the telephone is important. Consequently the telephone *ih*, figure 41, was mounted on a stout flat steel spring controlled by a micrometer-screw. By actuating this the face of the magnet could be approached as near g' as permissible or withdrawn to a remoter position, with precision. The experiments made at length showed that within a wide range of tensions and of telephone positions a particular degree of approach of the telephone corresponded to each particular stress of wire. Unless these paired positions are selected, the bifilar system does not respond. Tense wires require a nearer telephone; less tense wires a more remote telephone, within wide limits. Therefore the condition of resonance may be reached either by adjusting the telephone on the micrometer-screw for a given tension of wire or by changing the tension of the wires for a fixed telephone. Curiously enough, there is no marked difference of sensitiveness within the range in question. Naturally, if the wires are too loose or the telephone too remote there will be no response. It is not, therefore, possible to increase the sensitiveness by approaching the telephone magnet to the armature as one would naturally suppose. If resonance is reached the position of the telephone is rather a matter of indifference. It is very important, however, that the tension of both wires is the same; otherwise there is reduced sensitiveness.

If the lens carriage gg', figure 41, is loaded appropriately, the sensitiveness may be increased. The tension of the wires of the loaded system is increased until resonance appears. On the other hand, such a system is apt to be annoyingly subject to tremors, particularly during adjustment, and the maxima are reached more slowly. A light carriage is thus in general preferable, unless very low frequencies are to be matched.

With a distance of not more than 50 cm. between slit and ocular, a judicious disposition of parts eventually gave me 40 ocular scale-parts per virtual milliampere, viz:

n	$i \times 10^3$	$s \times 10^2$ cm.
15	0.67	27
20	.67	27

so that here the above effect of frequency, n, is no longer apparent.

Carrying the telephone from a remote distance (say 0.5 cm.) to the nearest distance compatible with a free vibrating system (this eventually sticks to the magnet), the deflection s in a given case increased from about $s=16$ through 21 then falling to 7 for a very tense wire. The maximum is flat. Tight wires require very accurate adjustment as to tension, but the full deflection builds up very slowly in view of the remoter telephone. The system is more subject to shaking during adjustment.

33. Coil tester.—An interesting application of the above apparatus, where a definite frequency is usually assigned, is its possible use for measuring the magnetic fields of different coils. For this purpose I wound a long, thin solenoid or primary of an induction coil, which when inserted into the coil to be tested, should, from the measurement of the current induced in the secondary in question and in the absence of other mutual inductions, give the constants of the secondary. As many of the coils to be tested were of small internal diameter, the primary was wound on a long, thin iron tube, fine wire being necessary. The dimensions were for the iron tube: diameter outside, 0.635 cm.; inside, 0.470 cm.; length, 55 cm.; walls, 0.08 cm. thick. For the helix: diameter outside, 0.7 cm.; wire, 0.034 cm. in diameter; $n_1/l_1 = 21$ turns per linear centimeter.

If L is the coefficient of induction per turn of primary, the total induction is

$$B = Ln_1 i \tag{1}$$

Hence the electromotive force induced in the secondary becomes

$$e = Ln_1 n_2 (di_1/dt) \tag{2}$$

If the field of the secondary per unit current is put

$$H_2 = 4\pi n_2/l_2 \tag{3}$$

for n_2/l_2 turns per linear centimeter, and r_2 the resistance of the coil and its circuit, we may compare any two coils by the equation

$$\frac{e_2}{e'_2} = \frac{n_2}{n'_2} = \frac{H_2 l_2}{H'_2 l'_2} = \frac{i_2 r_2}{i'_2 r'_2} \tag{4}$$

e_2 and e'_2 being the electromotive forces, and i_2, i_2' the currents induced in the two secondaries in question. Thus

$$\frac{H_2}{H'_2} = \frac{i_2 r_2/l_2}{i'_2 r'_2/l'_2} = \frac{s_2 r_2/l_2}{s'_2 r'_2/l'_2} \tag{5}$$

A measurement of s and the *total* resistance of the secondary circuit should therefore give us the unit field within, when the winding is unknown. It is assumed that the resistance r_2 added to the secondary circuit is so large that the inductive resistance vanishes, so that $r_2 = r'_2$.

Thrusting this coil tester or primary in the helix No. D heretofore used ($H_0 = 113i$) the results of figure 52 were obtained. R is the resistance added to that of the coil and the inductive resistance, both of which are small relative to R.

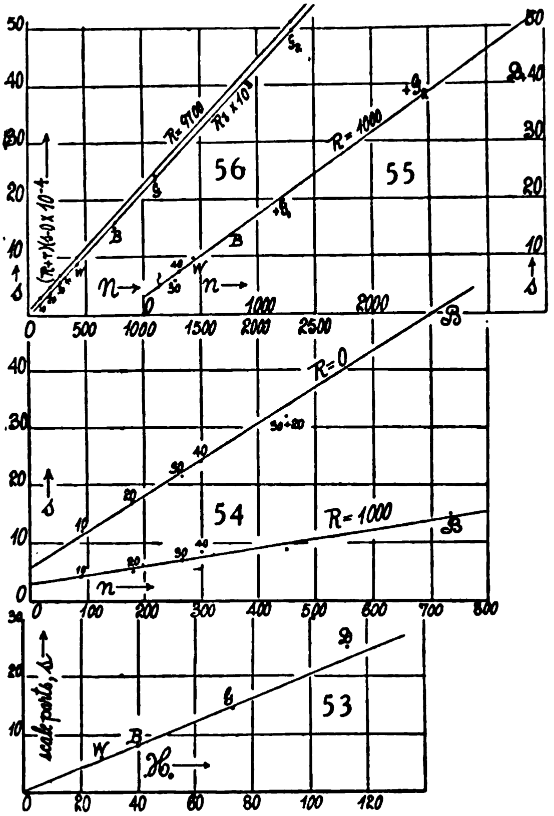
Unfortunately, the constant resistance of the circuit (about $r = 540$ ohms) was not carefully taken; but the products $(R+r)\ i$, show that with an additional resistance above 1,000 ohms the inductions may be disregarded.

The coil-tester was now thrust through a variety of helices, as follows: (l length of helix, d internal diameter, n/l turns per cm. of length per layer, S number of layers, d diameter of wire):

No.	l	d	n/l	S	d	H
D	37 cm.	1.5 cm.	11.2	8	0.08 cm.	113
G	20	2.5	5.8	10	.07	73
B	24	1.4	10.6	3	.08	40
W	19.5	3.0	5.5	4	.095	27

The last column shows the ratio of the field within and the current, i, in amperes, if $H_0 = 4\pi n/l$ is considered sufficient.

Figure 53 gives the results from observations read off for these coils at the dynamometer (φ) and at the vibrator (s), when different resistances R were added to the circuit. Ri was practically the same both for $R = 1{,}000$ and 2,000 ohms, to the degree in which i is measurable by the Siemens apparatus. Moreover, the independent relation of H_0 and s as seen in figure 53, is nearly linear for W, B, G. The data are very much more satisfactory than I had ventured to expect, except for the case of helix D.



These coils were wound on non-conductors or split brass and of about the same length. Coil D, however, was wound on a rather thick brass tube without a longitudinal fissure. Hence it is probably the mutual induction which accounts for the low value of s/l for this coil, which was about 1.8 times longer than the others.

To test these questions further I wound coils on glass tubes as follows, to be compared with B already treated above:

No.	l	Layers	d_i	d_w	n/l	n	H_0
10	9.8 cm.	1	1.2 cm.	0.08 cm.	9.1	89	11.4
20	19.7	1	1.2 cm.	.08	9.2	181	11.6
30	29.0	1	1.2 cm.	.08	9.2	267	11.6
40	39.0	1	1.2 cm.	.10	7.7	300	9.7
(B)	23.7	3	......	......	31.2	735	40

H refers to amperes. The Siemens dynamometer, with its large resistance (480 ohms) was excluded. The circuit resistance was now about 180 ohms.

The data of s and number of turns n_1 found for an inclosed resistance of $R=1000$ ohms and $R=0$ are given in figure 54. They are mean results, as the current fluctuated for incidental reasons difficult to enumerate. This was particularly the case for coil B. When newly inserted ($R=0$) the band-width was apt to be above $s=55$; but after long vibration it fell off to $s=40$ or even less. Fresh washing of the interruptor made no difference.

As this discrepancy disappears in the lapse of time, if the coil is not used, and as it is present markedly only in connection with the larger coils, it is probably a temperature effect resulting from heating the coils of the bobbin of the telephone.

Figure 54 shows that for $R=1,000$ ohms additional resistance, the vibrator indicates 0.016 scale-part per turn of wire in the secondary; for $R=0$ the datum is 0.063 scale-part per turn. In each case, however, the graph which is fairly linear, begins with an initial s of 3 and 5 scale-parts, respectively. This recalls the similar experiences above. It can not be initial band-width, as this did not exceed 1 scale-part.

34. Heavier armature and less damping.—With the observer still within easy reach of the apparatus, a further increase of sensitiveness may be gained by removing the iron plates gg' of the lens carriage (figs. 41, 42) and soldering to it on the side of the telephone i, a heavier piece of soft iron (about a square centimeter in area and 0.2 cm. thick). The maximum band-width was reached about as soon as before and the instrument was not much more subject to tremors during manipulation. It was curious to observe that when the current is closed any existent band-width sometimes expands further, while at other times it first closes and then expands, depending on the phase in which it happens to be caught. Having been adjusted for resonance at the beginning of the work, the wires were thereafter left without further interference, assuming that the same resonance adjustment belongs to each observation. Though this is not strictly true, it greatly facilitates the measurements. With the new apparatus, moreover, the cyclic phenomena of figure 45 were not noticed.

In this way the sensitiveness was about doubled, as compared with the former values, though in the repetition of the experiments with coils the band-width of about $s=30$ scale-parts per milliampere was not exceeded. The results of the new tests of the coils 30, 40, W, B, G, $2G$, and D, are given in figure 55, the resistances inserted being respectively 1,000 and 2,000 ohms, while the resistance of the circuit itself (in the absence of the Siemens dynamometer) did not exceed 200 ohms. The G coil contained two helices wound side by side, which here could be used in parallel and in series. The curve for 1,000 ohms is fairly linear (without beginning at the origin, however), with the exception of the datum for the D coil, which is small for the reasons given above. The series result $2G$ exceeds the individual results for G. At 2,000 ohms the graph was less satisfactory and, even if the D coil is excluded, could not be considered straight. One reason for this is the fact that but a single adjustment for resonance was made at the beginning and not repeated for each individual coil; another is the decrease of sensitiveness in the lapse of time, attributed above to temperature, so that initial results are always relatively large.

35. Further magnification.—To improve the apparatus further in the way indicated in §34, it would be necessary to put the vibrating system in vacuo, in order to reduce the damping coefficient. This would have been inconvenient with the present form of apparatus, though steps will be taken toward this end later.

I therefore proceeded to increase the magnification, using a small microscope objective about one-eighth inch in diameter and weighing but 1 or 2 grams. Its focal distance was 2 cm. The carriage, etc., were of form already discussed. Some difficulty was experienced in finding a suitable kind of slit. Fine lines ruled on smoked glass, bon ami glass, photographic plate, etc., were much too wide and irregular, subtending about 10 scale-parts in the ocular. Similarly a quartz fiber, seen on a field of transmitted light, was 5 scale-parts broad. The same fiber seen in a relatively dark field and illuminated by *side light*, gave a bright line, not more than 1 scale-part broad, and was quite satisfactory. The ocular of the telescope was placed at about 40 cm. from the objective. Using one helix of the G coil, and a total resistance of 2,700 ohms, the Siemens dynamometer showed a deflection of 0.3 cm. and therefore a current of $10^{-3} \times 0.41$ ampere. This current was now weakened by a total resistance of 30,000 ohms when the vibrator still recorded 10 scale-parts. Thus it follows (the inductive resistances being negligible) that a single scale-part corresponded to about 5×10^{-6} ampere, which is equivalent to 200 scale-parts per milliampere. The band admitted of an extension to 50 to 1,000 scale-parts before seriously losing demarcation, and there was no greater difficulty in making the adjustment than heretofore. In a non-exhausted environment and close ocular this seems to be the limiting performance.

TABLE 5.—Deflections s of dynamometer. Microscope objective; $C = 0.75 \times 10^{-3}$; $r = 700$ ohms; $s_0 = 1$.

Coil No.	No. of turns n	$R \times 10^{-3}$	s	10^{-3} $(R+r)(S-1)$	$e = i(r+R)$	e per turn
		ohms			*volt*	
10	89	9	3	19.4	0.093	0.00105
		4	5	18.8		
		2	8	18.9		
		1	12	18.7		
		0	28	18.9		
20	181	9	5	38.8	0.191	0.00106
		4	9	37.6		
30	267	9	7	58.2	0.286	0.00107
		4	13	56.4		
40	300	4	14	63.4	0.37	0.00116
W	419	9	10	87.3	.44	.00105
		19	5	88.6		
B	754	19	9	158.	0.75	0.00100
		9	16	145.		
G_1	1150	29	9	238	1.18	0.00103
G_1+G_2	2300	29	18	505	2.50	0.00109

I now used this vibrator in connection with the coil-tester (§33, primary) obtaining trustworthy results throughout. The data (table 5) obtained for the additional resistance of 9,000 ohms (resistance of circuit, 700 ohms) are shown in the upper curve of figure 56. A few of them (40, *B*) are low, due to incidental reasons, but the line as a whole indicates a quiet band-width of 1 scale-part. Hence if the inductive resistance is relatively negligible,

$$(R+700)\ (s-1)$$

is the effective voltage and should be constant for each coil, increasing from coil to coil with *n*, the number of turns. The result is shown in the lower curve, figure 56, which is quite as nearly straight as the inevitable inaccuracies (*s* data too small, interrupted induction, irregularities in winding, insufficiencies of the elementary equation) can possibly permit one to expect. The current in *B* at 9,000 ohms could just be detected in the dynamometer and could not have exceeded 7×10^{-5} ampere, so that $s=1$ is again equivalent to less than 5×10^{-6} ampere, as before. A fraction of this was quite observable.

36. Organ-pipe.—It seemed worth while (in connection with the present dynamometer) to ascertain whether the node in an open organ-pipe could be detected by bolometric measurement of the temperature alternations adiabatically produced at the node of the sounding-pipe. To test this a c-pipe, 4 feet long and about a square decimeter in section, giving a frequency of 130 per second, was harnessed to an appropriate blower, avoiding overtones. The first bolometer was a grid of fine platinum wire 0.003 cm. in diameter and in all about 48 cm. long, stretched out on a square frame of wood. By aid of a stem attached to the latter, the grid could be lowered at pleasure into the organ-pipe. The bolometer was put in circuit with a storage-cell, an ammeter, a telephone, and a key. The resistance of the grid was about 56 ohms and that of the telephone 87 ohms.

The bolometer was now supplied with current up to 0.16 ampere, but no note whatever could be obtained from it. At times high-pitched noises (probably of microphonic origin) were heard, but nothing else.

The platinum wire bolometer was now replaced by one of gold leaf, the part between the terminals on the frame being about 1 cm. long and 1 cm. broad. Contrary to my expectations, the resistance was here unfavorably low, not exceeding a few ohms as compared with the 87 ohms in the telephone. On lowering the bolometer into the organ-pipe and closing the circuit (currents up to 0.16 ampere) a loud clatter was always apparent, but no certain evidence of the note c. I heard microphonic noises, often very loud, due to the wind-currents. Cutting down the breadth of the gold foil to a few millimeters made no difference of consequence. A similar experiment was then tried with very thin silver foil with the same negative results.

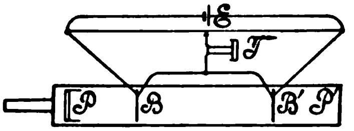


Fig. 57.

A more promising form of the experiment was for the present postponed, because of the noise of the pipe, which would here have to be blown hard to bring out the first overtone. In this case, figure 57, there are two nodes at

B and *B'*, alternately dense and rare. Hence two bolometers, *B* and *B'*, within the pipe *PP'* can now be joined to form a Wheatstone bridge, as indicated in the figure, where *E* is the battery and *T* the telephone or vibrator.

This, therefore, corresponds to the usual bolometric adjustment and may be pushed to much greater sensitiveness, particularly if the nearly equal resistance in *B* and *B'* are unavoidably small.

37. Equations.—If τ denotes absolute temperature, p pressure, r resistance, and i electric current, we should have in succession

$$\frac{d\tau}{\tau}=\frac{k-c}{k}\,\frac{dp}{p}=0.29\,\frac{dp}{p} \tag{1}$$

where k and c are the specific heats of air;

$$\frac{dr}{r}=\frac{d\tau}{\tau}=0.29\,\frac{dp}{p} \tag{2}$$

if the electrical temperature coefficient of the wire is the same as the coefficient of expansion of a gas. Thus finally, by Ohm's law for large non-inductive resistances,

$$\frac{di}{i}=-\frac{dr}{r}=\frac{d\tau}{\tau}=0.29\,\frac{dp}{p}$$

or

$$\frac{dp}{p}=3.5\,\frac{di}{i} \tag{3}$$

This assumes that all the resistance is in the bolometer. As this is not the case, let R be the other resistances (telephone). Then

$$\frac{di}{i}=-\frac{r}{R+r}\,dr/r=0.29\,\frac{r}{R+r}\,\frac{dp}{p} \tag{4}$$

or

$$\frac{dp}{p}=3.5\,\frac{R+r}{r}\,\frac{di}{i} \tag{5}$$

Thus if $di=10^{-7}$ ampere can still be heard in the telephone and the total current is $i=0.1$ ampere, and if $(R+r)/r=2$

$$\frac{dp}{p}=3.5\times2\times\frac{10^{-7}}{10^{-1}}=7\times10^{-6}$$

so that $dp=10^{6}\times7\times10^{-6}=7$ dyne per square centimeter. This is the virtual pressure increment, the maximum being $\sqrt{2}$ larger. At all events, as the actual acoustic dp is much below this, it would imply a virtual acoustic energy of less than 7 ergs per cubic centimeter at the node.

Lord Rayleigh* finds 42.1 ergs/sec. issuing from a tuning-fork just audible. This is equivalent to an energy residence of $42.1/33{,}100=1.3\times10^{-3}$ erg per cubic centimeter. Again, according to Rayleigh's later estimate, the $dp/p=6\times10^{-9}$ is at the beginning of audibility. Hence the above excessive superior limit would be over 1,000 times larger.

Two reasons thus suggest themselves for the failure of the present experiments. Either the note at the node is not loud enough to produce an impression sufficient to actuate the telephone, or the alternations of pressure (130 per second) are too rapid to enter the wire or foil appreciably as a heat current, even when this wire or foil is the thinnest available.

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* Rayleigh, Phil. Mag., 1894, xxxviii, p. 369.

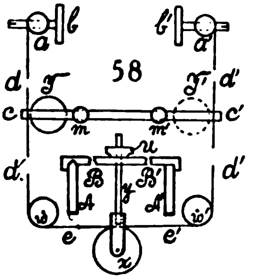
CHAPTER VI.

THE RAPID TELEPHONIC VIBRATOR ON THE INTERFEROMETER.

38. Introductory.—The preceding apparatus,* with telescopic or microscopic enlargement of the telephonic vibrations, behaved on the whole so satisfactorily that it seemed worth while to try a similar design on the interferometer. I was inclined to doubt the feasibility of the plan; but it appeared on trial that the high-tension wires actually keep the auxiliary mirrors of the interferometer practically quiet, so that in the absence of alternating current it is not difficult to find the fringes. Tense wires are out of step with the usual laboratory tremors. The system needs no special damping.

The displacement of the achromatic fringes due to induced secondary current is normal to their direction. The objective of the vibration telescope, oscillating in the direction of the fringes, is to be coupled with the impressed alternating voltage. In such a case a full account of changes of phase evoked in any galvanic system under examination may be inferred from the fringe ellipses obtained in the interferometer. The interpretations will therefore be facilitated if a diagram like figure 72 (p. 66), for instance, is kept at hand.

39. Apparatus.—This is in large measure a modification of the apparatus described heretofore, except that special attachments have been added for sharply reaching the resonance tension of the wire. The latter is shown at *d*, *e*, *e'*, *d'*, in figure 58 (front elevation), being the thinnest steel music wire,



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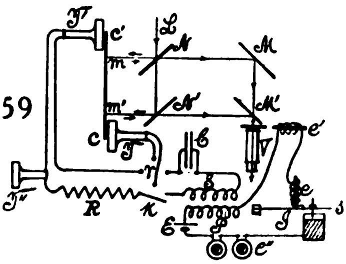

about 0.023 cm. in diameter. Its ends are wound around the stiff screws *b*, *b'*, provided with lock-nuts, and rotating in horizontal short, strong rods *a*, *a'*, attached to stout standards (not shown) fixed to the bed-plate *AB* of the interferometer. The wire *dd'* passes around the grooved pulleys *w*, *w'* and above the grooved pulley *x*, carried in a fork and screw-stem *y*. The latter may be raised or lowered by the bolt *u*, which rests upon the massive carriage *BB*, supported by the slides *AA'* of the apparatus. Provision must be made (slotted sheath and pins, not shown) to prevent *y* from turning on its axis. Tension is roughly given to the wire at the screws *b*, *b'*, and the fine adjustment is thereafter made at the nut *u*. This worked very satisfactorily.

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* P. N. A. S., IV, 328–333, 1918. Carnegie Inst., Wash. Pub. No. 249, III, chap. v, 1919.

The vibrator proper, *cc′*, is attached at the middle of the wires *d*, *d′* and carries the parallel auxiliary mirrors *m*, *m′* of the quadratic interferometer. A thin steel umbrella-rib seemed well adapted to fulfill the requirements of *cc′*, though a light soft-iron tube would have been preferable.

The telephones *T* and *T′* are adjustable on special standards, attached to the bed-plate (carriage *BB*) and placed horizontally, one in front and the other toward the rear of the vibrator *cc′*. It is desirable that one be adjustable on a micrometer screw and spring, so that the distance of the poles of both from *cc′* may be regulated to correspond with each other and the tension of the wire, as explained in the preceding paper.

The achromatic fringes in the fine slit-image of the telescope field of the interferometer must be observed with a vibration telescope, and it was found desirable to control the latter by a special electromagnet. Figure 59 is a diagram of the parts of the apparatus as a whole, *M*, *M′*, *N*, *N′* being the mirrors of the interferometer (*M′* on a micrometer-screw *s*); *m*, *m′* the auxiliary mirrors on the vibrator *cc′*; *T*, *T′*, are the telephones (one provided with a switch *r*), *V* the vibration telescope, *I* the mercury interruptor. *T″* is an auxiliary telephone for the ear.

Thus the primary consists of the linear coil *P* described in the preceding paper, the storage-cells *E* (usually four), and the two small electro-magnets *e*, *e′*, for controlling the interruptor and the objective of the vibration telescope *V*. The secondary *S* was the coil "*B*" of the preceding paper, wound on glass. This was in circuit with a rheostat *R* (up to 40,000 ohms), the telephones *T*, *T′*, *T″*, and the key *U*. The condenser *C* (up to 4 microfarads) is available when needed.

The whole of the parts shown in figure 58 could be slid fore and aft on the carriage *BB* and slides *A*, *A′* to accommodate the interferometer. Inductive resistances *e″* were later added to the primary and the key *K* was replaced by a switch or commutator.

40. Observations with the slit-image.—It will be seen that if an ordinary telescope is used at *V*, figure 59, the sharp slit-image must widen to a band here, as in the preceding apparatus; but it is much less sensitive because the rays are parallel throughout. It nevertheless suffices admirably for finding the resonance tension. For this purpose the screw *b* or *b′*, figure 58, is first manipulated till the image begins to widen. The fine adjustment is then made at *u*, *y*, till the maximum band-width is reached. One easily recognizes in this way three harmonics—the fundamental at lowest tension and small band-width, the octave at larger tension with maximum band-width, and the next overtone at still larger tension and diminished band-width. Above this I did not go, as the stress on the wire would have been excessive. The reason for the prominence of the octave here is not clear to me. It does not occur again in the later work. While the fundamental showed a band width of but 5 scale-parts, the same for the octave was 42 scale-parts wide; and for the next harmonic it was 26 scale-parts wide.

The remarks made presuppose that the telephones are acting in concert, on opposite sides of the vibrator. We may refer to this as an arrangement in series. When the telephones are acting in opposition, the band-width decreases to a few scale-parts, 2 or 3, depending on the symmetry of adjustment, etc., which, if perfectly made, should throw the differential s out entirely.

A large number of experiments were carried out with soft-iron armatures, cemented to the vibrator cc' in front of the telephones; but no result of consequence was obtained in this way, so that I returned to the simpler unarmed vibrator. Care must be taken to obviate vibration about the axis of cc', which betrays itself by producing wave-lines across the band-width for every obstruction in the slit. These would be a serious annoyance were the interferometer used.

41. Observations with the interferometer.—As has been stated, the fringes are easily found, because the rapid motion of the vibrator implies considerable damping. The slit-image is thus quite stationary and the fringes clear and strong. On starting the inductor, the fringes at once vanish and after breaking circuit only slowly reappear, unless the vibration telescope is used. If the period of the latter differs from that of the induction (to be very weak), the even band of fringes, *a*, figure 60, changes to wave-lines *b* traveling in opposed directions and of continually increasing amplitude. Eventually the crests or troughs only are seen (and these but on one side *c*, figure 60), as some micrometer adjustment for one or the other will be necessary to keep them in the field) again traveling in pairs, in opposite directions, through each other. On breaking the circuit these pulses slowly coalesce into the wave-band *b* and finally into the even band *a*.

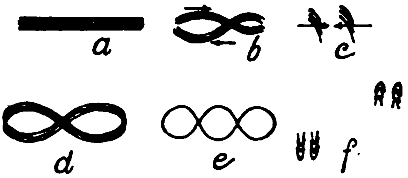


FIG. 60.

The case here presented is that of a relatively slowly vibrating telescopic objective at V. If the frequency of the latter can be counted, the frequency of the alternator may be deduced from the number of moving crests in the field. Thus, when the frequency of the objective was $n' = 5$, there were four crests in motion, implying a frequency of $n = 20$ for the interruptor of the coil. In this respect the case of different periods is advantageous.

The fringes in these preliminary experiments were chosen small. It is of interest, nevertheless, to compute the sensitiveness reached. The effective voltage of the coil was about 0.7 volt; the resistance inserted 42,000 ohms. Hence the mean current was $0.7/4.2 \times 10^4 = 10^{-4}/6$ ampere. The net double amplitude of the waves measured by the ocular micrometer was about 4 scale-parts, so that $i = 4 \times 10^{-6}$ ampere comes to a scale-part in case of the present small fringes.

The next step in advance consisted in adding an electromagnet (e', fig. 59) at the objective of the vibration telescope V, in series with the electromagnet e of the interruptor. The two springs at V and I, moreover, were adjusted to about the same period. The electromagnet *e′* could be rotated on a vertical axis, so that by moving it closer to or farther from the steel spring of *V* any degree of band-width was obtainable in the telescope. Magnificent octave fringes (*d*, figure 60) were obtained in this way. They moved merely on opening and closing the circuit. Otherwise either the stationary type *a* or *d* permanently occupied the field. The fringes were still small, but a double amplitude of about 5 scale-parts was registered as before. Again, by changing the tension of the wire, one may pass from *a* through *d* back to *a* again.

The fundamental has too small an amplitude to be striking, but the second overtone develops well. The form *e* is producible, but what usually appears is apt to resemble *f*. Changes of tension may be used to generate the stationary figure.

Fringes which have vanished (from slight disadjustment) may frequently be recovered by changing the focus of the telescope. In fact, care must be exercised against the possibility of vibration of other parts of the system, the telescope for instance.

42. Decreased bifilar distances.—The easy accomplishment of the above experiments, where the distance between the bifilar threads was about 30 cm. and their length 60 cm., encouraged me to reduce the distances between the threads until the system was virtually torsional. In such a case the displacement at the ends of the vibrating beam *cc′*, figure 61, is no longer limited to that of the bifilar wires. Figure 61 is a front elevation of the new apparatus, the steel-wires *dd* being at a distance of about 6 cm., their length 60 cm., the distance between the auxiliary mirrors *m*,*m′* 10 cm., and between the magnets of the telephones *T*, *T′*, 16 cm. The vibrator *cc′* was (as above) a steel umbrella rib. The ends of the wires *dd′* were again wound about the stiff screws *b*, *b′* held in posts *a*, *a′* rigidly attached to the bed-plate of the interferometer (not shown). The wires were stretched below by the pulley *w* and screw *y*, controlled by the nut *u* pressing against the rail *v*, also rigidly attached to the bed-plate of the interferometer. Provision must be made (slotted sheath *z* and pin *p*) to prevent *y* from turning on its axis. One of the telephones is to be adjustable on a screw-spring device (not shown) to regulate its distance from the vibrator *cc′* to correspond with the other telephone. Again, a variety of braces are to be introduced to obviate synchronous vibrations of parts of the apparatus (especially of the telephones), in so far as needful.

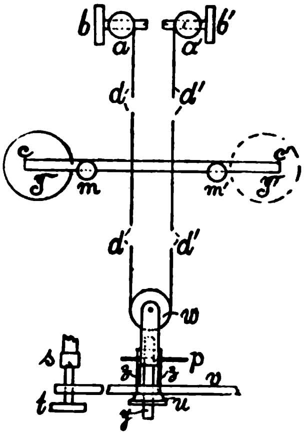


FIG. 61.

The adjustment for resonance is here more difficult than in the preceding case, and *y* must be a fine or a differential screw and the nut *u* work smoothly. Resonance should first be established by aid of the slit-image in a telescope with fixed objective and with small resistance (200 ohms) in circuit. After this a large resistance (10,000 ohms and more) may be inserted and the work continued with the interference fringes when further adjustment is possible. The vibrator *cc'* is more liable to become unstable and stick to the magnets of the telephone. But the advantages gained are manifest, as the sensitiveness will at once have increased 50 times or more.

In a later design of apparatus, the bar *v*, rigidly fixed at one end, was controlled at the other by the screw *t* (fine-threaded) pushing against the rigid socket *s*. This makes an excellent fine adjustment for resonance and does not disturb the interferences when used. The fine adjustment may therefore be made with the interference fringes at once.

43. Observations with the new apparatus.—The first trials were made with the white slit-image. Using the coil with a virtual electromotive force of about 0.7 volt and a small resistance (200 ohms), the resonance conditions are easily reached in a well-braced apparatus. The resistance was then increased to 10,000 ohms and the corresponding band-width found. This is more difficult, for the maximum deflection is painfully sensitive to slight differences of tension or to slight changes in the position of the telephones. When obtained, however, the band-width s persists. The sensitiveness obtained is thus, the observed s being 18 scale-parts, $0.7/18\times 10^4 = 4\times 10^{-6}$ ampere per scale-part, or 4×10^{-4} per ocular centimeter. Hence, per $\Delta N = 10^{-4}$ cm. of the micrometer of the interferometer, where $b\theta = \Delta N \cos i$ and $\theta = s/2f$ ($f = 23$ cm. being the focal length of the telescope and $s = 10^{-2}$ cm. the value of 1 scale-part, $b = 10$ cm. the breadth of the ray parallelogram, $i = 45°$), the current would be estimated as

$$4\times 10^{-4}\times s = (4\times 10^{-4}\, 2f\Delta N \cos i)/b = 1.3\times 10^{-7} \text{ ampere.}$$

A fringe would usually represent much less than this.

In the interferometer experiments now begun with this adjustment the fringes were found much more easily than was anticipated. The slit-images soon became adequately quiet. There will be considerable difficulty, however, in testing the degree of resonance by means of the fringes alone, unless the slide *z* of the tension-screw *y* is very true. Any rotation of *y* around its axis will send the fringes out of the field. Adjustment may sometimes be made at the telephones as specified, or the wire may be stretched by lever mechanism. In the present experiments I made the resonance adjustment with the slit-image before using the fringes. Later the device *v*, *t*, *s*, figure 61 was adopted.

The *B*-coil of mean voltage 0.7 volt was tried first. But this was rather too strong for these measurements, so that coil No. 10, with about 90 turns giving 0.001 volt per turn was substituted. With the slit-image alone the sensitiveness was 4.5×10^{-6} ampere per scale-part; with the sliding micrometer of the interferometer, $\Delta N = 10^{-4}$ cm. was equivalent to 8×10^{-7} ampere; with the ocular micrometer I obtained 10^{-7} ampere per scale-part. This is somewhat less than what was estimated; but a shortage is here inevitable. The fringes used were fairly large, say 2 scale-parts, but completely under control.

Unlike the preceding case, the fringes now obtained were of the elliptic type, so that there is unison between interruptor and vibrator. The ellipses were often magnificent. There was little difficulty in measuring their breadth normal to the direction of fringes when quiet, as this is the fringe displacement. On making or breaking the circuit the ellipses oscillate in the well-known way, and it may take part of a minute or more before they subside into the bands (circuit broken) or into stationary ellipses (circuit made). They remained in the field with resistances up to 10,000 ohms, after which the ΔN micrometer had to be adjusted to bring either axial extremity into view. Notwithstanding the feeble current, the telephone was still audible, so that the sensitiveness of the ear has not been exceeded. It is convenient to set the fine slit-image normal to the fringes. The objective vibrates parallel to the fringes, but the major axis of the stationary ellipses is nevertheless usually oblique to these directions, in view of the phase differences of currents. The appearance of the ellipses is often that of disks. When the bands occur, the impression is that of disks seen edge-on. In the absence of resonance the ellipses are imperfect and appear as overlapping half-wave curves. When the degree of resonance between the telephonic vibrator and the vibrating objective is exceptionally perfect, marked ellipses may appear in the absence of current, in spite of the fact that the telescope has an independent mounting. This very annoying phenomenon is apparently hard to eliminate with the observer near at hand. A trace of wax on the vibrator is sometimes a remedy at a sacrifice of sharpness of resonance. Whether the coupling in such a case is merely mechanical or else magnetic remains to be seen.

In the next experiments another smaller coil with but 10 turns, giving 0.001 volt per turn, was installed with the object of ultimately approaching a condition of silence in an audible telephone. Good vibration ellipses were obtained, both with large and smaller fringes, the latter preferable, because they are usually adequate and more easily controlled. The results were throughout striking and may be exhibited by the aid of figure 62, where *ak* is the direction of the originally linear slit-image, broadened by the objective vibrating in the direction *ab* (parallel to the fringes when at rest) to the band *abgk*, prolonged. The displacement of fringes due to the alternating current being in the direction *ak*, the ellipse *e* results. This is often very brilliant, and, except on making and breaking the circuit, quite stationary. If the resistance in circuit is reduced, the ends near *a* or *g* of the ellipse may leave the field, unless restored by the micrometer. For large resistances (5,000 to 10,000 ohms) the ellipse shrunk to a line like *hf*, oblique to *ab*. The vibration of fringes has not therefore ceased in the absence of current; but the two components are now in the same phase and the coupling is apparently mechanical, although the vibrating telescope is on an independent table insulated from the vibrator. As indicated below, it may, however, be magnetic.

The occurrence of the oblique stationary ellipse indicates a difference of phase between *ab*, in step with the objective and therefore with the current produced by the impressed voltage in the primary, and *ak*, in step with the induced current in the secondary. This is in keeping with theory, which demands an angular phase difference of the form $\tan^{-1}(L\omega/R)$. On changing the resistance R, these ellipses showed little tendency to oscillate. They rather expand or contract. A measurement of their extent between the tangents *ab* and *kg* is difficult, because of the initial displacement corresponding to *hf*, for the difference $ah+fg$ only is effective. The full ranges may be stated as

at R = 1,250 ohms,	25 scale-parts,
2,200 "	20 "
4,200 "	13 "
10,200 "	10 "

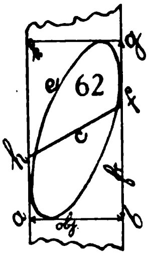
With lower or higher resistances the change of size necessarily (Ohm's law) increases or decreases asymptotically; at 20,000 ohms the effect of breaking the circuit is still quite observable. In fact, the bands for $R=\infty$ are slightly overcompensated, as though they corresponded to an electrostatic capacity, so that truly linear fringes were obtained on inserting high resistances.

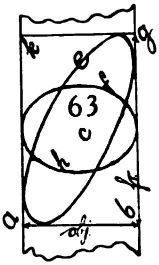
The auxiliary telephone in circuit was now nearly silent. If at 2,000 ohms the net range of elliptic displacement is taken as 10 scale-parts, the current would be (an average of 10^{-2} volts being impressed)

$$i=\frac{10^{-2}}{10\times2\times10^{3}}=5\times10^{-7}\text{ ampere per scale-part.}$$

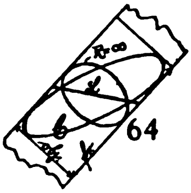
But much less than this is observable in the changes of form and disposition of fringes.

The conditions of figure 62, in which for $R=\infty$ the bands *hf* appear, merely make a special case. Figure 63 shows another of many similar cases observed. Here, when $R=\infty$, the symmetrical ellipses *hf*, implying a phase difference of 90°, due to the so-called mechanical coupling, was observed. On decreasing R from ∞, *kf* passed into the oblique ellipse *e*, the range of which, *bg*, increases as R decreases. At 25,000 ohms no change was observed; but at 20,000 ohms there was definite change of inclination apparent. At 5,000 ohms the ellipse showed a new increase of range of over 10 scale-parts. The occurrence of a variety of conditions, exemplified by figures 62 and 63, shows that a very variable cause is productive of phase difference between the vibrating objective and the vibrator of the interferometer.





44. Capacity and self-induction in the secondary.—The phase differences thus far observed are attributable to the self-induction of the secondary. It is interesting, therefore, to test whether the lead of the form $\tan^{-1}(1/CR\omega)$ due to capacity can be equally well observed. The circuit (fig. 59) was therefore provided with a condenser C, containing up to one microfarad, in steps of tenths. An auxiliary telephone T'' was also inserted as a detector. The results were successful at once, as shown in figures 64 and 65. In figure 64, $R = \infty$, or the symmetrical ellipse is obtained on open circuit. This changed rapidly into the oblique ellipse C when 0.5 microfarad is inserted, and the latter into the bands L (with a range of 30 scale-parts) when the circuit was closed with about 3,000 ohms. In another adjustment (fig. 65) of primary, $R = \infty$ gave normal bands (*i.e.*, the fringes do not vibrate); the capacity 0.5 microfarad now gave the oblique bands C and the self-induction ($R = 5{,}000$ ohms) the nearly symmetrical ellipses L.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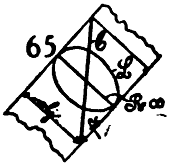
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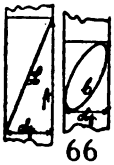
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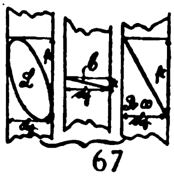
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The passage from L to C was always through bands, thus indicating the probability of an opposition of phase change of the relation of lag to lead. This is instanced in figures 66 and 67, where the direction of fringes (like the vibration of the objective) happens to be horizontal, and the displacement of fringes therefore vertical. L is obtained by closing the circuit with $R = 3{,}000$ ohms. When 0.5 microfarad was then inserted, the oblique bands L changed into the ellipse C by first passing through nearly horizontal bands of duplicated fringes. In figure 67 (owing to a change in the primary) L has the elliptical form and C is banded or duplicated. In both cases the detecting telephone was audible to about the same degree. On breaking circuit ($R = \infty$) the telephone was silent; but the fringes vibrated, as shown in the figure. It is not unusual for these patterns to occupy the greater part of the field of view. The ellipses are sharply visible at the ends of maximum curvature, where there is partial cessation of motion. The connecting lines may escape detection. It is frequently necessary to re-focus the vibration telescope.

45. Self-induction in the primary.—The present experiments contain an element of uncertainty, owing to the so-called mechanical (possibly magnetic) coupling of the vibrator cc', on the interferometer (fig. 59), and the objective of the vibration telescope V. Some relevant information, it was thought, would be gained by inserting additional self-induction into the primary. The two small electromagnets e'' (fig. 59; about an inch long), which could be used either separately or in series, were available for this purpose.

The different elements of harmonic motion involved in the experiment may therefore be analyzed as follows: The whole is fundamentally subject to the vibration period of the spring at the interruptor of the primary, which gives the impressed electromotive force

$$e = e_0 \sin \omega t \qquad (1)$$

The current induced in the primary controls the objective of the vibration telescope, which thus moves with a lag α subject to

$$i = i'_0 \sin(\omega t - \alpha) \tag{2}$$

and this may be modified by the resistance and inductance in the primary.

The objective is, as stated, either mechanically or magnetically coupled with the vibrator on the interferometer in a way yet to be ascertained. Hence the vibrator displacements s are subject to an equation with a lag or lead

$$s = s_0 \sin(\omega t - \beta) \tag{3}$$

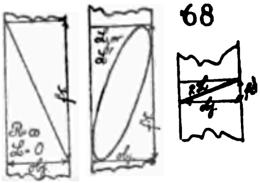
in the absence of current in the secondary ($R = \infty$).

Finally, the secondary, if carrying current, has its own lag or lead γ, depending on the R, L, C, there inserted, and is thus subject to an equation

$$s = s_0 \sin(\omega t - \gamma) \tag{4}$$

where γ is essentially associated with β, as seen in the preceding paragraphs.

If we suppose the coupling implied in equation (3) to be uniform, the lag in equation (2) may be made obvious. In this way vibration figures were obtained, examples of which are given in figures 68 and 69. Calling the two nearly equal auxiliary self-inductions in the primary L and L' ($R = \infty$ in the secondary), in the case of figure 68, the bands obtained in the absence of L or L' changed to ellipses of about the same range for either the L or L' insertion, and these to nearly horizontal bands when both L and L' were inserted. On removing the $L + L'$ the first figure returned. In figure 69 the initial form ($L = 0$) was an ellipse, changing to bands with but little difference of phase between the L and $2L$ insertions.



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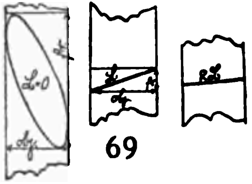


69

In all these cases the vibration figures were very large and very definite in the successions of their changes with L, however frequently repeated.

The insertion of L (choking coil) naturally somewhat reduces i and therefore the amplitude of the objective of the vibration telescope. It appears from figures 68 and 69 that at the same time the vibration of fringes is lessened and ceases practically under $2L$ (horizontal bands). Here, in other words, the apparently mechanical coupling has been eliminated; but by closing the secondary with a resistance R, the effect of C and L in the secondary (which might now be investigated without the mechanical discrepancy) proved to be insignificant. There was no appreciable induced current left in the secondary. Again, since these vibration figures appear and are modified when $R = \infty$, the coupling roughly called mechanical must be magnetic. In other words, all the quivering stray magnetic fields in the room directly influence the vibrator cc', figure 59. Further corroboration will be found in §48.

Following the usual theory, if L has the form $4\pi n^2/(l/\mu A)$, n being the number of turns of wire around an electromagnet of length l, area A, and permeability μ, and if the coefficient of mutual induction of the transformer is $M=\sqrt{L_1L_2}$, and if $e=e_0 \sin \omega t$ is the impressed voltage, the secondary voltage will be

$$e_2=-M\omega e_0\frac{\cos(\omega t-\varphi_1)}{\sqrt{R_1^2+L_1^2\omega^2}}$$

Hence the secondary current, when L_2R_2 and C_2 are there inserted, is

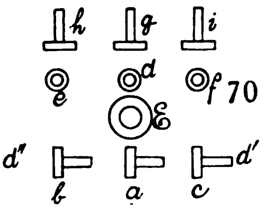
$$i_2=\frac{e_2\cos(\omega t-\varphi_1-\varphi_2)}{\sqrt{R_2^2+(1/C_2\omega-L_2\omega)^2}}$$

where

$$\varphi_1=\tan^{-1}L_1\omega/R_1 \qquad \varphi_2=\tan^{-1}(L_2\omega/R_2-1/C_2R_2\omega)$$

This is further to be modified with reference to the amplitude and phase of the extraneous coupling.

46. Direct telephonic induction.—The influence of the oscillating magnetic field on the telephone is much more pervasive than one is apt to suppose. The effect, moreover, is particularly marked if the telephone is open, *i.e.*, with no connection between the clamps. A coil of high resistance of telephone wire, implying many turns, is naturally preferable. The stray vibrating field produced by a small electromagnet (say 0.25 inch iron, 2 inches long) is quite audible even beyond 50 cm. from the electromagnet. The degree of response depends, moreover, on the orientation (fig. 70) of the telephone relative to the electromagnet E. If we take the three cardinal positions of the plane of the coil or the diaphragm, the vertical positions e, d, f, and the fore-and-aft horizontal positions h, g, i, have their maximum response in the plane of symmetry gdE. The right-and-left horizontal positions d'', b, a, c, d' give minimum response (telephone silent) in this plane (Ea), with maxima at symmetrical positions, b and c. Near E (20 cm.) the sounds may be quite loud.



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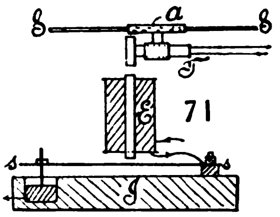


71

Although all telephones show the phenomenon pretty well, since it is more distinct on open circuit (which implies a current oscillating from clamp to clamp) it would be well worth while to wind a telephonic bobbin provided with a capacity for the particular purpose of catching the stray magnetic field, such as is here encountered. Without proceeding to this extent, I used the telephone as a secondary, as shown in figure 71, where E is the electromagnet of the interruptor I, S being the vibrating break-circuit spring. The telephone depending adjustably from the sleeve *a* may be slid right and left or rotated into any horizontal position relative to *E*, and the current obtained measured by the vibrator.

Instead of sliding the telephone it is sometimes convenient to rotate it into various positions. One may observe that the cases *b*, *a*, *c*, figure 70, are the more interesting, since the current alternations of *b* and *c* are necessarily opposite in phase, whereas *e* and *f*, *h* and *i* are not so. Hence the first design *b*, *a*, *c*, is to be chosen.

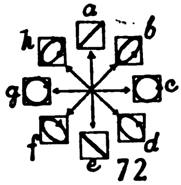
In the endeavor to minimize the mechanical coupling, the vibration telescope was first put on a massive standard and additionally weighted. This, however, had no advantageous effect of consequence. The telescope (separately mounted) was then placed at about a meter from the vibrator. In such a case the phase difference of the vibrations of fringes and objective was annulled, but the bands in the absence of current were nevertheless somewhat oblique to the direction of the vibration of the objective, showing that the fringes still vibrated.

With this exception (presently to be removed) the behavior of the telephone inductor was admirable. In passing from the positions *b* to *c* by sliding the telephone, the ellipses regularly passed through the oblique bands, indicating that these successive ellipses, even if of nearly equal size, were opposite in their phase rotation. This was the case when the secondary was closed with 5,000 ohms and the inevitable inductance; also when a capacity was placed in the secondary, and finally on passing from an inductance to a capacity in the secondary. The effect produced by changes of capacity of 0.5 microfarad was marked. The alternate-current effect, moreover, was still apparent when the circuit was closed with 25,000 ohms and the telephone practically silent.

There are, however, some peculiarities of behavior when a capacity is inserted which need elucidation. The telephone responds strongly for capacities exceeding 0.05 microfarad, but the vibrator is often less influenced than when the same loudness of telephone is produced by closing the circuit with a resistance. Moreover, large ellipses are more apt to pass through bands than small ones, which may be interpreted as an oscillation, or as the mere dying away of strong antecedent vibratory motion and does not necessarily involve changes of phase in the final result.

The most immediate criterion as to changes of phase is the rotation of ellipses as indicated in figure 72. If this rotation is in appearance counterclockwise when the telephone passes from positions *b* to *c*, figure 70, it will be clockwise on the return passage from *c* to *b*, remembering that marked changes of amplitude are involved and that the changes are gradual in correspondence with the mass of the vibrator. Nevertheless, figure 72 and its suggestion of the rolling of ellipses into place is not adequate, as it leaves out the usual oscillation of ellipses before they become stationary. One must train oneself to follow the sliding of any one of the four points of contact, within the given vibration rectangle. I shall give a few examples of what is observed in the telephone displacement in question.

In the absence of current the fringe bands were nearly horizontal parallel lines. The secondary was closed with 5,000 ohms and the inductance of the three telephones. From the position *b* (ellipse 1, fig. 73, quiescent) the inductor telephone was quickly displaced to position *c*. The enormously eccentric, finally linear ellipse 2 follows, which then rotates and contracts counterclockwise through the figure 3 and 4 into the sharp bands (usually but not always) No. 5. These duplications then separate on further rotation into the final quiescent form, 7. The arrows indicate the drift of one of the four points of tangency. On returning from *c*, by quickly sliding the telephone inductor into the position *b*, the figures roll clockwise from 7′ to 1′. Number 7′ passes at once through the highly eccentric ellipse 6′, though in other slower adjustments intermediate sharp duplicates like 5 may be detected between 6′ and 7′.



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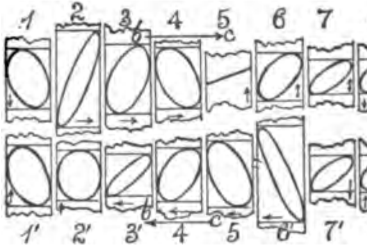

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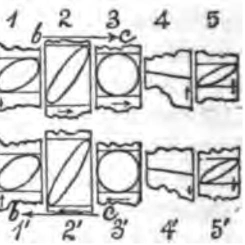

The stretched ellipses, which follow immediately after the change of aspect of the telephone bobbin to the magnetic lines, are noteworthy. They indicate the inductive effect of the reversal of the magnetic field, impressed on the vibrating system and observed in spite of it. Ellipses cross over, or change sign of rotation, at 2 and 6′, but not near 3′ or 5, the latter being oscillations.

At a large resistance, $R = 25{,}000$ ohms, the range of the ellipses was incremented by about 4 scale-parts; at 5,000 ohms by 12 scale-parts. Figure 73 as interpreted by 72 would suggest a passage from *f* to *b* of the latter, the case *a* corresponding to No. 2 of the former, so that 4 to 7 are oscillatory. The interpretation is difficult, because the four points of contact of the ellipses lie in a vertically expanding, contracting, and eventually oscillating rectangle.

The corresponding cases for capacity (fig. 74) are similar on the whole, though less pronounced. From telephone positions *c* to *b* the ellipses 5′ pass through strong, duplicated fringes 4′, when 1 microfarad is in circuit. With 0.5 microfarad in circuit, the ellipse 5′ remains banded. The telephone continued to be appreciably audible, even when the capacity was reduced to 10^{-4} microfarad; but the vibrator was not influenced by less than 0.1 microfarad. Moreover, the forms 1 and 5 are not in opposed phases.

An example of the change of ellipses for a passage from self-induction to capacity (1 m.f., resistance 5,000 ohms) need not be given, as it merely involves a smaller degree of rolling with variation of size. Thus, for instance, in figure 74, the case 3 for capacity became the case 1 for inductance; or 3 passed to 5.

47. Narrow bifilar.—After obtaining the favorable results just described, it seemed obvious that the sensitiveness could be further increased by diminishing the distance between the bifilar wires. Accordingly, with the same inductor (fig. 71), the above wires (diameter 0.022 cm.) were adjusted at but 1.5 cm. apart, by decreasing the diameter of the lower pulley. A few other modifications were added. The results, however, were disappointing throughout. In view of the relatively greater inertia of the system, the resonance deflection (band-width in the telescope) was therefore difficult to find. Any incidental quiver of the vibrator decayed very slowly. It was necessary, moreover, to use relatively slow vibrations ($n = 10$ and less). The wires broke before the higher frequencies were reached. The maximum sensitiveness obtained was about 32 scale-parts for coil No. 10 and $R = 300$ ohms, so that

$$i = \frac{0.09}{300 \times 32} = 10^{-6} \times 9.4$$

go to a scale-part of band-width. This is less than the above result.

The preceding vibrator (umbrella-rib) was now replaced by a thin iron wire about 1 mm. in diameter. This with coil No. 10 and $n = 10$ to 15, gave but 23 scale-parts for the maximum band-width, so that this result is worse than the preceding. A glass vibrator made of a thin-drawn light tube, with iron plates cemented to the end, was next tested. The maximum in this case was reached much more rapidly, but in a great variety of tests and modifications it did not exceed 10 scale-parts. Fast and slower vibrations were tried, without much difference. This vibrator would have been steady enough for use on the interferometer; but because of the lack of sensitiveness, trial was not worth while.

I next inserted a thicker bifilar wire (diameter 0.036 cm.). The results were not much better. In fact, with the iron-wire vibrator I obtained a maximum band-width of 23 scale-parts, the same as in the preceding case. The vibrator was much steadier. As there seemed to be no promise of success, further experiments were here also abandoned.

A few incidental tests were made. Thus with the telephones in series, the band-width of 23 scale-parts fell off to about one scale-part when the telephones were joined oppositionally. This is a good test on the symmetry of adjustment, as the rotation of vibrator is changed to translation, broadsides onward.

Again, the auxiliary audible telephones responded with about equal loudness when the telephone circuit was closed with 250 ohms and when a condenser of 1 microfarad capacity was inserted. But the vibrator reacted in the former case (resistance and self-induction) with a deflection of 23 scale-parts, whereas in the latter (capacity) the response was at most 2 scale-parts; and this required a slightly different tension of wire. The metallically closed circuit therefore affected the vibrator at least 12 times more strongly than the oscillation through the capacity of 1 microfarad. Small capacities like 0.1 microfarad fail to influence the vibrator, though to the ear the sound is quite loud. The capacity would have to be increased to 10 or 20 microfarads for equal effects on telephone and vibrator. In another experiment the closed circuit gave 20 scale-parts. The insertion of 4 microfarads decreased this to 4 scale-parts, which is again a demand of about 20 microfarads for an equality of behavior. On the other hand, while the telephone responds for a phenomenally small capacity, it soon ceases to increase in loudness (for 1, 2, 3, 4 mf. or even 250 ohms), whereas the deflections of the vibrator increase regularly.

48. The vibratory stray magnetic fields.—As the narrowed bifilar of §47 had virtually failed, I returned to the apparatus of §42, with bifilar distance of about 6 cm., this being the more serviceable. The vibrating needle was here a steel wire with adjustable mirrors on bits of cork. The fringes were easily found. Instead of the telephonic inductor of §46 (which, though interesting in itself, gives a tumultuous effect on the reversal of the magnetic field), a simple spring commutator (old Ruhmkorff style) was put in place of the key K (fig. 59), receiving the poles of the secondary. In such a case any secondary could be used at S, and its alternating currents either reversed or broken at pleasure. This proved to be an excellent method, for the ellipses now merely glided into new positions without passing through oscillations. The secondary system was, as it were, dead beat, though still somewhat slower than would have been desired in the interest of expeditious work.

Before proceeding further it was necessary to learn to control the so-called mechanical coupling. The probable reason has already been suggested in § 45. By placing the primary adjustably on a table at a distance of about a meter or more from the vibrator, the fact that the latter is subject to all the quivering stray magnetic lines in the room was soon verified. I may recall that the primary coil consisted of a thin (walls 0.08 cm.) iron tube, 55 cm. long, 0.635 cm. in diameter outside, wrapped with a single layer of wire (0.034 cm. in diameter), so that about 21 turns came to the linear centimeter. On this stick-like solenoid any secondary could be slid at pleasure. The primary was now fastened in a horizontal position to a standard, in a way admitting of rotation around a vertical axis into any fixed position. With the secondary broken ($R=\infty$) it was possible to obtain any phase or amplitude of ellipse within limits by rotating the primary solenoid. Thus either of the cases, figures 64 and 65, for $R=\infty$ could be produced at will. If the solenoid is too far off, etc., the figure of minimum amplitude may be a symmetrical ellipse, so that the bands parallel to the vibration line of the objective are unattainable. Otherwise it is as easy to compensate the vibrator as it would be, for instance, to compensate a galvanometer by an astasizing magnet.

When this compensation has been made the vibration figures obtained by switching the secondary current at K are in opposite phases and the measurement of the range of ellipses may be made with the same result in either phase. It is also possible to change the phase of ellipses produced by a secondary current, obtaining a series of oblique bands corresponding to the current.

The amplitude of the ellipses for the case of direct magnetic action (secondary current absent, $R=\infty$) depends both on the strength of the alternating field and on the degree of resonance, while the phases change with the latter. Thus if the resonance is not adequately perfect, linear fringes only are obtained and these change through inclined fringes, through normal fringes, into fringe-lines with an inclination opposite to the first, while the inducting solenoid changes its position continuously through the symmetrical position. Thus the solenoid may either be moved parallel to the vibrator, between positions on opposite sides of it; or it may be correspondingly rotated in any given direction, so that the effects of its opposite ends respectively supervene.

When the degree of resonance is sufficiently accurate, the axes of the fringe ellipse are parallel to the sides of the vibration rectangle. In other words, there is a phase difference of 90° between the vibrating objective of the telescope and the vibrator (*cc*, fig. 61). This may be used as a sharp criterion for resonance. Thus, I placed the solenoid in a non-symmetrical position, so that the vibrating magnetic field was seen at the vibrator. Then by very gently changing the tension of the wires (system *v*, *t*, fig. 61) from excess to deficiency, fringe-lines inclined upward on the right with a range of 5 scale-parts changed to the orthogonal ellipse with a range of 20 scale-parts, into fringe-lines inclined downward on the right, ranging 5 scale-parts. When the tension was thereafter gradually increased, the same sequence was observed in the opposite direction. Again, if the resonance is adequate, the orthogonal ellipse does not change in phase when the solenoid is moved, as specified. The passage is through normal fringe-lines; but if resonance is only slightly imperfect, the oblique ellipse passes through oblique fringe-lines into a symmetrically inclined ellipse.

The criterion is sufficiently sensitive to indicate a change of period with the amplitude of the interruptor. Thus an orthogonal resonance ellipse was produced with a range of 25 scale-parts. An inductance was now put into the primary, decreasing the image band-width (vibration of the objective) and decreasing the fringe-lines to a range of 5 scale-parts. The tension of the wires of the vibrator was now carefully decreased, little by little, until an orthogonal ellipse with a range of 25 scale-parts was obtained. Thereafter the inductance was again removed. The band-width of vibration increased again, but the fringes became lines of symmetrically opposite inclination. Increasing the tension of the wires slightly brought back the original orthogonal ellipse with a range of 25 scale-parts. Many other experiments similarly striking might be instanced.

The orthogonal ellipse is clearly compatible with maximum fringe displacement. For the magnetic field changes sign at the maximum elongations of the vibrator. If the ellipse is permanent, the periods of field and vibrator are rigorously equal.

49. Resistance, capacity, inductance, in case of the compensated vibrator.— With the fringes for $i=0$ in the secondary reduced to bands, a number of experiments on the effect of resistance were made, examples of which are given in figures 75 and 76. In the former case a coil with 10 turns ($e=10^{-3}$ volt) was slid over the primary and large additional resistances up to $R=$

30,000 ohms inserted. The ordinates *s* (scale-parts) show the range of ellipses in the direction of the fringe vibration. The fringes being about a scale-part in width, the sensitiveness at high resistances was about 10^{-7} ampere per fringe. In case of figure 76 the secondary consisted of but one turn ($e = 10^{-3}$ volts) and relatively low resistances were admissible. The sensitiveness here is about 10^{-8} ampere per fringe. In case of the large fringes used in series 7 and 8 below, I found about 0.5×10^{-8} ampere to the scale-part, which is probably the limit of the present apparatus. At $R = 20,000$ ohms the telephone was practically silent to the ear, but the vibrator gave distinct evidence of current even above 30,000 ohms. The curves (figs. 75 and 76) are so nearly hyperbolic that the inductance must be negligible compared with the high resistances.

In the first series Rs is practically constant, a result which might be used for standardizing *s* in case of these particular fringes. In series (3) *r* is computed from $(R+r)s = \text{const}$. For *R* below 500 ohms, the results for the effective resistance of the circuit come out between 400 and 500 ohms. To compute $L^2\omega^2 = \Delta(s(R+r))^2/\Delta s^2$ would require a more accurate specification of *r* and better individual values of *s* than the method provides in the secondary. Thus, if $r = 250$ ohms, $L\omega = 640$ to 660; but this is clearly over five times too large. Probably *s* and *i* do not pass through zero together. In fact, if $r = 280$ ohms and if $1/(R+r)$ and *s* be constructed as in figure 76, the relation is so nearly linear that *L* will be merged in the errors of observation. But the line suggests an initial $s = 1$ scale-part.

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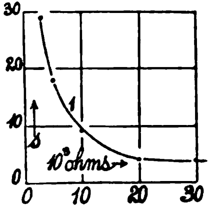

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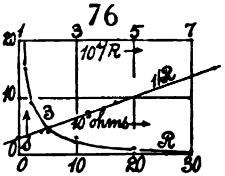

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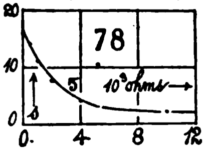

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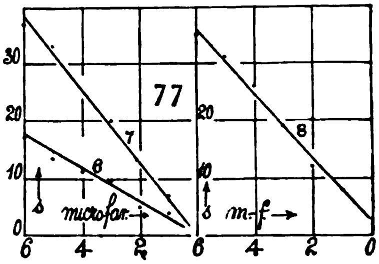

In a later adjustment I reduced the induction till the ellipses remained in the field in the absence of extra resistance *R*. The resistance of the circuit itself with its 3 telephones was $r = 280$ ohms. Results were obtained, for example, as follows:

R	*s*	*Lω*
0	15	120
100	10.5	

After removing the auxiliary telephone ($r = 187$):

0	24	73
100	15	

This makes $L\omega = 40$ and 37 per telephone and is a fair result. Data of this kind, however, which depend on differences of the squares of s, are necessarily crude, unless some method of smoothing the observations is first resorted to.

Examples of the results of s varying with capacity at constant resistance are given in figure 77. In both instances the additional R was zero, but the effective resistance of the circuit (telephones, etc.) about 400 ohms. In case of series (6) the fringes were smaller than a scale-part and the sensitiveness about 3 scale-parts per microfarad. In case of series 7 and 8, the fringes were larger and the sensitiveness about 6 scale-parts per microfarad. In these adjustments therefore the detection of 0.1 microfarad would be very easy.

Within the range of observation and the values for L, R, and C, the capacity loci are linear, so far as can be made out, although subject to a well-known equation.

$$i = \frac{e_0}{\sqrt{R^2 + (1/C\omega - L\omega)^2}}$$

If L be neglected or associated with the constant r of the circuit, the latter may be computed from

$$(R+r)^2\omega^2 = \Delta(s^2/C^2)/\Delta s^2$$

where R in the present experiments is zero.

Taking the first four observations of figure 77, series 7, ωr comes out $10^4 \times 14$, $10^4 \times 8$, $10^4 \times 12$, in the successive pairs of data. We would thus compute r from the mean value of $n = 25$ per second as 700 ohms. The observations are again too crude for this method of treatment. Series 8 comes out similarly if the individual data be taken. The results must first be smoothed as in the figure. Thus is $s = 36$ at 6 microfarads and 19 at 3 microfarads, and if $n = 25$ per second, $r = 400$ effective ohms. The line in series (7) similarly treated gives $r = 240$ ohms. These are reasonable values.

Finally, it seemed interesting to trace the effect of additional resistance R *through* a capacity. This is done for a capacity of 6 microfarads in figure 78 for resistances up to $R = 10{,}000$ ohms. The results below 5,000 ohms are very marked; above 5,000 ohms the curve flattens. In many respects these observations may be regarded as elucidating some of the anomalous results obtained in the capacity experiments in the earlier parts of this paper. Unfortunately it is impracticable to reduce the resistance of the circuit itself below a few hundred ohms, unless the telephones are in parallel (§53).

It is obvious, however, that the measurement of a capacity can be carried out in the secondary by direct comparison with a known capacity, or even in comparison with a given resistance R, if the inductance is relatively negligible. For, if we neglect the L, the C would be given $C^2\omega^2 = \Delta s^2/\Delta(s(R+r))^2$ from the data of figure 78. Here Δ is a differential symbol and r was measured as 280 ohms. The values of $C\omega$ came out 780, 970, 1130 for the first 4 pairs of data taken in succession, so that an initial s is again in question. Since $C=6\times 10^{-6}$ farad, if the mean value of $C\omega$ be inserted.

$$\omega=\frac{960}{6}=160$$

and the frequency therefore $n=160/6.28=25$ per second. This is not in bad agreement with facts.

To measure a self-induction in the secondary is difficult, unless it is large in comparison with the effect of the resistances there. (§ 53.) It might however, seem to be measurable in the primary if adequate provision were taken to guard against the direct effect of the stray magnetic lines in the vibrator, in the way suggested by the experiments of § 45; for here the non-inductive resistances are but a few ohms. I made a variety of interferometer experiments of this kind, but without success.

50. Ring transformer.—A little ring transformer, consisting of about 11 turns of wire per centimeter wound on an iron ring 7.5 cm. in average diameter and 1 cm. thick, was used in place of the above solenoid. In this case there is of course no external magnetic field and the vibrator is not influenced. With an open secondary (1 to 10 turns were used) the fringes are normal bands. The results obtained with this apparatus were similar to the above and may therefore be omitted here.

In one respect, however, this apparatus is disadvantageous, as (owing to the thickness of solid iron of the ring-body) the inductive effect of a reversal of the magnetic lines within on the secondary is almost tumultuous. In other words, in commuting the secondary current (even if induced in a single turn of wire with high resistances, R, inserted), the ellipses enlarge enormously (similarly to fig. 73) and then roll through bands several times. All this takes considerable time, even when the final ellipse is of small range. In the case of the solenoid with tubular core, the ellipses under like conditions merely expand or contract with change of phase. A hollow annulus therefore would best meet all conditions, or it may be even preferable to dispense with iron altogether. Thus a coil was wound on a split 0.25-inch lead tube, which was thereafter bent into a ring. A few turns of secondary may then be used and the ellipses kept in the field without additional resistance.

51. Magnetic screens.—A number of special observations were made by screening the primary linear solenoid with metallic tubes (capable of sliding over it) from the influencing vibrator. For this purpose the primary was so placed as to give a large deflection by direct magnetic induction, while the secondary current was broken ($R=\infty$). A 3/8-inch gas-pipe slid over the primary then reduced the deflection, s, about 50 per cent, though a few centimeters at one end of the primary had to be left uncovered. Such a result would, in general, be expected from the theory of screening.

On the other hand, when a thick brass tube was slid over the solenoid, the deflection s was increased 20 or 30 per cent. If we suppose the more marked induction occurs on breaking the primary circuit, the equatorial currents induced in the thick brass tube would coincide in sense with the magnetization and this would be made obvious by an increase of s. In a repetition of this experiment some time afterward the brass tube behaved exactly like the iron tube, the deflection being diminished over 50 per cent in both cases.

Finally, with the solenoid so placed that there was no displacement of fringes (normal bands), both the iron and the brass tubes or screens produced fringe displacement, the iron without change of phase (oblique bands), the brass with change of phase (ellipses).

52. Amplitude of the interruptor.—One curious result, however, was obtained by inserting additional resistances $R=1$ to 10 ohms in the primary circuit. It was found that the secondary current passes through a maximum and does not as a rule decrease with the resistance R added to the primary. Naturally the band-width, which marks the amplitude of the objective, does regularly decrease with R. An example may be given (ring transformer) the *ohmic* resistance of the circuit being below 1 ohm:

$R=0$	1	2	3	4	5	10 ohms.
$s=10$	10	12	18	15	11	7

With the solenoid similar results were found, for instance (internal ohmic resistance 1.7 ohms):

$R=0$	1	2	3	5	10 ohms.
$s=13$	16	18	15	12	5

The maxima here are sharp, almost cuspidal. Hence the only explanation which occurs to me is a consideration of the resonance of the interruptor of the primary circuit and the vibrator in the secondary. As the interruptor plays through insulating water into the mercury contact, its period will probably increase with its amplitude, because of the damping effect of the water. The sharp maximum is determined by coincident periods at the interruptor and the vibrator.

53. Telephones in parallel.—The telephones actuating the vibrator were, in the above work, connected in series, because they could then be used either in concert or in opposition, as indicated. There may, however, be an advantage in joining them in parallel and excluding the auxiliary telephone, when a low resistance is needed in the secondary. This would fall to about one-fourth and thus be below 50 ohms. I made a number of experiments in this way with results essentially like the above. In case of the ring transformer and a single turn of secondary, the ellipses remained in the field without external resistances ($R=0$). Under these circumstances it was possible to exchange an inductive and a non-inductive resistance allowing for the difference of phase. The sensitiveness obtained was about 5×10^{-8} ampere scale-part.

CHAPTER VII.

THE SELF-ADJUSTING INTERFEROMETER IN RELATION TO THE ACHROMATIC FRINGES AND REFRACTION.

54. Introductory.—This apparatus was first used for the case of coincident ray systems by Michelson and Morley * in their famous experiments on the Fresnel-Fizeau coefficient and has since been similarly employed by Zeeman.† It has so many practical advantages that a special reference here is justified. It is shown in an essentially modified form in figure 79,‡ where *L* is a pencil of white light, preferably from a collimator. It is separated by the half-silvered plate *N* into the two beams *L12345T* and *L16785T*, both of which are recombined at *5* and then enter the telescope at *T*. It is merely necessary to rotate any of the mirrors, say *N′*, around a vertical axis until the two vertical white, wide slit-images coincide in the telescope, when brilliant fringes will be at once obtained on the coincident white fields. The central fringe is achromatic, for the system is self-compensating, or the glass-paths are rigorously equal. The fringes may be enlarged to infinite size and then reduced in size again, the phenomenon being symmetrical, by rotating any mirror, say *N′*, about a horizontal axis.

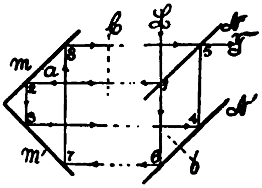

FIG. 79.

The mirrors *N* and *N′* are rigidly fixed to a carriage capable of sliding right and left parallel to the lines *8*, *5*, etc. Hence the rays *8*, *5*, and *2*, *1* and the rays *3*, *4* and *7*, *6* may be brought to coincidence (cf. fig. 94) or separated in any degree at pleasure. If the slide were perfect the fringes would not be disturbed by this process; but few are perfect to this degree, and the fringes will change size somewhat, since there is rotation. Practically this is of no consequence.

The fringes, which when sharp are necessarily horizontal, may also be changed in size by inserting a plate-glass compensator, *C*, about 5 mm. thick, in the paired beams *8 5*, *2 1*, or *3 4*, *7 6*. When this is rotated on a horizontal axis the fringes pass through infinite size, and this arrangement is particularly adapted for the detection of the character of the fringes and will be so used presently.

If a direct-vision prism or grating is placed in front of the telescope, the spectrum is seen to be crossed by intense black lines, very nearly parallel and horizontal, but actually diverging from blue to red symmetrically up and down from the horizontal central black line. It is not necessary here, that the slit be fine. In fact, it may be several millimeters broad without destroying these spectrum fringes, if essentially horizontal.

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* Amer. Jour. Sci., XXXI, p. 377, 1886.

† Proceedings Amsterdam Acad., vol. XVII, 1914, p. 445; and vol. XVIII, 1915, p. 1.

‡ The Michelson-Morley apparatus does not admit of appreciable ray separation, for on displacing any of the mirrors, the rays entering the telescope separate. This is not the case in figure 79.

If the linear phenomenon of reversed spectra, coinciding on a certain line of the spectrum, is wanted, a prism may be inserted between *m* and *m'* suitably adjusted. These fringes then also appear at once and may be put in any color, at pleasure, on rotating any mirror, say *N*, on a vertical axis. Rotation around a horizontal axis enlarges them.

Finally, if separate plate-glass compensators are placed in the paired beams *8 5* and *2 1*, for instance, and rotated around a horizontal axis, independently, the fringes may be displaced up or down the slit-image for the purpose of measurement. A double-offset air-compensator, consisting of 3 right-angled V-mirrors with their corresponding faces parallel, the central V-mirror movable on a micrometer (described in my paper on gravitation) is available. Such a compensator would be placed normally to the rays *8 5* and *2 1*, for instance, to give them lateral path-length. In these cases the spectro-telescope may also be used where the strong bands register the displacement in any wavelength. Since the slit may be broad there is a great abundance of light.

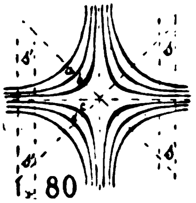
The rays *8 5* and *2 1* may be made of almost any reasonable length and distance apart, if the mirrors *N*, *N'*, *m*, *m'* are broad. To secure greater length the mirrors *m*, *m'* (rigidly connected) may be moved at pleasure in the direction *5 8*, without disturbing the fringes, good slides presupposed. The rays may be separated, if x is the available breadth of mirror, to an extent $x \cos 45°$, by moving the rigid system *NN'* in the direction *8 5*.

If either of the mirrors *m* or *m'* is separated (as, for instance, at *a* in fig. 79), the part may be placed on a micrometer; but the apparatus would not then be quite self-adjusting, for the parts will not in general be coplanar. But facility in adjustment is nevertheless increased.

To secure the best conditions for sharp, strong fringes the two slit-images seen in the telescope must be of equal intensity, and this depends on the half-silver *N*. On testing a number of plates it is usually easy to find one which fulfills this condition nearly enough. The fringes are still good, even when the intensity of images is noticeably different. The secondary images due to the reflection from glass faces are either weak or (if thick plates are used) these reflections may be blotted out by small opaque screens suitably fixed to the mirrors *m* and *N*. It is interesting to observe that with proper adjustment these secondary reflections carry their own fringes, some of which are modified in like conditions more rapidly than the main set. Intersecting fringes producing a beaded structure and fringes moving in opposite directions are also observed.

55. Character of the achromatic fringes.—Since the achromatic fringes are quite symmetric, consisting of a central white or fringe, flanked on either side by three or four colored fringes rapidly decreasing in intensity, it is obvious that (practically) they must consist of superposed monochromatic confocal hyperbolas. This may be well shown in the present apparatus, where the fringes are stationary and are displayed relative to horizontal and vertical lines of symmetry. To carry out the experiment, it is best to insert a single plate compensator (say *C*, fig. 79, 5 mm. thick) normally into the rays *8 5* and *2 1*, preferably in the same vertical plane. When the plate is not perfect it may be necessary to adjust for coincidence of slit images.

If, now, this plate is rotated about a horizontal axis (normal to the lines *8 5*, *2 1*) the fringes walk laterally through the broad coincident slit-images in such a way as to clearly outline a moving design of the form given in figure 80. In other words, as a first approximation (for the case is, of course, essentially more complicated) the achromatic fringes may be assumed to be a family of confocal equilateral hyperbolas, referred to given horizontal and vertical axes. When the rays *8 5* and *1 2* are at the same level, the broad slit-image is in a position of symmetry relative to the hyperbolas (fig. 80). When this is not the case, the image is at *ss* or *ss'*, with the fringes very rapidly becoming horizontal. Since this design is similarly carried out with decreasing coarseness from red to violet, it is clear that a single characteristic central achromatic fringe results, invaluable for purposes of displacement interferometry from its smallness, and since from the breadth of slit it can be made so intensely luminous. When the path-difference of the rays *8 5*, *1 2*, in figure 79 is changed by the micrometer or by independent compensators, the figure 80 shifts bodily up or down the slit-image. It is also obvious that when the fringes with white light are horizontal they must appear as horizontal black bands in the spectro-telescope, regardless of the width of slit used; and hence these fringes also are excessively luminous, while their displacement may be referred to a definite wave-length. If the interferometer is not self-adjusting the axes of figure 80 are as a rule inclined, and fringes are obtained in all angles of altitude needing special adjustment. The spectrum fringes then demand a fine slit,* but are also horizontal. The shift of achromatic fringes due to micrometer displacement may therefore be at once expressed in terms of the spectrum fringes rigorously in a given wave-length.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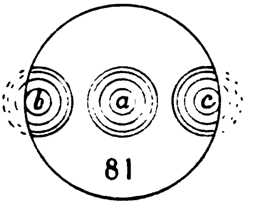

81

A very interesting transformation of the design (fig. 80) will be noticed, if by rotating any of the mirrors, *N*, *m*, figure 79, for instance, the two white slit-images seen in the telescope are passed horizontally through each other. During this motion the originally vertical, nearly linear, achromatic fringe passes through the form of the area between the hyperbolæ *a* and *b*, figure 80; next through the area between *b* and *c* (coincidence of slit-images); then into the area between *c* and *d*; finally again into a vertical hair-line, always retaining its individuality among the surrounding colored fringes of similar shapes.

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* A method of avoiding this condition will be given in Chapter IX.

The whole is particularly vivid if the fringes are observed with the ocular drawn well forward, quite out of focus. The same end may be reached by adding a diopter spectacle lens, convex or concave, to the objective. The bearing of this will be seen presently.

56. Curvilinear compensators.—If the rays *8 5* and *2 1* in figure 79 are brought to coincidence, it is obvious that a lens, either convex or concave, may be inserted between the mirrors *m* and *m'* and normal to the rays, without destroying the interferences, though they must be greatly modified in form. If the lens is symmetrically inserted, the two broad slit-images will be equally wide, so that coincidence is perfect. The fringes so obtained (fig. 81) are usually large, brilliantly colored circles, while in case of imperfect plate they become oval and coarse. The large central disk *a* is achromatic. To center the fringes the mirror *m'* may be rotated on a vertical and horizontal axis until the symmetrical circular figure is obtained. Here again the individuality and even the approximate position of the achromatic fringe is retained on passing the broad slit-images through each other; but the sequence of types of fringe is peculiar. As the slit-images separate (see fig. 81) toward the right or the left, as a result of the corresponding rotation of *m* on a vertical axis, the originally colorless disk *a* of the circular fringes moves to the right or left, but at the same time becomes very vividly colored (*b* and *c*). The coarse fringes now show considerable resemblance to the coronas of cloudy condensation, in which there is also a colored disk. When the slit images have been markedly separated, the disk vanishes and thinner lines appear, at first as complete circles surrounding the fading disk, but rapidly losing curvature to become vertical. Throughout the whole transformation there has been a grouping of symmetrically concentric colored circles on both sides of the achromatic circle. To state this differently, each originally linear fringe in turn, on expanding (slit-images approaching coincidence), contracts vertically and broadens horizontally into a disk, which retains the color of the fringe out of which it originates. The same result may be obtained by moving the lens inserted between *m* and *m'* into both rays, fore and aft (direction *8 5*). Similarly the corresponding sequence between horizontal fringes appears on moving the lens up and down. If the *m*, *m'*, mirrors are moved bodily fore and aft, however, the circular fringes merely pass horizontally through the field, without appreciably changing form.

It makes little difference whether concave or convex lenses are introduced between *m* and *m'*, except that the objective of the telescope will have to be armed with a convex or a concave lens (of corresponding strength), respectively, to assist in focussing. But here again the most vivid effects are obtained with the ocular drawn out of focus. Sunlight falling on the slit without a condenser gives the best definition. I examined lenses of 1, 2, and 3 diopters of focal power, concave and convex. There would be nothing against the treatment of stronger lenses; only the secondary adjustments become increasingly difficult, unless special devices are resorted to. Figure 81 shows the case as observed with 3 diopter concave lenses between *m* and *m'*, by an ordinary telescope without additional objectives. The slit-images being quite out of focus, the field is uniformly illuminated. Through it pass the succession of forms, *b*, *a*, *c* (as described), when (for instance) *m* is rotated slowly on a vertical axis. Many of the forms are quite visible to the naked eye. If the fringes are moderately fine, complete hair-like circles may be produced at *b* and *c*. Moreover, beginning with the symmetrical position of the lens, the rays *2 3* and *8 7* need not be coincident.

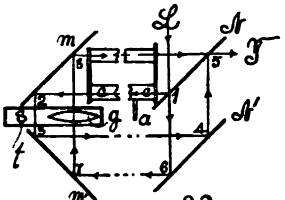
If the lens is not symmetrical in form, *i.e.*, for plano-convex meniscus and other lenses, the simple figures above discussed become more complicated and the fringes multi-annular.

57. Index of refraction, irrespective of form.—If a plane-parallel trough, *t*, containing a solution of mercury potassic iodide, *s*, is placed between *m* and *m'*, figure 82 normally to the rays, neither the achromatics nor the spectrum fringes (broad slit admissible) are affected. Inclination to the normal position will change the size of fringes only. Hence if a piece of glass, *g*, is inserted into the trough with the rays separated as at *2 3* and *8 7*, in figure 79, one of them only, say *8 7*, passing through the glass, the spectrum fringes will change form or vanish, except at that part of the spectrum, in which the index of refraction of the glass and the solution are identical, provided the dispersion coefficients are also the same. It is thus of interest to determine to what degree the method can be practically utilized.

In the case of bodies of regular form, like lenses, spectrum fringes and even achromatics will usually appear when the sharply seen, fine slit-images coincide in the principal focus, *i.e.*, the position of the ocular for parallel rays. But the fringes will as a rule be in other focal planes. When one slit-image is not quite white, and if there is a small angle between them, the fringes will run obliquely through the spectrum from top to bottom. Different regions will be brought out by different focussing of the ocular. By trying out the four edges of the half-silver, one may usually be found in which the vertical coincidence of the slit-images is perfect and these annoyances disappear.

58. The same. Glass Plate. Lenses.—In relation to the principle (§57) in question, if a plate of glass of higher refractive index is introduced into the solution, *s*, figure 82, and transversed by one ray only, the original intensely black, nearly horizontal bands in the spectrum are changed to much finer lines, at a considerable angle (45°, etc.) to the horizontal. This inclination is symmetrically down toward the red, or up toward the red, according as one or the other beam traverses the glass. Moreover, the size of the fringes now decreases in much more pronounced ratio from red to violet. The achromatics have necessarily vanished. It would need special compensation (horizontal spectrum fringes) to restore them, and from this compensation the difference of index between solution and glass could be computed. Had there been no difference at any color, the horizontal fringes would have been retained in that part of the spectrum. Such experiments may be made with astonishing ease and accuracy, but this interpretation is involved.

Now, as every lens is practically a plate for a small area near its optical center, it is obvious that the same results must be obtained for a limit horizontal band of the spectrum, if lenses are used as at *g* in figure 82. I tested a concave spectacle-lens of about 2 diopters focal power, suspending it in the solution from a pair of hard-rubber forceps, *f*, figure 83, slotted at the end to receive the lens *g*. As one beam only passed through the lens, which was here stronger in refractive index than the solution, the slit-images were not in the same focal plane, but the corresponding parts were of the same height in the field. On bringing the two slit-images into coincidence, the spectrotelescope showed the fringe design in figure 84, the spectrum being clear from violet *v* to red *r*, except at the longitudinal band *ab*. On raising and lowering the lens in the solution, this band, *ab*, rose and fell correspondingly, and when the lens was withdrawn the horizontal fringes at once appeared. As a rule the band *ab* is not seen sharply in the principal focal plane and the ocular must be drawn out or in, or an objective lens added. If the other beam (*2 3*, fig. 82) is made to pass through the lens, the same phenomenon is observed, except that the fringes are now upward toward the red. If the ocular is drawn out in one case it will have to be drawn in for a sharp display of fringes in the other. This seems to be due to the fact that the middle position (equidistant from the half-silver mirror *N*) coincides with the mirror *m'*, so that the lens is not quite symmetric in either position. Sharp fringes occur when the area of the washed slit-images (irrespective of sharpness of focus for slit-images) coincide.



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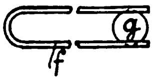

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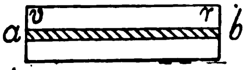

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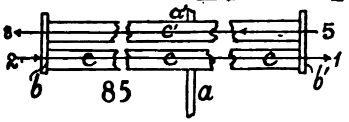


85

59. Adjustable compensators.—It is difficult to install a V-compensator into either beam *8 5* or *2 1* in figure 82, as the rays are usually too close together. A Billet glass wedge compensator suggests itself, but the introduction of glass-path excess modifies the fringe number and would ultimately obscure or wipe out the achromatic fringe. For the case of spectrum fringes, however, it proved to be a very serviceable instrument.

The air compensator (fig. 85) is also available. Here *cc*, *c'* are a pair of narrow rectangular brass tubes, about 1 cm. wide, 2 cm. high, 10 or more cm. long, closed at each end with the glass plates *b* and *b'*. Each glass plate is common to both tubes, so that both beams *8 5* and *2 1* may penetrate identical glass-paths. The tube *cc* is exhausted to a known degree through the tubulure *a*, similarly *c'* through *a'*. The compensator is shown in place in figure 82. Brass tubes about 15 cm. long are usually of convenient size and will incline the horizontal spectrum fringes about 30°, or restore them from 30° of inclination, if exhausted.

60. Observations of refraction.—The methods of procedure have been elucidated in the preceding paragraphs relating to the self-adjusting interferometer. It is there inferred (in connection with figure 82) that we may either make a solution like mercury potassic iodide (for instance), optically identical with that of the glass at any given wave-length of light, by diluting the concentrated solution; or that we may interpolate between two given solutions, or from one given solution up or down, by aid of the Billet or the air compensator. In case of a plate about 3 mm. thick, an air compensator (fig. 85) about 15 cm. long will throw the center of spectrum ellipses from the extreme red into the yellow and correspondingly for the other colors. A second solution would thus admit of interpolation from the green toward the blue, etc., by aid of the same compensator. Three solutions should suffice. A longer air compensator could of course be used; but this would correspondingly elongate or else complicate (if introduced on both sides, in rays *2 1* and *3 4*, fig. 82) the apparatus and thus militate against easy manipulation. The range of the Billet compensator is longer and its control much easier.

To bring the center of ellipses into the field of the spectrum, it suffices to use the trough containing the solution *s* with or without the body *g*. The trough need merely be rotated on a horizontal axis until centers of ellipses appear or (if centers are beyond the limits of the visible spectrum) the fringes are vertical. In such a case the center of ellipses is not displaced longitudinally (red to violet) in the spectrum and there is no interference with the measurement based on longitudinal displacement. This easy possibility of adjustment is a great convenience in practical work. Allowance, however, must be made for the virtually increased thickness of the plate submerged.

When the mercury potassic solution is concentrated, almost any glass plate submerged and penetrated by but one beam will show only very fine spectrum interference fringes, hair-lines, which may be made vertical, as just specified. It requires considerable dilution (more than one-half) to obtain large fringes. These eventually show curvature, and finally the center of ellipses appears in the extreme red. On further dilution (which must now be made more cautiously) the center passes through the spectrum from red to violet.

If, in place of plate glass, ground glass is substituted, or in any case of window glass, the ellipses come out remarkably well, quite adequately for measurement. With very coarse ground glass and irregular fragments of glass this is not apt to be the case.

Lenses of several diopters, so long as the index of the glass is identical with that of the solution within the spectrum, show clear fringes throughout the height of the spectrum; but the lines are eventually apt to be doubly inflected.

The center of fringes is very definite throughout and the measurement nearly as easy as with plates.

If a lens of 15 or more diopters is placed in the solution (ellipses centered in the green or yellow ray), the fringes become very indistinct, because of the different divergence of the rays of the two slit-images. In this case it is interesting to let both beams pass through the lens, when the two slit-images will usually be short spectra, direct or reversed, according to the manner in which the beams pass the lens. In the latter case one obtains the interferences of reversed spectra at the line of color coincidence of the two spectra. If the phenomenon is enlarged by the spectro-telescope, an oval area of very sharp fringes may be obtained and the area passed from end to end of the spectrum by rotating the mirror *m* on a vertical axis. If the ellipses are centered, horizontal arrow-like figures are usual, much resembling a shark's head with the mouth strangely outlined in black.

61. Equations and data.—An example of data obtained when the Billet compensator was used for interpolation may now be given. In the earlier paper (l.c.) it is shown that as a rule the equation

$$e(\mu-\mu^1+2(B-B')/\lambda^2)=n\lambda \tag{1}$$

adequately represents the experiment. Here μ, B, μ', B', are the indices of refraction and dispersion constants

$$\mu=A+B/\lambda^2$$

of the glass and solution, respectively; e the thickness of plate; λ the wave-length of light; and n the number of λ-fringes which pass a given fixed point in the telescope. We may write

$$n\lambda=\left(\frac{\lambda}{\Delta x/n}\right)\Delta x=C\Delta x \tag{2}$$

if Δx is the displacement of the wedge of the compensator corresponding to n wave-lengths. The factor C refers to the displacement of wedge per fringe, to be found experimentally. Hence

$$\mu=\mu'-\frac{2(B-B')}{\lambda^2}+\frac{C}{e}\Delta x \tag{3}$$

If the medium μ', B' is air, the equation becomes practically ($\mu'=1$, $B'=0$)

$$\mu=1-\frac{2B}{\lambda^2}+C\frac{\Delta x}{e}$$

Using this for the wedge of the compensator, if δe is the displacement of wedge for 100 fringes,

$$\delta e=\frac{C\Delta x}{\mu-1+2B/\lambda^2}$$

In an experiment 100 yellow fringes ($10^6\lambda=60$ cm.) corresponded to 22.65 scale-parts of the wedge micrometer-screw, so that

$$C=10^2\times6\times10^{-5}/26.65$$

is the conversion factor for any Δx. If $\mu = 1.525$ and $2B/\lambda^2 = 0.026$, we may therefore write

$$\delta e = 10^2 \times 6 \times 10^{-5}/(0.525 + 0.026) = 1.11 \times 10^{-2} \text{ cm.}$$

or the increment of glass-path due to 100 fringes is 0.0111 cm. at the wedge.

The wedge, calipered, showed an increase of 0.01 cm. per centimeter of length. The scale-part was 0.05 cm. Hence 22.65 scale-parts (100 fringes) correspond to an increase of thickness of

$$\delta e = 22.65 \times 0.05 \times 0.01 = 0.0113 \text{ cm.}$$

agreeing as closely as may be expected with the preceding optical result.

The first experiments were made with a trial plate of glass, $e = 0.202$ cm. thick, traversed by one or the other beam when submerged in aniline oil, purified by distillation. The test showed that 100 fringes corresponded to $\Delta x = 21.85$ scale-parts, so that $C = 2.7 \times 10^{-4}$. The values μ' and B' for the liquid were computed from Perkin's experiments (Landolt and Boernstein) and for the D line; $\mu' = 1.59073$ at 11.2° and $B' = 10^{-11} \times 12.5$ were taken (between D and F, B' increases to $10^{-11} \times 13.4$). Hence equation (3) becomes (as Δx is negative)

$$\mu = 1.59073 + \frac{2(12.5 - 4.6)10^{-11}}{(5.893 \times 10^{-5})^2} - \frac{2.7 \times 10^{-4}}{0.202} \Delta x$$

The displacement of wedge to keep the center of ellipses in the field when the glass plate was put in one beam of light or the other, successively, was found to be $2\Delta x = 158.0$ scale-parts of wedge. Hence

$$\mu = 1.5907 + 0.0452 - 0.1056 = 1.5303$$

For such an enormous interpolation, just in fact within the limits of the micrometer-screw, one can not have much confidence in the result. Aniline is an unsuitable liquid for this glass, though it would do very well perhaps for flint glass. Moreover, the temperature of the liquid ($d\mu/dt = 0.0006$) was not taken.

The next experiments were therefore made with benzol, also taking Perkin's data (l.c.). Here $C = 2.60 \times 10^{-4}$ for the D line and $\mu' = 1.50871$, $B' = 8.8 \times 10^{-11}$ at 8.5° C (between D and F, $B' = 9 \times 10^{-11}$); $e = 0.202$ cm.; $B = 4.6 \times 10^{-11}$. The experiment gave $2\Delta x = 18.9$. Hence

$$\begin{aligned}\mu &= 1.50871 + 2(8.8 - 4.6)10^{-11}/(5.893 \times 10^{-5})^2 + (2.6 \times 10^{-4}/0.202)9.45 \\ &= 1.50871 + 0.02421 + 0.01223 = 1.5451\end{aligned}$$

This is experimentally probably a good result. Its absolute value will depend on temperature conditions and the B value of the glass. The results show very well the relative importance of the terms in Δx and in B'. The large B' for all liquids which I have tried militates against the method.

A second experiment with the trough more carefully adjusted to the vertical and with the insertion of a glass-plate compensator to modify the size of fringes gave $2\Delta x = 19.13$ for the same plate. Hence $\mu = 1.5453$, coinciding closely with the first result. [illegible] liquid taken [illegible] obviously very large.

TABLE 6.—[illegible]

Liquid.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Aniline, [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Benzol, [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]					[illegible]	[illegible]	
Achromatic fringes									
Benzol, [illegible]	[illegible]	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]
Benzol, [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

To endeavor to reconcile [illegible] values with benzol, experiments were made at [illegible]. The [illegible] could be conveniently [illegible] whereas [illegible] benzol the displacements were taken at the [illegible] and F lines. The spectrum [illegible] was unfortunately [illegible] for many [illegible], but the results obtained are given for all the cases in table [illegible]. The displacement per [illegible] fringes was at first taken at the [illegible] line only, later [illegible] value, at the E and b lines also. These values are bracketed. From these the coefficient λ [illegible] follows. B was computed as above from [illegible] results and B' (glass, temporarily put [illegible] there being no direct means [illegible]). The value of μ at [illegible] corrected for the temperature of aniline and benzol still differs considerably in the two cases [illegible] as much as [illegible]. If the above equations are correct and the rather large [illegible] for [illegible] differences [illegible] to [illegible] and [illegible] be admitted, then there remains only the possibly insufficient purity of the distilled liquids or the assumed $B=4.6\times 10^{-[illegible]}$ for glass, to account for this.

The table gives Δx for benzol at the successive spectrum-lines [illegible]. If we suppose the displacement of wedge per 100 fringes does not vary in marked degree with the color, we may compute λ/Δx and consequently

$$B-B'=\frac{\lambda/\Delta x\cdot\Delta x-\lambda'/\Delta x'\cdot\Delta x'}{3e\left(\frac{1}{\lambda^2}-\frac{1}{\lambda'^2}\right)}$$

for intervals between the successive spectrum-lines. Here if λ/Δx is replaced by the mean value between two spectrum lines and considered constant and if $\delta x=(\Delta x)_C-(\Delta x)_D$, etc.,

$$B-B'=(\lambda/\Delta x)\delta x,\ 3e\left(\frac{1}{\lambda^2}-\frac{1}{\lambda'^2}\right)$$

for these intervals, and δx may be taken from the table. The results now come out more smoothly. In this way we find

Lines	C		D		b		F
$10^4(\lambda/\Delta x)$	2.84		2.55		2.24		2.10
δx		5.5		8.8		9.9	
$10^7(1/\lambda^2-1/\lambda'^2)$		5.57		8.58		4.95	
$10^{11}(B-B')$		4.4		4.1		6.77	

The datum computed above for $(B-B')$ 10^{11} is 4.2 to 4.4 as far as the b line and does not differ much from the tabulated values between C and b.

Later I made an actual count of wedge displacement for 100 fringes at three spectrum-lines with results as follows (δx and $\Delta\left(\frac{1}{\lambda^2}\right)$ being as above):

Lines	C		D		b	
Δx	24.9		22.9		20.1	(observed)
$10^4(\lambda/\Delta x)$	2.64		2.57		2.57	
$10^{11}(B-B')$		4.25		4.35		
$10^{11}(B-B')$		4.2		4.4		(as originally taken)

These values are so nearly in accord with the data used (particularly as the A and b lines are dark) that a discrepancy here can hardly be looked for. The results of the rigorous equation would be $10^{-11}\times 4.5$ and $10^{-11}\times 4.4$, respectively.

It is rather unfortunate that the achromatic fringes can not be used for the present purposes, as they are very clear; but in the first place the wave-length to which μ belongs is not implied. In the second place, with an increase of glass-path, they soon multiply beyond recognition. Using the nearly colorless central fringes of the group (flanked on either side by reddish fringes), the data, also given in table 6, were obtained. The value of μ computed therefrom lies about midway between the aniline and benzol results. This is much better than was anticipated.

Experiments were now made with the air compensator for comparison. In this case, if ΔN is the equivalent air-path removed by the exhaustion of the compensator of length E, we may write

$$\Delta N = E\left(\mu_0 - 1 + \frac{2B''}{\lambda^2}\right)\frac{p}{76}\frac{273}{\tau}$$

if μ_0 is the index of refraction of air under normal conditions and $B'' = 1.65\times 10^{-14}$ the dispersion constant of air. Thus the equation becomes in the above notation

$$E\left(\mu_0 - 1 + \frac{2B''}{\lambda^2}\right)\left(\frac{p}{76}\frac{273}{\tau}\right) = e\left(\mu - \mu' + 2\frac{B-B'}{\lambda^2}\right)$$

If for $\mu_0 - 1$ we write 292.7×10^{-6} and neglect B'' ($E = 13$ cm. being the length of the compensator used) we get

$$\Delta N = 292.7\times 10^{-6}\times 13\times 273p/76\tau = 1.367\times 10^{-2}p/\tau$$

On submerging the above plate of thickness $e=0.202$ cm. in benzol, it was found that the length $E=13$ cm. of air-tube sufficed to center the spectrum ellipses in the red when completely exhausted. Hence the limiting ΔN for one tube would be $293\times 10^{-6}\times 13$ or 0.0038, whence

$$\Delta N/e=0.0038/0.202=0.0188$$

as the range of possible accommodation. The ellipses entered on exhaustion at the violet and reached the yellow at above $p=50$ cm., the temperature being about 20° C. This would agree with the increment given by the Billet compensator (0.0123), at 20° and a pressure difference of 53.3 cm.

In a later experiment at the temperature 16.5°, $p=46$ cm. was observed giving for the μ increment

$$0.01367\times 46/289\times 0.202=0.0108$$

also agreeing with the corresponding datum of the Billet compensator (0.0107). Unless the ellipses are very strong, however, and not too large, it is difficult to set the air compensator to a centimeter of pressure, which is equivalent to about two units in the fourth place of μ.

The air compensator is thus not very convenient unless the experimental equipment is elaborate. A single tube will only give Δx and not $2\Delta x$, unless it is shifted from one ray to the other; but this is dangerous, as it is liable to modify the fringes. Hence the double-tube method is almost essential, which implies a definite distance apart of the tubes and the same annoyance introduced by the Billet apparatus. The latter, though it requires special standardization, is much more easily manipulated.

Work of the same kind as that given in table 6 was carried out at greater length, but with no essential improvement in the results as a whole. As there are three terms, viz, the one in Δx, the one in $(B-B')$, and the temperature correction from 8.5°, all about of the same order, to be added to the μ' of benzol, better agreement is hardly to be expected, unless a plane parallel optic trough is used.

62. Micrometer measurements.—The use of a screw micrometer in case one of the separated mirrors in figure 79 was suggested above. Experiments of this kind are given in table 7. The mirror N' was in this case preferably separated at b, each half being on a screw normal to the face. The adjustment (first made for trial with an unseparated mirror) proved to be very easy. The two mirrors replace the single mirror, the apparatus being nearly in adjustment.

In this case, if the plate is put successively in the two beams, the displacement of micrometer is again twice that (ΔN) which corresponds to the thickness e of plate, and if i is the angle of incidence (here $i=47°\ 10'$)

$$\mu=\mu'+2(B-B')/\lambda^2+\frac{2\ \Delta N\cos i}{e}$$

Two plates of glass were examined without difficulty, the absolute value of μ depending on the purity of the benzol. This appears in the last result for the first plate. The screw micrometer has a much wider range than the Billet compensator. It admits of the trial of thick plates. It does not require standardization. On the other hand, however, it is not so sensitive. A comparison of the two shows, for instance,

Screw....	3.1	6.4	9.8	18.4	10^{-3} cm.
Billet....	95.9	79.0	61.0	17.1	scale-parts

so that about 5 scale-parts go to one 10^{-3} cm. of the screw, both of which can be read to tenths of these divisions.

TABLE 7.—Refraction of plates submerged in benzol. $i = 47° 10'$.
Constants as in Table 1.

e	Temp.	$10^3 \times \Delta N$	$10^3 \times 2\frac{B-B'}{\lambda^2}$	$10^3 \times \frac{2\Delta N \cos i}{e}$	μ	Line.
0.202	23.0°	+3.23	19.7	21.7	1.5317	*C*
		+2.02	24.5	13.6	1.5333	*D*
		+ .46	31.8	3.1	1.5377	*b*
		− .35	36.0	− 2.3	1.5411	*F*
.307	23.2	5.45	19.7	24.0	1.5339	*C*
		3.86	24.5	17.1	1.5367	*D*
		1.61	31.8	7.1	1.5416	*b*
		.30	36.0	1.3	1.5446	*F*
.501	23.2	7.15	21.0	19.3	1.5354	*D*
0.202 *	22.4	2.30	24.5	15.5	1.5359	*D*

* Fresh benzol.

63. Summary.—The present method of using the self-adjusting interferometer has an advantage as compared with the older one (in which but one of the interfering beams traversed the trough), inasmuch as the wall of the plane-parallel trough and the liquid contained are of no influence on the results. Such an effect would be added to both beams under the same conditions. Moreover, by putting the glass-test objective into the beams successively twice ($2\Delta x$) the micrometer displacement is obtained and the null position is of no consequence.

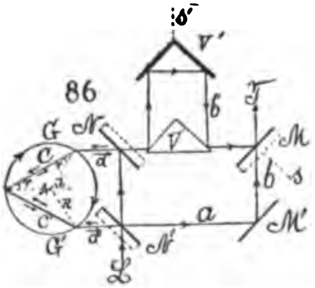
On the other hand, there are distinct disadvantages, because the range of displacement of the compensation micrometer (vacuum air compensator, Billet wedge micrometer) are limited. The spectrum ellipses are rather too large for convenience, therefore. One might add glass relay plates counterposed by air-paths to the Billet wedge; but these (even if of optic plate) are liable to modify the fringes. For the same reason the dispersion constants of a single glass plate can not be easily found, a problem for which the older apparatus was particularly well adapted.

Some other liquid with a refraction between benzol and aniline (probably a mixture of these), would have to be used for the examination of glasses in general. The difference could then be expressed by the Billet or the air-micrometer. The former has a large range, controls the fringes with remarkable facility, but has to be standardized in terms of them, for each color. The vacuum air-micrometer has a small range and is cumbersome. Finally, there are two annoyances incident to the method as a whole, viz., the relatively large dispersion coefficient of liquids and their relatively high temperature coefficient. The experiments therefore require extreme care as to details throughout. Otherwise the present method is quite self-contained for $B-B'$, the difference of dispersion coefficients, is found on passing the ellipses through the spectrum.

CHAPTER VIII.

AN ADJUSTMENT IN RELATION TO THE FRESNEL COEFFICIENT.

64. Apparatus, one internal reflection.—Figure 86 gives an application of the quadratic interferometer for the possible demonstration of ether drag. Here *M*, *M′*, *N*, *N′* are the mirrors of the interferometer and *L* and *T* the collimator and telescope of this design, the rays passing as shown. In place of the auxiliary mirror to the left of *NN′*, the transparent glass cylinder or disk *GG′*, capable of rotating rapidly around an axis *A* normal and symmetrical to the ray parallelogram, has been installed. The mantle of the cylinder is supposed to be carefully ground. If the index of refraction *GG′* is adequate, the component ray reflected at *N′* takes the path *N′C′CNM* and then enters the telescope *T* after reflection. The ray traversing *N′*, on the contrary, takes the path *N′NCC′N′M′M* and also enters *T* to interfere with the preceding ray. In other words, the two rays are refracted in like manner by the cylinder *GG′*, at the angles of incidence and refraction *i* and *r*, respectively, where $i = 2r$, since *R* is the radius of the cylinder.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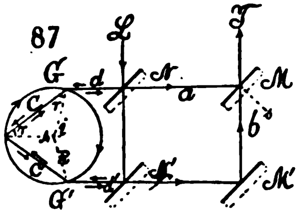
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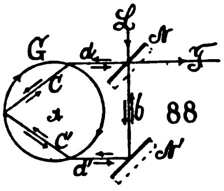
88

If now the cylinder rotates clockwise to the eye, as shown by the arrows, the rays *N′C′CN* will be accelerated and the rays *NCC′N′* retarded by the same amount while in the cylinder, and the question is to what degree such an effect should be observable.

The adjustment is made for reversed spectrum or linear fringes as presently to be indicated.

If, in figure 86, the half-silver mirrors, all of equal thickness, are set with their silver reflecting faces as indicated, each ray traverses the glass-path thrice, and since *M* and *N* are to be quite equal in thickness, the glass-paths are equal. Not so the air-paths, for these differ by $2b$. Hence the V-compensator *VV′* must be introduced in the ray between *N* and *M*, so that the missing $2b$ may be thus inserted, as shown in figure 86. The right-angled mirror *V* is stationary, while the reentrant mirror *V′*, with its sides respectively parallel to *V*, is on a micrometer, the screw *s′* being directed in the direction of *b*. The displacement is thus virtually normal and the rays *CC′* are not dislocated on operating *s'*. This is the case when other screws, except those normal to *M* and *M'* are used. *V' s'* is conveniently near the observer at the telescope *T*.

If the offset or V-compensator is to be avoided, a design (fig. 87) must be used in which all mirrors are parallel. If the half-silvers are equally thick and placed with their reflecting faces as shown in the figure, each ray traverses the glass-path twice and the compensation is complete, seeing that twice the glass-path in *N* is identical with the glass-path in *M* and *N'*. The air-paths are the same. Moreover the mirror *M* (for instance) may be displaced by a micrometer-screw in the direction *s* (within limits) without changing the adjustment at the cylinder *GG'*.

If micrometer facilities are to be dispensed with (and that is permissible in the present experiment), the design shown in figure 88, which is now a modification of Michelson's interferometer, suffices. The white light *L* from the collimator takes the respective paths *dCC'd'b* and *bd'C'Cd*, the plate *N* being half-silvered and *N'* an opaque mirror. The telescope or spectro-telescope is at *T*. The glass face at *N* may be turned either way.

Such an interferometer is self-adjusting. In the design, figure 88, two reversed spectra will be visible in the telescope, which if superimposed by rotating *N* or *N'* on a vertical axis will show the linear phenomenon at once, in any color at pleasure. The fringes may be enlarged by rotating *N* or *N'* on a horizontal axis, and they are symmetrically equal in size on the two sides of the adjustment for infinitely large fringes.

If the achromatics are wanted, a prism must be inserted into the rays *b* (preferably between *N* and *N'*, figure 88, with a prism angle selected to counteract the refraction of the cylinder *GG'* in the manner indicated in figure 93 of paragraph 68.

65. Apparatus. Two internal reflections.—As the fringes were found without much difficulty (68) in case of one internal reflection, it seemed desirable to determine whether this would still be feasible in the apparently more favorable but also more difficult case of two internal reflections. In figure 89, white light arrives from a collimator at *L* and strikes an auxiliary mirror *m*, before reaching the half-silver *N*. If *m* is capable of rotating both on a horizontal and vertical axis, as well as sliding right and left in the diagram, it greatly facilitates adjustments of angle and location of rays. The two beams *bcdef* and *bfedc* reunite at *b* after passing the glass cylinder *G* (rotating around the axis *a*) and are observed by the telescope at *T*. As the spectra (after refraction at *c* and *f*) are reflected three and two times respectively, the fringes of non-reversed spectra will be obtained covering the whole length of spectrum. A glass *G* of low index of refraction will here usually be preferable.

In case of a half-silver mirror at a small glancing angle, there are usually two pairs of bright spectrum images, and one fainter pair, apart from very faint ones. One bright and one faint pair carry identical fringes and the spectrum images may be small enough to be separated. In case of clear glass, however, there is practically but one pair of bright images, and they carry fringes when properly superposed.

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* Cf. Michelson and Morley: Amer. Journal, XXXI, p. 377, 1886. Also Zeeman, below.

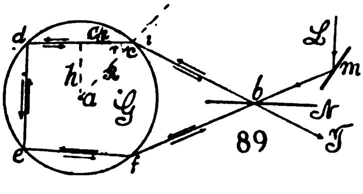
66. Equations. One internal reflection.—The first question to be elucidated is the nature of the conditions of refraction. From figure 88, in view of the symmetry of the arrangement, if b is the breadth of the ray parallelogram and R the radius of the cylinder and h the distance of the chord C from the axis A,

(1) $$\sin i = \sin 2r = b/2R$$

(2) $$\sin r = h/R$$

and the index of refraction

(3) $$\mu = \sin i/\sin (i/2) \text{ which reduces to } \mu = 2 \cos r = b/2h.$$



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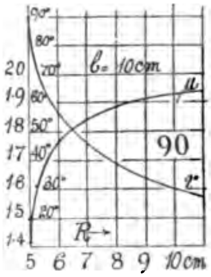

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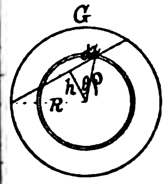

91

The relations remain the same if $b/2R$ is constant. If the (small) value $b = 10$ cm. is inserted into the equation, the results are such as shown by the graphs for i and μ in figure 90. It is seen, therefore, that for diameter $2R$ between 10 and 11 cm., the available indices of refraction of the glass would increase from 1.4 to 1.7 roughly, while the angle i falls from 90° to about 65°. Hence the experiment requires the interfering rays to impinge near the outer limits of the cylinder; but it is otherwise quite feasible. By narrowing the outside beam, only a small part of the caustic within the cylinder will be used.

It is next in order to consider the possibly observable conditions of apparent ether drag. The velocity within the refracting medium of index μ is usually written (or follows from the theory * of relativity) in the form

(4) $$c/\mu \pm v(1 - 1/\mu^2)$$

where v is the velocity of the medium in the direction, or contrary to the direction of the velocity of light c. It remains therefore to determine the average speed of the beam along the chord C of figure 86. From this figure

(5) $$C = 2R \cos r = \mu R, \text{ and } b = 2\mu h$$

whence,

(6) $$b = 2C\sqrt{1 - C^2/4R^2} = 2\mu R\sqrt{1 - \mu^2/4}$$

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* The insufficiency of this equation has been shown by Zeeman (Amsterdam Acad Sept. 1914 and Sept. 1915). But an estimate only is above in question.

In figure 91 let ω be the angular velocity of the cylinder and dx an element of the chord C at a distance ρ from the axis A. Let the minimum distance of this chord from A be h, and θ its angle with ρ. Then

$$dx = \rho\omega dt/\cos\theta = \rho^2\omega dt/h$$

if dx is described in the time dt. Hence

$$dx/dt = (h^2+x^2)\omega/h \tag{7}$$

To find the mean speed v along C, we may multiply dx/dt by dx, integrate between 0 and $C/2$, and divide the result by $C/2$. Thus

$$v = \frac{2}{C}\int_0^{C/2} \omega(h^2+x^2)/h\,.\,dx = \omega(h + C^2/12h) \tag{8}$$

Reducing this by equations (1), (2), (5), eventually

$$v = R\omega\frac{1-\mu^2/6}{\sqrt{1-\mu^2/4}} \tag{9}$$

or the mean speed along C may be expressed in terms of R, ω, μ; v is naturally proportional to R and ω.

The ratio of the speed in equation 9 (seeing that it is respectively + and − for the two interfering rays) to the velocity of light is thus $2v/c$. Since these rays traverse a path $2C$ in the rotating cylinder in opposite directions, the path difference resulting will be

$$\Delta P' = (2v/c)2C = 4\frac{Cv}{c} = \frac{4\mu\omega R^2}{c}\,\frac{1-\mu^2/6}{\sqrt{1-\mu^2/4}} \tag{10}$$

so that the path difference for a given μ and ω increases with the square of the radius R of the cylinder or disk.

But equation (4) introduced another factor $(1-1/\mu^2)$, so that finally the path-difference is

$$\Delta P = 4\omega R^2\frac{\mu}{c}\,\frac{(1-\mu^2/6)(1-1/\mu^2)}{\sqrt{1-\mu^2/4}} \tag{11}$$

We may now take the above data ($b = 10$ cm.) from figure 91, for a small cylinder, making $n = 100$ turns per second.

$R = 5.3$ cm. $\mu = 1.63$ $\omega = 628$ $i = 70.6°$ $r = 35.3°$ $b = 10$ cm.

In accordance with equation (10) therefore, since $\sqrt{1-\mu^2/4} = 0.58$ and $1-\mu^2/6 = 0.44$ nearly, the uncorrected path-difference is

$$\Delta P = \frac{4\times1.63\times0.628\times(5.3)^2}{3\times10^{10}}\,\frac{0.444}{0.577} = 2.95\times10^{-6}\text{ cm.}$$

The corrected path-difference $\Delta P = \Delta P'(1-1/\mu^2)$ thus is finally

$$\Delta P = 2.95\times10^{-6}\times0.623 = 1.84\times10^{-6}\text{ cm.}$$

The fringes which appear in the above interferometer are primarily those of reversed spectra. If the yellow parts of the spectra ($\lambda = 60\times10^{-6}$ cm.) are superposed, $\frac{1.84 \times 10^{-6}}{60 \times 10^{-6}} = 0.031$ of a fringe would pass for the given radius of cylinder ($R = 5.3$ cm.) at 100 turns. A cylinder 30 cm. in diameter (about a foot) would therefore show 0.28 fringe, and since this may be doubled by reversing the rotation of the cylinder (by which the strains due to centrifugal force are also eliminated), something short of two-thirds of a fringe should be observed.

With an ocular micrometer divided in 0.1 mm., there should be no difficulty in making the fringes 3 mm. apart, so that a displacement of 20 scale-parts may be expected, 10 for each of the directions of rotation.

In the present experiment the reflection within the cylinder can not be total, for it is obvious that if a ray gets into the cylinder it will under like conditions come out again. Some advantage would be obtained from a thin coat of silver. If x is the fraction of light reflected, that entering the telescope should be proportional to $x(1-x)^2$, which is a maximum when $x = 1/3$. The experiments, however, show no serious difficulty from deficient light,

67. Equations. Two reflections.—The equations for this case are somewhat more involved than the preceding; but it suffices to accept for the angle of incidence i at the cylinder G, figure 89, the value given by the old-fashioned theory of the rainbow, viz,

$$8 \cos^2 i = \mu^2 - 1 \tag{12}$$

The chord C from c to d, etc., and its distance h from the axis a will be, as before, $C = 2R \cos r$, $h = R \sin r$, where r is the angle of refraction and R the radius of the cylinder. Finally, equation (8) for the average speed v along a chord also applies. Hence with the inclusion of equation (4), the path-difference on rotation may be written, c being the velocity of light.

$$P.D. = 3 \times 2C(v/c)(1 - 1/\mu^2) \tag{13}$$

since there are three chords, C, on sequence. This equation may be reduced by the equations for C, h, v, to

$$P.D. = 4 \frac{\omega R^2}{c} \frac{\cos r(1 + 2 \sin^2 r)}{\sin r} \left(1 - \frac{1}{\mu^2}\right) \tag{14}$$

and by equation (12) to

$$P.D. = \frac{9R^2\omega}{c}\left(\frac{3}{\mu^2} + 1\right)\sqrt{\frac{(1 - 1/\mu^2)^3}{(9/\mu^2 - 1)}} \tag{15}$$

a form convenient for computation.

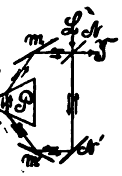
Data similar to the above may now be inserted, viz, for a small cylinder of water (to be used in the experiments below), $R = 5$ cm.; $\mu = 1.33$; $\omega = 6.28$; $c = 3 \times 10^{10}$, whence

$$P.V. = 9 \frac{25 \times 628}{3 \times 10^{30}} \times 2.70 \times \sqrt{0.082/4.09} = 1.82 \times 10^{-6} \text{ cm.}$$

This, curiously enough, is about the same value as was obtained in case of equation (11), so that the same deductions apply. The conditions are somewhat more favorable for larger values of μ. Thus in the limiting case $\mu=\sqrt{3}$, the path-difference would be about doubled.

68. Experiments.—To carry out these experiments at the present time is of course out of the question; but a number of contributory observations may be made with advantage. The case of figure 88 is in a measure similar to figure 92, where the dispersion of the cylinder G in the former case is simulated by the prism P and the auxiliary mirrors m, m', of the latter. If the slit of the collimator at L is not too coarse, two reversed spectra will be seen in the telescope at T, which on being superposed by rotating m or N on a vertical axis will show a vivid linear phenomenon in the lines of symmetry of the two superposed spectra. On rotating m or N on a horizontal axis, the distance apart of the fringe dots along this line may be given any reasonable value at pleasure. With simple means, therefore, the experiment can be made quite sensitive. These displacements are at once referred to the definite wave-length in which the linear phenomenon is put. The dispersion of the prism has no bearing on the clearness of the phenomenon; 30° and 60° prism were tested with like results.

92



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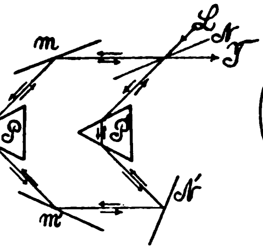
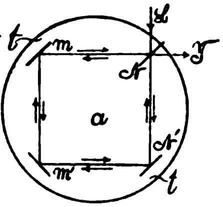




94

To obtain the achromatics and increased luminosity in the spectrum fringes (now to be horizontal bands throughout from red to blue), the rays of the spectrum will have to be reassembled, and that may be done as in figure 93, by inserting a second prism P' in a way to counteract the effect of the first. If the achromatics are to be obtained, the glass-paths of the two rays in P and P', respectively, must be coincident. Hence, the axis of the collimator at L must be inclined to accommodate the angle of minimum deviation of the identical prisms P, P'; and while N and m are parallel, N' and m' normal to each other, L and T have their axes symmetric to N. The adjustments are not difficult, as they need not be perfect to secure good achromatics; but if they are not made, the fringes are numerous, colored, and unsatisfactory.

The experiments, figures 92 and 93, differ from the case figure 88, because the rays are parallel in the former case and condensed to a caustic by the eccentric refraction of the cylinder in the latter. Hence with these a short-range telescope with strong objective is necessary; but, as has been stated, the lines of the solar spectrum nevertheless come out surprisingly clearly. Experiments were therefore made by simulating the glass cylinder *GG'* by a thin cylindrical glass shell, closed below and above and containing a solution of mercury potassic iodide, with an index at pleasure between 1.5 and 1.7. It was not difficult to meet the conditions of figure 88 so far as mere refraction is concerned, and certain incidental results obtained in this work are given elsewhere (Chapter X, § 79).

The active slit in this experiment is the image within the cylinder, *G*, of the slit of the collimator and the former is sufficiently fine to show Fraunhofer lines even when the latter is a millimeter or more broad, so that there is no deficiency of light.

But in relation to the detection of the interferences, the two reversed spectra, strongly divergent in their homogeneous rays, introduced certain grave difficulties. For it will appear that the spectrum issuing at *d'*, figure 88, passes over the distance *b* farther than the spectrum issuing at *d*, before they reach the telescope together. The result is that the two spectra lie in different focal planes, unless the telescope *T* is very remote. In other words, there must be parallax between the apices of the spectrum wedges. This makes the adjustment very difficult and I failed after long searching and with many devices to obtain any results, though I see no reason why the fringes should not occur. Identical spectra issue at *d* and *d'*.

To obviate this annoyance the symmetrical adjustment (Michelson and Morley) with an additional mirror at *d*, figure 88, corresponding symmetrically to *N*, and a symmetrically placed cylinder *G*, suggests itself. In such a case the spectra lie in the same focal plane, and since they have undergone two and three reflections, respectively, before reaching *T*, the interferences of non-reversed spectra are obtained without much difficulty. In my experiments, owing to the irregularity of the glass cylinder used, the fringes were also irregular, but otherwise clear and strong, as a wide slit is admissible. To adjust for coincidence it suffices to rotate the half-silver *N* on a vertical and a horizontal axis. The fringes are modified as to size, etc., by rotating *N* on a vertical axis and displacing it at the same time. They lie rather sharply in a definite focal plane.

The same difficulty is attached to the designs in figures 86 and 87, which also give reversed spectra and a path excess at *T*, of *b* for one of them, after issuing from the cylinder.

The case of two internal reflections is complicated by the occurrence of multiple images from *N*, figure 89, even when one side is half-silvered. This is particularly the case when the cylinder *G* contains water, as in my first experiments; for the glancing angle at *b* is then but 25°. There is an advantage, however, inasmuch as *N* may be placed at a correspondingly large distance from *G*. In spite of the duplicated images, the fringes were found more easily and were less irregular than anticipated. They are liable to be reproduced usually in a different size and orientation in each of the images. They will be found in the colored edge and even in the white glare (caustic) which emanates from the cylindrical surfaces. They could be made quite large, clear, and strong, moreover, although the cylinder used was (as above) an ordinary glass shade. As in case of the triangular interferometer (Chapter X, § 82), the fringes rotate when *N* is displaced parallel to itself or rotated on a vertical axis. To control their size, *N* is to be rotated on a horizontal axis.

69. Modification of the experimental design. Rotating, self-adjusting interferometer.—Inasmuch as the experiment (fig. 88) may suffer from inadequate light, one may notice that there would be no difficulty in rotating the interferometer as indicated in figure 94. Here *tt* is a stout metallic plate or wheel, capable of rotating about the axis *a* at any reasonable speed. The mirrors *N*, *N'*, *M*, *M'*, which may be as thick as desirable, are rigidly and firmly fastened to the plate. Any displacement from centrifugal force is equally effective in case of both of the rays, and must therefore vanish in the self-compensated interferometer. Any flexure outward of *N'* would be balanced by the outward flexure of *m*; *N* would not be flexed and the outward flexure of *m'* is equally effective on both rays. Hence there should be no appreciable change of size of fringes. Any other stresses due to centrifugal forces could be eliminated by reversing the rotation. The illumination at *L* is intermittent, but so rapid in succession that a continuous effect is produced to the eye at the telescope *T*, *L* and *T* being adjusted independently of *tt*. The experiment is again favored by the high luminosity of the achromatic fringes. Here, however, it is necessary that an identical glass-path or path of high index of refraction intervenes between each of the mirrors, *N*, *m*; *m*, *m'*; *m'*, *N'*; *N'*, *N* (fig. 94). For the effect depends on $1-1/\mu^2$; and as this coefficient may be varied between about 0.3 and 0.7 as extreme limits, it should not go unnoticed.

CHAPTER IX.

SHARP SPECTRUM FRINGES WITH AN INDEFINITELY WIDE SLIT, INCLUDING THE SUPERPOSITION OF FRINGES DUE TO THE COLOR AND TO THE OBLIQUITY OF RAYS

70. Introductory.—To obtain sharp spectrum fringes it is necessary, as a rule, to use a slit narrow enough to show the Fraunhofer lines. Hence there is sometimes a deficiency of light from this reason alone. It occurred to me, on producing identical fringes of inclination (achromatics or monochromatics) and of color (dispersion), that by their superposition a slit of any width (or an entire absence of slit) would be admissible, without destroying the fringes in the impure spectrum resulting.

Furthermore, if the edge of the prism is rotated 180° around the axis of the spectro-telescope, the inclination of all spectrum fringes must be symmetrically reversed, *i.e.*, inclination up toward the right (positive) will become inclination down on the right (negative) to the same amount. The identical result may also be reached, independently, by displacing one of the mirrors of the interferometer parallel to itself (path-difference) until the fringes passing through their maximum size reach the opposed inclination and size. Hence there must be a relation of a periodic kind between the displacement of mirror ΔN and rotation of the spectro-telescope $\Delta\varphi$, by which sharpness of fringes in the absence of a slit is conditioned.

This device of locating an angle of rotation of the telescope by sharpness of fringes may possibly be used for other purposes, somewhat after the manner of the halfshade or the sensitive tint; for, if small, they jump suddenly out of an intensely brilliant unbroken spectrum band when a definite $\Delta\varphi$ is reached.

Finally, the fringes, being examples of interference of intense non-reversed spectra, should be available in such experiments as described in the last paper, Chapter VIII, for instance. No deficiency of light need therefore be apprehended.

71. Apparatus.—To fix the ideas it will be necessary to give a diagram of the apparatus (fig. 95) employed. It is the self-adjusting interferometer, very serviceable here because of the large number of separate adjustments to be made, each of which might otherwise require long searching for fringes. White light L from a collimator takes the paths $12345T$ and $16785T$, N being a half-silver. The telescope T is provided with the direct-vision grating g, capable of rotating around the axis $5T$ (angle $\Delta\varphi$). T and g are preferably rotated together, as a rigid system. The mirror MM' consists of two independent, nearly coplanar parts as shown, one of which, M for instance, may be displaced parallel to itself by the micrometer-screw along the

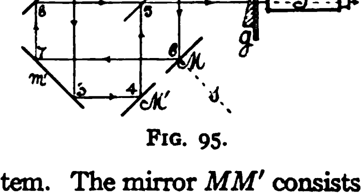


FIG. 95.

normal s (displacement ΔN). Path-difference to the amount $2\Delta N \cos 45°$ is thus introduced, more than sufficient to pass the spectrum fringes through their maximum sizes between extremes of hair-lines. By rotating m on a horizontal axis and m and M' on vertical axes, fringes of all sizes and inclinations when at their maximum may be obtained. The character of the fringes due to inclination is shown by the achromatics and hence the adjustment is made with reference to them. They depart but little, relatively speaking, from their slope throughout the experiment.

72. Equations.—The full analysis of the phenomena of coincidence would have to refer to the whole area of spectrum and would therefore be complicated. It suffices here to exclude the oblique fringes and to consider vertical fringes only, in which case the distribution along the longitudinal axis, r to v, need only be treated. We may therefore begin with the equation

$$n\lambda = 2e\mu \cos r - 2N \tag{1}$$

referring to n fringes of wave-length λ, the thickness, index of refraction, and angle of refraction of (or within) the half-silver being e, μ, r, respectively. N is the air-path excess of either ray, a coordinate independent of λ. The thickness e is virtual, resulting from the fact that the two rays do not traverse the half-silver along identical paths. If they did so e would be zero and the fringes infinite and useless. One ray is usually a little above or below the other, so that a small virtual e is implied and not complete compensation.

If equation (1) is applied to two successive fringes n and $n+1$ in the spectrum and the difference of equations taken, since μ and λ vary,

$$n(\lambda' - \lambda) + \lambda' = 2e(\mu' \cos r' - \mu \cos r) \tag{2}$$

When the difference of order n to $n+1$ is produced in homogeneous light by difference of inclination, λ and μ are constant, and r varies only. For this case the difference equation will be

$$\lambda = 2e\mu(\cos r' - \cos r) \tag{3}$$

We may now impress this on equation (2) by subtracting it therefrom, whence,

$$(n+1)(\lambda' - \lambda) = 2e \cos r'.(\mu' - \mu) \tag{4}$$

Hence if the fringes are small so that λ' and λ, μ' and μ, are nearly the same, equation (4) becomes

$$\frac{d\mu}{d\lambda} = \frac{n}{2e \cos r} = \frac{\mu - N/e \cos r}{\lambda} \tag{5}$$

if the value of n is introduced from (1). The first member may be reduced by the simplified Cauchy equation $\mu = A + B/\lambda^2$, so that

$$-2B/\lambda^2 = \mu - N/e \cos r$$

or

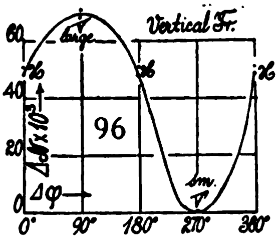
$$\cos r = \frac{N}{e(\mu + 2B/\lambda^2)} = \frac{N}{e(A + 3B/\lambda^2)} \tag{6}$$

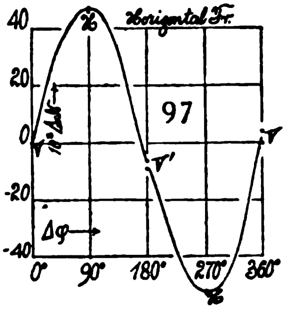
an equation by which the relations of r, N, λ, are determined. But because of the occurrence of r and e the equation is of very little aid in the experiments.

73. Observations.—For the present purposes the case of achromatic fringes, horizontal, vertical, and at about 45°, respectively, will suffice. Moreover, relatively small fringes, requiring much larger displacements (ΔN) than very large fringes, will generally be preferable.

Figure 96 gives the results for vertical achromatic or monochromatic fringes, the ordinates showing the displacement of micrometer ΔN (at M fig. 95) in 10^{-3} cm. and the abscissas the corresponding rotation of spectro-telescope, gT, needed to produce sharp fringes in the spectrum of an indefinitely wide slit. When the fringes are small a few degrees of excessive rotation $\Delta\varphi$, either way, will cause them to vanish completely, so that the orientation for sharp fringes is quite sensitive. The symbols H (horizontal) and V (vertical) refer to the orientation of the edge of the prism or the lines of the grating. The plane of dispersion is thus normal to H and V.

Hence it appears that vertical fringes are left unchanged when the plane of dispersion is vertical (edge of prism horizontal), which is to be expected; for in such a case the light is permanently absent at the absorption bands due to the inclination fringes. On the other hand, when the plane of dispersion is horizontal, the nearly vertical fringes have to pass from the positive to the negative inclination through their maximum size, when the telescope is rotated over 180°, and hence ΔN is very large, particularly so when the fringes are relatively small. In this large displacement of mirror $\Delta N = 0.070$ cm., nearly, small monochromatic fringes will not change their inclination much; but their size will change considerably, and thus at $\Delta\varphi = 90°$ they are large and at $\Delta\varphi = 270°$ small.



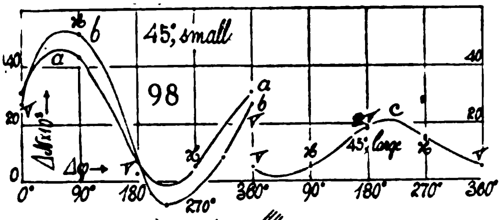


Exactly the opposite conditions are met with when the fringes are nearly horizontal, as in figure 97. In this figure V' lies somewhat below V, as I could not (for incidental reasons) obtain adequately horizontal fringes without extreme difficulty. But this amounts merely to a slight shift of phase in the diagram, which is otherwise the counterpart of figure 96. The fringes were smaller and hence a much larger double amplitude of displacement ($\Delta N = 0.1$ cm.) is here recorded.

Finally, figure 98 gives the results for achromatic fringes at about 45° (estimated by the eye), the curves a and b referring to small fringes, whereas c corresponds to large fringes. The maxima are somewhat near $\Delta\varphi = 45°$ and

225°, though the results are less smooth here from deficiencies in the orientation (45°) of the achromatics. There was no fault to be found with the clearness of fringes or with their abrupt evanescence.

If the spectro-telescope *Tg* (fig. 95) with a very fine slit is rotated, the fringes, as in figure 99, remain parallel to the length of the spectrum passing through the forms *a*, *b*, *c*, *d*, *e*, where at *c* the spectrum is reduced to a single colored line parallel to the slit. The fringes remain parallel to the edge of the prism.



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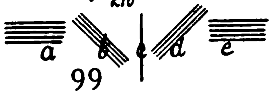

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Hence if the form *b*, for instance, coincides with the achromatic or monochromatic fringes, it will be retained sharply on opening the slit wide, whereas *a*, *c*, *d*, *e*, requiring a fine slit, will vanish with the Fraunhofer lines. In the absence of a slit the whole colored field bursts into sharp fringes whenever the proper angle $\Delta\varphi$ of the telescope is reached. If the slit is a little too broad to show the solar lines distinctly, the monochromatic fringes may often be detected cross-hatching the vague Fraunhofer lines, even when the spectrum fringes are still strong.

If the fringes of a fine slit are at say 45° to the axis of the spectrum, their inclination will change to 135° on passing the stage *c*. However, there is, in such cases, a considerable change of angle relative to the spectrum as well as of size, so that the conditions of compensation are complicated.

74. Summary.—It has been shown in the experiments that the fringes (monochromatic) due to differences of inclination of rays, and the fringes (dispersion) resulting from differences in wave-length of rays may be made of nearly equal size by displacing any mirror of the rectangular interferometer normal to itself (ΔN). The fringes will not, however, generally have the same inclination. This may be imparted to the spectrum fringes by rotating the spectro-telescope (prism edge) on its axis, until the inclinations also coincide. In reality the phenomenon is more complicated as the spectrum fringes change both size and inclination on rotation of the spectrum. In case of the completion of this twofold adjustment the slit of the collimator may be made indefinitely wide or removed altogether (undesirable light is to be screened off). The spectrum fringes may thus be given any intensity of illumination at pleasure, while the wave-length corresponding to any fringe may be found by narrowing the slit until the Fraunhofer lines reappear. When the fringes are small the orientation of the spectro-telescope revolving around its axis may be determined by the appearance and evanescence of fringes. On the other hand, the spectro-fringes, particularly if large, remain clearly enough in the field for the observation of the motion of a large number (*i.e.*, for interferometry) before they vanish.

75. Reversed spectra.—If the linear interferences are produced by placing a grating between *m* and *m'* (fig. 95) and suitably adjusting the apparatus, little effective modification is possible. As the breadth of this phenomenon is independent of dispersion, a prism (preferably 30°) may be placed between *m* and *m'*, as in figure 92, Chapter VIII. The narrow but very luminous spectra, on superposition, etc., then show an intense string of interference beads at the line of symmetry if the slit is also narrow. If *M* and *M'*, figure 95, are quite coplanar and the fringes therefore horizontal, the slit may now be broadened with a corresponding effect on the interferences, but eventually they are lost in the glare of light. When viewed through the spectro-telescope, *gT*, however, they come out distinctly and the slit may now be broadened indefinitely. The striations are found to be very sensitive to the degree of verticality of the slit and slight departures from the vertical throw the striations into opposite inclinations to the horizontal. This accounts for frequent occurrence of arrow-headed forms, as these correspond to a vertical slit.

If the mirrors *M* and *M'* are separated as in figure 95, the fringes are no longer horizontal, as a rule. The spectro-telescope is ineffective and the slit can not be broadened. On displacing *M* or its micrometer-screw *s*, however, the strong duplicated fringes are soon found, and when viewed through *gT* any breadth of slit is permissible. In this way the linear phenomenon may be expanded laterally to an indefinite area and the character of its individual fringes (which were originally point-like) observed in detail. If the reversed spectra are passed through each other, the fringes undergo marked changes, such as from horizontal maxima in a blue field of coincidence to fine vertical lines in a red field, for instance. Each color, moreover, requires a particular adjustment, ΔN, to secure the maximum sharpness of the design. If the fringes are not duplicated they are nearly invisible in the spectro-telescope.

For a given line of coincidence the fringes admit of but little displacement, ΔN. When this is varied the fringes appear in one size and vanish in a markedly different size and inclination.

As a whole the advantages gained in duplicating and enlarging the linear phenomenon are not as striking as is the case with non-reversed spectra, a result to be anticipated from the increasing presence of non-interfering light.

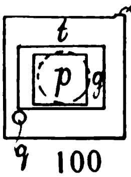
76. The same, continued.—I have already instanced that the roof-shaped or arrow-headed forms of interference patterns also occur with homogeneous light along the line of contact when a cylindrical lens is placed on a plate. One may therefore suppose that the effect of cylindrical variation of thickness normal to the line of contact when wave-length is constant is (formally) closely like the effect of variation in color when thickness is constant (linear phenomenon of reversed spectra). The pattern in the last case is of course apt to be enormously finer or narrower. From this point of view the enlargement of the linear phenomenon may have a direct bearing on the question of interference of light of slightly different wave-length.

When produced by a single dispersion the linear phenomenon is independent of the amount of dispersion, being alike in width for the spectra of a 30° prism and of a strong grating, for instance. It depends, therefore, for its width on the diffraction of the telescopic system. Thus, if one decreases the aperture of the observing telescope by a circular screen, the linear phenomenon increases in width with the loss of resolving power. When the linear phenomenon is viewed through a spectro-telescope (*i.e.*, subjected to a second independent dispersion) this is no longer the case. The phenomenon broadens and shows much more variety of detail, even though there is liable to be deficiency of light.

It is necessary that the original phenomenon shall be symmetric (arrow-heads, or closely packed very eccentric ellipses), otherwise the enlargement merely brings out an awned structure difficult to interpret. The symmetrical linear phenomenon enlarged by the second dispersion broadens out so that the fine lines from each arrow-head may be seen to about five times the width of the sodium lines, on either side of the apices. On widening the slit only those parts which are parallel to the second plane of dispersion are accentuated. Hence, if this is horizontal, the broad-slit phenomenon is bead-like in structure. For oblique planes it becomes more and more linear. Fine fringes may thus be detected.

With the non-symmetric linear phenomenon, the enlargement resulting from the second dispersion sometimes brings out the arrow forms on widening the slit. One or the other side is cut off on closing the slit.

77. Monochromator.—The use of two identical direct-vision gratings (prism gratings) for the purpose of obtaining approximately homogeneous light is not only very convenient but has certain ulterior advantages, provided the light is not deficient. Each of these consists of a cap *C* (fig. 100, front view; fig. 101, sectional plan) fitting the end of a telescope *T* like an ordinary cap, but provided with a plate *rr* in front, to which the swiveling plate *t* is attached by a spring and bolt at *q* and a stop at *s*. The plate *t* carries the grating *g* and prism *p* for direct vision. When not wanted the plate *t* is rotated on *q* to one side. The spring keeps the plate in any position when the telescope *T* (and with it the grating *g*) is rotated on its axis.



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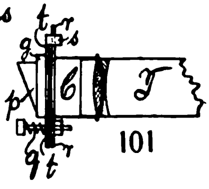

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One of these gratings is to be attached at the collimator (grating I) and the other at the telescope (grating II). It will be seen that if the gratings are similarly oriented the dispersion is summational (D_1+D_2); when either is rotated 180° the dispersion (D_1-D_2) is zero, but the illumination is nevertheless colored, because only the undeviated beam gets through the interferometer, or strikes the second grating. This undeviated beam is usually green; but it is advantageous to have several of these gratings in which the undeviated beam (alteration the prism angle p) is red, yellow, blue, etc. If there is an angle between the planes of dispersion of gratings I and II, the two correronding spectra will cross at an angle and the interferences will be found within the quadrilateral resulting.

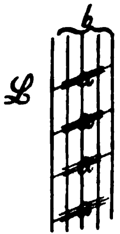
To indicate the uses of these paired apparatus, suppose that the interferometer shows horizontal achromatics in the absence of gratings I (collimator) and II (telescope). Then if II is introduced (dispersion plane horizontal) a luminous spectrum with strong interference bands (horizontally fan-like, opening from blue to red) throughout its length and breadth will be seen. These extend, of course, above and below, far beyond the limits of the achromatic fringes. If the fringes were not horizontal the slit would either have to be fine or the telescope (with II) would have to be rotated on its axis sharply into the right position.

If both gratings I and II are used in parallel, a less luminous ellipse of green light, more highly dispersed and carrying the fringes, will be seen. If I and II are used in opposition a sharp image of the slit in green light carrying the fringes appears. If I is used alone, the green ellipse, less extended and with fringes, appears again. All admit of an indefinitely wide slit on proper rotation of the spectro-telescope.

On applying this apparatus to the linear phenomenon, one may note that there are now three dispersions, D_1, D_2, and d, the latter symbol applying to the prism within the interferometer (fig. 92, Chap. VIII). Hence the dispersions $D_1+D_2\pm d$ or $\pm d$ alone are available; in other words, the spectra corresponding to d are necessarily reversed. The spectra due to D_1 are not. Hence if D_1 and D_2 are opposed, one obtains two sharp slit-images in green, but of different width in view of the $D+d$ and $D-d$ effect. At their line of coincidence is the linear phenomenon, in green light, which on proper focussing will not otherwise differ from the phenomenon with white light. If the gratings I and II are in parallel, the dispersion $D_1+D_2\pm d$ so much exceeds d that we approach the case of non-reversed spectra. The fringes are parallel curved arcs in green, covering a wide vertical region of the spectrum. The curved lines do not admit of a very broad slit; *i.e.*, the fringes suddenly appear, as soon as the Fraunhofer *b*-lines (for instance) coincide; otherwise they vanish at once, precisely as in the case of non-reversed spectra of slightly unequal lengths.

78. Quartz prism.—As there was a good quartz prism in the laboratory, it seemed interesting to place it between the mirrors *m* and *m'* of figure 92, Chapter VIII. The linear phenomenon thus obtained did not differ from the usual form after a single dispersion; but on second dispersion with the spectro-telescope a much coarser laterally broadened pattern was obtained, which in some adjustments (spectrum coincidences) seemed to separate into two vertical and parallel strings of beads. It is natural to refer these to the two indices of refraction of quartz.

The distinctive and interesting feature of quartz is the appearance of a new set of interferences superimposed on the normal set just referred to. An illustration of this is given in figure 102, where *a* represents the duplicated normal group and *b* the new interferences. If the conditions were not elliptic it would be natural to suppose that the new set is due to the interference of the two normal groups side by side, so that the *b* group is a linear phenomenon of the second order under high dispersion. The pattern (fig. 102) is merely one case; very frequently the *b* group of fringes consists of oblique lines, or lie on one side or the other of *a*. They are also much modified as to size by enlarging the slit. The *b* group may even appear alone, closely resembling Fresnellian fringes.



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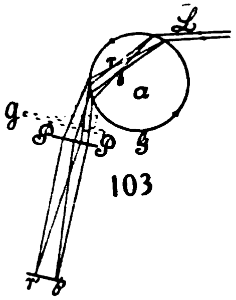
By using a nicol at the collimator, either the right or left side of the group *a* could be eliminated, and the *b* group seemed to appear for an oblique position of the nicol when the two sides of the *a* group were distinct.

In other adjustments, however, two groups of fringes, side by side but in different focal planes and of an entirely different pattern, were obtained, among which the occurrence of three groups was rare. Though I made many experiments I did not come to a definite conclusion as to the phenomenon as a whole. Two groups, of course, are to be anticipated.

CHAPTER X.

MISCELLANEOUS RESULTS OF THE PRECEDING EXPERIMENTS.

79. Spectrum phenomena due to moving motes.—In connection with this work I incidentally came upon a curious phenomenon which seemed to repay special investigation. To describe it, it will be advantageous to first indicate the disposition of apparatus used, as is done in figure 103. Here *L* is a pencil of white light, preferably from a collimator, impinging on the thin cylindrical glass shell *G* about 10 cm. in diameter and containing a solution of mercury potassic iodide about half an inch deep and not quite concentrated. The rays are thus both refracted and dispersed, and on emerging enter the strong objective of a short-range telescope (magnification above 15) of which *PP* is the principal plane and *rb* the narrow spectrum seen in the ocular of the telescope. Properly focussing the latter, the spectrum may be contracted to a vividly colored vertical line.



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If now a strong direct vision grating *g* is inserted in front of the objective, and the telescope is focussed anew, a *sharp* solar spectrum may be obtained. This was a surprise to me, as the cylinder *G*, though thin and clear, was obtained from samples of ordinary glass shades, such as are prized by the lover of stuffed birds. In other words, the cylinder supplies its own slit, as at *r* or *b* in the figure, by refraction. With a narrow beam of sunlight no collimator is needed.

The spectrum will now be found to be filled with short, slender, horizontal shadows, all moving endwise in a common direction, but at different speeds. On pushing the ocular in somewhat further, these shadows become sharply defined lines, all nearly horizontal, of all lengths, from mere points to black lines half the length of the spectrum or more. On attentive observation the black lines are seen to be associated with narrow areas of accentuated brightness, so that diffraction patterns are in question. Occasionally a beautifully complete, slender, spindle-shaped black body with a brilliant narrow frame of light around it will be seen. Arrow-heads holding patches of light on their notched sides are not infrequent, but as a whole the spectrum appears to be intersected with an interminable array of horizontally flying arrows, all shot in a common direction from end to end. With regard to the motion, this is (more usually) horizontally from red to blue; in the middle layers and in the lapse of time always so and not permanently from blue to red. Sometimes both motions were seen to occur together in different levels, the retrograde motion being relatively slow, less pervasive, and confined to the top or the bottom layers. All degrees of speed occurred from a passage through the spectrum in a fraction of a second, to passage lasting over a minute. Under the latter conditions it may happen that the particles actually stop and then begin a retrograde movement, soon to be accelerated in turn. During this period of transition, particles may be seen also to rise and fall, but with relatively great slowness as compared with the usually swift horizontal motion. Some of the arrows are somewhat oblique to the horizontal. Under rare conditions I noticed a line of light instead of shadow. Breadths differ greatly and would naturally depend on focussing.

Usually the motion persists with apparently undiminished swiftness for hours, so that it much outlasts one's patience. Often a single particle can be observed for a minute or more; but after 10 or 20 hours all particles disappear and the spectrum is clear. From this I concluded that the diffractions are not due to local difference of density, etc., of the solution, as I first supposed, but actually originate either in minute solid particles (or, in case of other liquids, in minute air-bubbles) entrapped in the liquid. The slow subsidence and persistence of particles indicate this state of things.

Moreover, I eventually found that the motion of particles as a whole from red to blue or blue to red could be controlled by *rotating* the cylinder G on its axis, a, either counter-clockwise or the reverse, respectively. Brownian motions are excluded, since these are promiscuous and since the magnification is inadequate. It is difficult to conceive how the angular momenta impressed on this solution can persist for hours within it, after the solution is apparently quite at rest, even if the solution is of large density (dense enough to float glass). Probably, since the internal friction of liquids vanishes with the relative velocity of layers, and since the apparent motions are magnified, there is eventually no friction torque left to absorb whatever angular momenta may be renewed or survive. The occurrence of direct and retrograde motions at the same time, separated sharply by a plane of demarcation, is suggestive of vortices. Above this plane, particles move with about the same speed in one direction; below the plane with a very different speed in the opposite direction. A particle which happens to be in the plane in question does not move at all. After a long interval the direction of the motions above and below a plane of demarcation may be found to have reversed, respectively. If a solution is cleared of particles by the lapse of a sufficient time for subsidence, they may be restored by brisk rotation. The number, size, density of color, and speed of the particles increases with the violence of rotation. Gentle rotations in opposite directions leave the particles in a curious state of indecision, after which the definite direction red to blue is adopted.

I have also tried the method where there is symmetrical reflection within the cylinder (as in the case of the rainbow). The results are similar, but less luminous.

To conclude: After the cessation of the initial disturbances, the liquid, left to itself and owing to the presence of motes, shows a persistent motion of its middle layers in the general direction of the impinging beam of light, while the motion of the relatively thin layers at the top or bottom (one or both) is usually persistently retrograde, but slow in comparison. This continues until after the lapse of hours the particles have practically subsided, when the retrograde motion seems to be equally prominent. Even when the liquid is manually rotated clockwise with violence this motion ceases in a few minutes, whereupon the counter-clockwise, red-blue motion, in the direction of the impinging beam sets in vigorously.

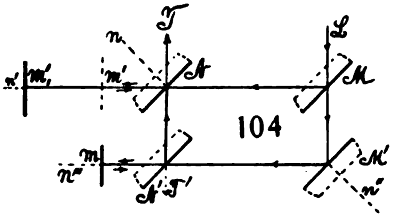
It suffices to add a few statistical remarks. The telescope may be adapted for small distances by placing three diopter spectacle lenses in front of it. Its external focal plane is then only about a foot off and within the liquid. The ray seen in the ocular of the spectro-telescope may be regarded as coming from a virtual slit within the cylinder; or else, on narrowing the incident beam L to within a centimeter (in case of a cylinder 10 cm. in diameter), the diffuse internal caustic has already been similarly narrowed down to a short internal spectrum rb in the figure. Hence, if the solution rotates slowly about the axis A, particles enter the red (r), and leave the blue (b) end, and are therefore seen sharply in the spectrum traveling from red to violet. The reverse is the case if G rotates in the clockwise sense. The small distance rb is thus virtually magnified by the immense dispersion of the grating g (15,000 lines to inch). Since the rays cross *within* the cylinder G, the motion from red to blue will characterize all particles distinctly seen (focus) and rotating counter-clockwise. Finally, this rotation corresponds in a general way with the direction of advance of the light transmitted through the cylinder.

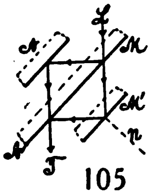
To obtain some idea of the size of particles we may take the breadth of the diffraction arrows in the ocular, which breadth will not usually exceed 0.01 cm. The particles may then be estimated as a fraction (say ½, ⅓) of this.

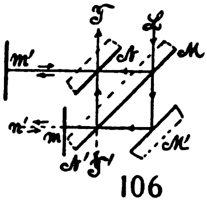
It would be simplest to refer the cause of persistence to a case of vortical motion in the wake of the beam of light traversing the solution. But the invariable occurrence, in the lapse of time, of motion in the middle layers of the liquid in the direction of the impinging light, no matter how the liquid is artificially rotated in the beginning, leaves this explanation unsatisfactory. Such vortices would not be orderly and persistently equivalent to the effect of a pressure in the direction of the beam of light. In case of a black body and a solar constant of 3 gram-calories per minute, the energy per unit of volume or the light pressure in question may be roughly estimated at 7×10^{-5} dynes per square centimeter. Even if but a part of the energy is absorbed by the liquid, this is by no means an insignificant pressure in a medium whose internal friction vanishes with its motion. In fact, if the given estimate be treated as a tangential force relative to the surrounding dark liquid, of about 0.01 viscosity, a speed of 7×10^{-3} cm./sec. (under normal conditions) would correspond to the shear. One may therefore infer that speeds within a tenth millimeter per second, about of the order observed, are not impossible. The very slow but persistent regressive movement at the top and bottom of the layer of liquid remains unexplained.

Furthermore, I was unable to find any adequate correspondence between the swiftness of the motion and the intensity of the impinging beam. Again, the molecular radiometer, in which the thermal gradient is at the same time a pressure gradient, would fall under the same objections. I can only conclude vaguely, therefore, that in some way the local vortices evoked by therma-distribution resolve themselves into a persistent ordered rotation of the cylinder of liquid around its vertical axis, with the regressive motion specified confined to one or two relatively thin layers. In other words, the conditions of hydrostatic equilibrium imply an inclined surface of the liquid, with its maximum head in the region of the illuminated part. But such a structure, with its forces oblique to the surface, is gravitationally unstable. It is difficult to see, however, why the flow which must result should be an orderly rotation of nearly the whole cylinder of liquid.

80. Separated Jamin plates.—In the course of my work the desirability of an interferometer like figure 104, in which the mirrors *M*, *M'*, *N*, *N'*, are parallel (all but *M'* being half-silvered) and an auxiliary mirror, *m*, *m'*, has often presented itself. In such a case the observer at the telescope at *T* can control the slit, being within reach of the collimator at *L*, and at the same time control the micrometer-screw at *n*, normal to *N*. If this screw is insufficient, since the auxiliary mirror is made of two parts, *m* and *m'*, one of these may be on a normal micrometer-screw *n'*. There is no difficulty in obtaining fringes when *m*, *m'*, is removed and observation made at *T'*; or at least the difficulties may be overcome if *M* and *N'* are parts of the same sheet of plate glass, as in figure 105, the parallelogram of rays having been adjusted for a suitable angle (cf. Carnegie Inst. Wash. Pub. No. 249, Part II, § 60). In a test made at random, this gave the spectrum fringes and the achromatics superbly. To equalize the glass-paths in the presence of *m*, *m'*, in order that the achromatics may appear in the minimum number, the half-silvered plate may be set (all at 45°) as indicated in figure 104, where each component ray traverses the plates of equal thickness four times. In the absence of *m*, *m'*, the silver faces of *M'*, figure 104, should be reversed, so that each ray penetrates the glass twice, as shown in figure 105. If this is not done, special glass compensators will have to be used, which usually are an annoyance.







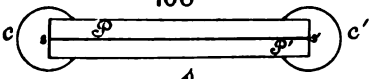
The case of figure 104, however, is apt to resist the endeavors to find either the spectrum fringes or the achromatics, even if *N'* is made continuous as in figure 106. In each case, on reversing *M'* and observing at *T'*, the fringes will be found at once. The reason for this is not far to seek. True, there are supernumerary rays, quite vivid and therefore giving a strong spectra. If the position for fringes were found, they would not, however, escape detection for this reason, and I have often seen good fringes under worse conditions.

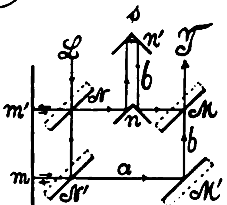
Figures 104 and 106 show that the interfering rays are non-symmetric; one of them, *LMNm'NT* is confined to but one side, the other, *LMM'N'mN'T* takes in the other three sides of the rectangle. Apart from the glass-paths, the latter ray thus has an optic path in excess of the other equal to twice the breadth of the rectangle. Hence the mirror *m'* is to be moved to m'_1, behind *m'* by the equivalent of the breadth in question. This makes it more difficult to locate *m'*; but for some purposes (for instance, if a bulky apparatus, placed between *m* and *m'*, is to be traversed by one ray only) it is a decided advantage. In the case of figure 106, the center of spectrum ellipses may be brought into the field or the achromatic fringes rotated at pleasure, by rotating *MN'* on a horizontal axis. In most cases *L* and *T* may be exchanged at pleasure. The glass reflections from *N* and *N'* (non-silvered sides) may usually be blotted out with small screens on *m* and *m'*. It is more difficult to get rid of the glass reflection from *M*, but it is usually sufficiently one-sided not to be much of an annoyance, if ordinary plate glass is used. As a rule, perfect achromatics are more difficult to obtain by the present method. They are liable to be not quite symmetric, greenish on one side and reddish on the other. The compensator adjustment, if it can be used, avoids much of this.

Another method of using this interferometer, frequently convenient, is shown in figure 107. Here the auxiliary mirrors *m*, *m'*, are coplanar, or nearly so. To elongate the short ray, *LNm'NMT*, a double V-compensator *n*, *n'*, is inserted, each consisting of two mirrors at right angles to each other, with corresponding parts parallel. These mirrors are therefore to be parallel or normal to the set *M*, *M'*, etc. They are conveniently made by silvering a pair of Fizeau bi-plates, inside and outside, and the surfaces must be brilliantly polished. The V-mirror *n* is stationary, whereas *n'* is on a micrometer-screw, actuating it in the direction *s*, preferably parallel to the *b*-side of the ray rectangle. Each ray-path between *n* and *n'* is obviously equal to *b*. The adjustment for parallelism of *n* and *n'* need not be very rigorously made; but in proportion as it is so the direction *ns* may be slightly inclined on any side without destroying the fringes, for the rays enter and issue from the system *n*, *n'*, in parallel. The system *M*, *M'*, *N*, *N'*, *m*, *m'*, may be first adjusted for parallelism and coincidence without *n*, *n'*. The latter may then be inserted and the adjustment repeated by its own leveling-screws at *n* and *n'*. The supernumerary white images (glass side reflections) may be removed at *m* and *m'* and also (partially) at *M*, by small paper screens covering them. The mirrors *M*, *N*, *N'* (half-silvers) must be of equal thickness, as above, and set with their faces in the directions shown by the dotted lines in figure 107. In this case each component ray traverses the identical glass path four times, and the achromatic fringes are obtained in a degree of perfection depending on the equality of the glass-paths. To find the achromatics, it is as usual necessary to find the spectrum fringes first, and to move the micrometer *n's* until these are horizontal. Fringes are enlarged or rotated by turning *M* and *M'*, slightly, on horizontal or vertical axes. Again, by rotating *n n'* (as a whole) on a vertical or a horizontal axis, fringes may be rapidly rotated or enlarged, respectively, while they remain in the field. It may be also done (sometimes) by rotating *M* and *M'* in their own planes 180° (supposing the mirrors to be of ordinary plate) and restoring the parallelism.

Achromatic fringes, if well produced, are capable of enormous enlargement. Thus I used a weak spectacle-glass (1 diopter) with a very strong ocular, obtaining a large telescope quite adequate for the purpose and enormous, flawless fringes with ordinary plate glass in the interferometer. In such a case, sunlight is to be used without a condenser. For the enlargement of the field in the telescope a ground glass screen and sunlight will do very well. In case of ordinary plate glass the fringes lie in a definite focal plane and may lose clearness when displaced.

108





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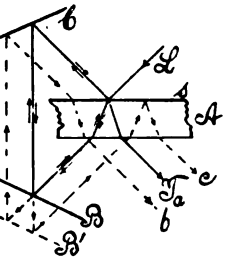


81. Interferometry with the aid of secondary and tertiary achromatics (satellites).—I have described the occurrence of repetitions of the achromatic fringes, on either side, at regular intervals, but with rapidly decreasing brilliance. In some adjustments as many as three distinct equidistant groups on each side of the primary had been noticed. The secondary set is of very common occurrence and may even be sufficiently clear to be mistaken for the primary group, unless a direct comparison is made. It is at first difficult to surmise a reason for such a phenomenon in the case of white light. Somewhere in the train of apparatus there is a second and independent cause for interference; *i.e.*, a special path-difference, which, when superimposed on either one or the other of the rays of the interferometer one or more times, produces the phenomenon in question.

I have since made some further investigations of these interesting fringes, showing their availability in interferometry. A pair of identical half-silver plates *P*, *P'*, were prepared as before (fig. 108). These were then pressed together on their silvered sides *s*, *s'*, by steel clips, *c*, *c'*. The plates hold between them what has been called a half-silvered air-film and thus offer the requisite path-differences, increasing in regular steps, in accordance with the number of reflection which occur within the film.

These plates may now be placed anywhere normal to the rays, either at the collimator or in front of the telescope, and it will be found that their presence is usually accompanied by the presence of the satellites in question. The latter may be made distinct by slightly rotating the plate (fig. 108) on a vertical or horizontal axis, which brings about a more perfect coincidence of the corresponding white slit-images. The edges of these may, in fact, sometimes be detected. Using this apparatus, I made a few measurements on the position of seven successive groups, obtained in a specially good adjustment. The micrometer reading ΔN corresponding to their position was as follows:

$$10^4\Delta N = 251,\ 234,\ 217,\ 200,\ 182,\ 165,\ 148 \text{ cm.},$$

the strong fringes being at $\Delta N = 0.0200$ cm., a normal micrometer as at m', figure 104, being used. On compressing the plates, figure 108, more tightly, I found $10^4\Delta N = 227,\ 214,\ 200,\ 186,\ 173$ cm.

The constant differences, 0.0017 cm. in the first example and 0.0013 in the second, can be nothing more (since both spaces count doubly) than the thickness of the air-film inclosed between the plates. If we call this thickness s, and denote the optic paths of the two rays of the interferometer by r and r', the following possibilities of interference present themselves:

	r and r'		primary.
$r+2s$ and r'		r and $r'+2s$	secondary.
$r+4s$ and r'		r and $r'+4s$	tertiary.

and it appears from the above data that three reflections within the air-film still produce an observable effect, even with the present ordinary plate glass.

This result, therefore, suggests the design of a peculiar displacement interferometer, using white light and the achromatic fringes; for it is merely necessary to put one of the two parallel plates P and P', figure 108, on a micrometer, in order to specify the distance apart of these plates in terms of the ocular distance of the primary, secondary, etc., achromatics seen in the telescope. They would all coincide when P and P' are in optical contact. Since these groups are here nearly of the same size, it becomes a rather interesting experimental question to see how far the separation of P and P' can be practically carried. I made a tentative experiment by inclosing a piece of paper, 0.1 mm. thick, between the edges of the silver doublet and obtained the two secondary fringes, one of which, however, was much larger and the other smaller than the primary fringe. A number of measurements were made, showing that the large secondary fringes corresponded to a position $\Delta N = 0.013$, 0.0112 cm., and the smaller to $\Delta N = -0.0112$ and -0.0111 cm. from the primary, agreeing, in view of the preceding measurements, with the surmise. The plates were not good enough to warrant further spacing.

82. The triangular self-adjusting interferometer.—This apparatus is shown in figure 109, in which A is a half-silver reflecting at the s face, B and C opaque mirrors. The white light a arrives at L from a collimator and is observed, when the rays are coincident, through the telescope at T. In order that such coincidence may be established the mirror B must be on a micrometer-screw, displacing *B* parallel to itself. The slit-images may be superposed by rotating *A* on a vertical axis, and the local coincidence is then secured at *B* as specified. The fringes should then be in the field. From the complete compensation they are liable to be large. Rotation of *B* or *C* on a vertical axis moves the slit-images through the field of the telescope. Horizontal axes (preferably at *A*) provide for vertical coincidence. Reflection from the glass face can usually be blotted out by a small screen on *C*, particularly when *s* is reversed.

It seemed reasonable to suppose that the rays of this instrument could also be appreciably separated, for instance, by moving *B* to *B'*, as the rays *b*, *c* would interfere in the telescope. But this attempt did not succeed to a useful extent. In fact, even slight displacement of the mirror *B* passes the fringes through a maximum with rotation. The same thing happens on rotation of any mirror, on either a horizontal or vertical axis, so that the fringes are large and fugitive and difficult to control. To obtain considerable displacement of *B*, the mirror *C* must be reciprocally rotated on a vertical axis with the fringes continually in the field. Similarly *B* and *C* may be rotated together, etc. The apparatus is nevertheless interesting, and if optic plate were used (any wedge angle in *A* is here of serious consequence) and small angles of incidence were chosen, it might be useful.

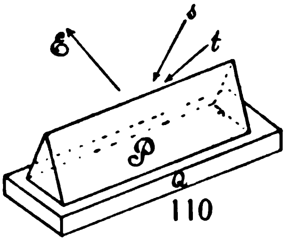
Thus, for instance, I had hoped to use the apparatus for an experiment on the Fresnel coefficient (see Chapter VIII) for the case of two internal reflections in the rotating cylinder *G*, figure 89. The cylinder is in such a case obviously used to greater advantage. But apart from the general difficulty of this installation, the low intensity of the twice-reflected spectra militated against the practical availability of the method, which is unfortunate, as the two spectra (unlike the case above) have the same focus in *T*. There being respectively two and three reflections, the fringes are those of reversed spectra.

To put fringes of any particular type in the white field, the mirror *B*, figure 109, may be rotated on a horizontal axis. If a vertical group is produced in this way, this may be enlarged indefinitely by displacing *B* parallel to itself. The fringes then remain vertical throughout, but their direction of motion changes (with abrupt rotation) at the maximum. If *B* is rotated on a vertical axis, the fringes in the now moving slit-image similarly change size, remaining vertical. The phenomenon is here not apt to be symmetric on the two sides of the maximum. Inclined fringes, while following the same scheme of enlargement, continually change their inclination, the fringe pattern being ultimately hyperbolic.

83. Herschel's fringes.—Herschel's fringes, as produced by the familiar apparatus consisting of a right-angled prism reposing with its broad face on a plate of obsidian, present the well-known group of achromatic fringes running parallel to the arc or limit of total reflection. Observation is made in a direction normal to the edge of the prism.

It occurred to me that the phenomenon could be made much more striking and of wider scope if a long 60° prism were used and observation made in a plane of symmetry *parallel* to the edge of the prism. In the interest of variety, moreover, it is preferable not to employ strictly accurate surfaces, so that the prisms with which grandfather used to decorate his gas-fixtures will, as a rule, suffice admirably. In figure 110, *P* is such a prism (truncated) on a plate of obsidian *Q*, the long edges being normal to a white window curtain at *L* near by, illuminated with sunlight or daylight; or any light toward the front overhead is good.

The rays that are wanted, *s*, will enter symmetrically at a mean angle of about 30° to the vertical and after reflection at the base of the prism and the plate reach in the eye in the direction *E*. The rays totally reflected, *t*, come from a greater angle to the vertical and are not wanted.



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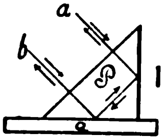

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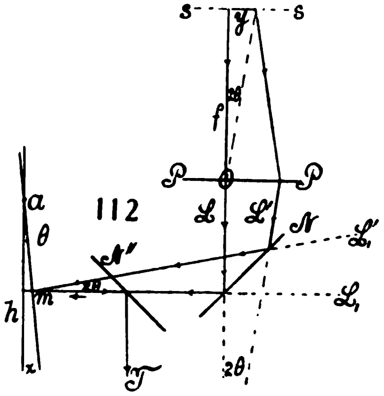

112

The limit of total reflection here (also easily recognized) is usually a sharp parabolic or cuspidal apex. The light seen through either face enters by the opposed face. On looking down from a steeper angle and with properly selected faces, brilliant groups of complete confocal ellipses (major axes 0.5 to over 2 inches), or of confocal hyperbolæ may be seen in each of the roof-faces. To find advantageous face combinations, the three faces of each prism should be examined in succession, and it is well to rub *P* on *Q* to improve the contact. On moving the eye fore and aft or using different pressures, any type of ellipse with white or colored disk may be produced at pleasure. It is usually preferable to use a shorter plate *Q* than is given in the figure, about one-half the length of the prism.

When well produced the ellipses may also be seen by side-light, with different patterns in the two roof-faces.

The type of interference figure clearly depends on micrometric differences of the faces in contact. The ellipses are Newton's rings modified by the color dispersion of the glass. The hyperbolæ, however, are about equally frequent; but their character is less easily stated. They probably originated in cylindrics.

84. The same, continued.—The case of the 45° to 90° prism (fig. 111) with the right-angles faces, respectively horizontal (on the plate) and vertical, is also interesting; for here the ellipses are apt to be circles with each of the two groups *a* and *b* seen after two reflections, one in each of the orthogonal faces. The light should enter nearly normal to the oblique face of the prism.

Reflection from the face of the observer, if in high light, is adequate, but it is preferable to look through a horizontal slot in a well-illuminated piece of white cardboard, held at a small distance parallel to the oblique face of *P*. The vertical side of *P* is therefore toward the source of light. Totally reflected rays are again to be avoided; but the circles about axes, such as *a* and *b*, open out through flower-like forms, into the usual parallel fringes at the limit of total reflection, when the angle is approached (successively) by the two faces. After this, in a well-adjusted apparatus, the black spot is apt to stand out with striking clearness on an intensely white surface.

I may add a correlative observation: If a cylindrical lens (say 1 diopter) is placed on a plate and illuminated with homogeneous light, the interference pattern consists of a succession of equidistant arrowheads along the line of contact, all pointing in its direction. Now, these are the very forms observed in the interferences of reversed spectra along the line of coincidence of spectra, except that the latter are apt to be far narrower than the former. It seems, therefore, as if the effect of color variation in one case and of the cylindric increase of thickness of air-film in the other were formally capable of like treatment.

85. Measurement of small angles by a half-silver plate.—In case of the rectangular interferometer the slit-images contain a second method of measuring small angles, which, though naturally inferior to the fringe method, is not insensitive and may often be used with advantage independently. I shall indicate this briefly by the aid of figure 112. Here LL' or L_1L_1' represent the directions of rays of parallel white light issuing from a collimator. If the latter are used the mirror N is superfluous; but it is an essential part of the interferometer. They pass through the half-silver N'', are reflected from the auxiliary mirror m in two positions, respectively, Θ radians apart, and then enter the telescope at T in parallel, T being the fixed line of sight. Let PP' be the principal plane of the objective of the collimator with the optical center at O and a focal length f, and let a micrometer plate *ss* replace the slit with its fine linear scale running parallel to the diagram. Then it is obvious that if y is the displacement of this scale seen in the telescope,

$$\Theta = y/2f = x/h$$

if an object at a distance h from the axle a moves over the distance x.

For instance, let a centimeter divided into 100 parts be used at *ss* and let f be a meter. If T is a strong telescope (magnification 25), 10^{-3} cm. may be estimated at T, so that $\Theta = 10^{-3}/2 \times 10^2 = 5 \times 10^{-6}$; *i.e.*, the limit of measurement is a second of arc. Good instruments are needed if f is a meter. I have more often worked with $f = 25$ to 50 cm. when ordinary facilities (common plate) suffice and the results are still very sharp. A small gas-flame suffices for illuminating *ss*. If $h = 10$ cm., x is determined to about a wave-length.

CHAPTER XI.

INTERFEROMETER OBSERVATIONS AND ACHROMATIC FRINGES IN CONNECTION WITH THE HORIZONTAL PENDULUM.

86. Apparatus and data.—The observations begun in my last report * were continued during the remaining summer months, and the graphs obtained may conveniently be considered here, particularly as they bring out more clearly than has hitherto been the case the effect of the temperature of the laboratory on the horizontal pendulum. Figures 57, 58, 59 of the apparatus of the last report should be consulted.

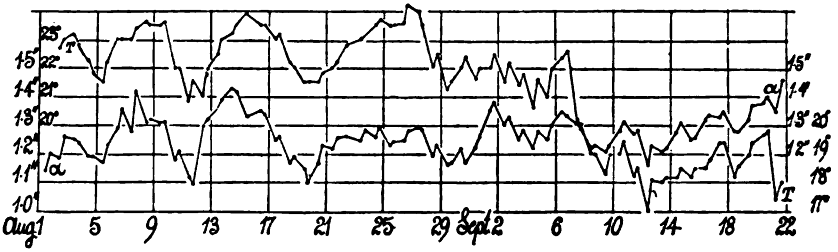

FIG. 113.

In the graph, figure 113, the initially lower curve shows the change of inclination of the pier in seconds of arc, the upper curve the corresponding temperature in degrees centigrade of the basement laboratory room. Observations were usually made at 10 a. m. and 6 p. m. Between August 1 and 21 the resemblance of the two curves is marked. After that the gross resemblance is no longer so striking, but it nevertheless remains in the details. There is a lack of quantitative equivalence only.

Toward September 8 the two curves cross each other, *i.e.*, the relatively enormous fall in temperature due to the cold season has had no corresponding effect on the pendulum. This is very curious, for thereafter the detailed resemblance of the two curves (the temperature graph being below the inclination graph) is in fact astonishing. The large fall of temperature toward September 21, however, again fails of similarly marked expression in the inclination graph. The alternations of temperature are thus sharply indicated in both curves, while large changes in one direction produce less pronounced resemblances. This behavior is very much like the backlash of a screw, but it is exceedingly difficult to surmise how such a discrepancy can occur when an ocular plate scale, only, was used for measurement. The effect of rain, apart from temperature, appeared so inconsistent that it is hardly worth considering.

The question is, therefore, where this temperature discrepancy has its seat. It can hardly be in the interferometer, where the parts are of the same metal, except at the virtually tetrahedral bracket, consisting of horizontal iron rods supporting the interferometer and the iron brace downward from their ends to the pier; for here the oblique part is of iron and the vertical part of brick, and there might be differential expansion. But the interferometer would not be sensitive to this motion, of which I convinced myself by bearing down on the ends of the bracket with the hand. No adequate displacement of fringes resulted. Hence it appears probable that what is observed is the warping of the pier, etc., as a result of the inward progress of the successive isotherms through it, beginning at the part least protected by surrounding walls. At all events, this temperature feature is so serious that a few tenths of a degree centigrade can not be overlooked. It is, then, upon a substratum undergoing continuous warping that any other phenomenon of more relevant interest must be superimposed. This would make their detection and interpretation so different that only under conditions of adequately constant temperature could the extreme sensitiveness realized be made practically available.

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* Carnegie Inst. Wash. Pub. No. 249, part III, §§47–51, 1919.

CHAPTER XII.

GRAVITATIONAL EXPERIMENTS.

87. Introductory.—In my last report * I inferred that it might be worth while to study the motion of the usual gravitational needle in vacuo during a brief interval after the attracting mass has been put in place. If this can be done before the torque of the quartz fiber has become appreciable, the weights at the end of the needle may as a first approximation be supposed to be in uniformly varied motion. It would then be possible to deduce the Newtonian constant in terms of the initial acceleration of the needle, or observationally in terms of its displacement during successive small times. It is necessary, therefore, that the smallest possible displacements of the needle should be measured, and hence the quadratic interferometer will be advantageously used for this purpose. It is my purpose in the present paper to pursue this investigation further.

88. Apparatus.—The apparatus used is essentially the same as that already described and consists of a very flat box with large plate-glass sides parallel to the needle inclosed and normal to the rays from the interferometer. In the present experiments this box is made air-tight with cement and is capable of being exhausted.

The needle consisted originally of a shaft of straw 25.6 cm. long, carrying a small shot of 0.61 gram at either end and suspended from a delicate quartz fiber 17 cm. long. Its air-damped period was about 18 or 20 minutes. Symmetrically to the middle of the needle two parallel small light mirrors were adjustably attached to receive the beams of the interferometer. The needle weighed 1.49 grams.

The attracting weight was a ball of lead 949 grams mass and could be moved expeditiously on a circular track to 4.2 cm. on either side of the shots. It was sufficient to act on one end of the needle only. This disposition was maintained in the earlier experiments, in which no exhaustion was attempted. The new modifications of the apparatus will be described in connection with the observations.

The arrangement of the gravitational needle relative to the interferometer is adequately shown in figure 117. White light L from a collimator is guided by the mirrors N, N', N'', N''' and m, m', on the needle, into the telescope at T. The mirrors N,N'', N''', are half-silvered. The mirror N' is on a micrometer with its screw in the normal direction n. The attracting mass M is moved alternately from M to M' on the track t, adequately supplied by a stout crank provided with adjustable stops. The breadth b of the ray parallelogram was about 10 cm. and is the same as the mean distance from m to m', the quartz fiber being between. In place of n a double offset micrometer n', n'', n''', virtually normal to the rays, was also tested. The achromatic fringes were used.

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* Carnegie Inst. Wash. Pub. No. 249, part III, Chapter IV.

89. Needle in air. The two methods.—To obtain some notion of the behavior to be expected, a number of experiments were made in series with the needle air-damped. Its motion was then practically aperiodic. With the quartz fiber used, the total deflections were so large that there are two methods for measuring the displacement of the small masses m, m'. For one can express the angle described by the needle either in terms of the displacement of the slit-image in the telescope relative to a fixed ocular scale or to a fixed collimator micrometer-scale; or again, with far greater accuracy, one can use the achromatic fringes moving within the slit-image and register their displacement either by the same ocular or collimator scale micrometer, or by the bodily displacement of the mirror N', figure 117, along the micrometer-screw n. Both methods give identical results, the former being coarse as compared with the latter. I first used a collimator micrometer in preference. When put in place of the slit of the collimator it may be adjusted laterally at pleasure and fixed in any position. Different scales may be used. (Compare Chapter X, § 85.)

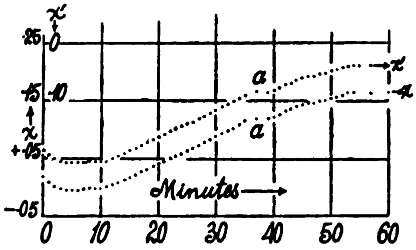


FIG. 114.

If x is the actual displacement of the mass m and θ the corresponding angular displacement of the needle of radius r (between centers of m and m'), $x=r\theta$. If y is the observed displacement of the image of the collimator micrometer in the telescope (since reflected rays are in question), $\theta=y/2f$, f being the (large) focal length of the collimator, so that $x=ry/2f$. If $f=44.5$ cm. and $r=12.6$ cm., $x=0.142\ y$.

Again, on the interferometer, if ΔN is the displacement of the mirror micrometer, $b=10$ cm. the breadth of the ray parallelogram and $i=45°$ the angle of incidence, $\Delta N \cos i=b\theta$ so that

$$x=r\Delta N \cos i/b=12.6\times\Delta N\times 0.707/10=0.891\Delta N$$

Both methods give identical results. To test this, experiments in table 6 and figure 114 were made over a wide range, but without aiming at special accuracy. The needle started from its position of equilibrium with the mass M placed on the left 4.2 cm. from m at zero minutes. Observations were thereafter taken at the end of each succeeding minute (t) for about an hour. The displacement x was computed from y and x' from Δx. In figure 114, x' is laid off downward in centimeters. The two curves are manifestly identical throughout, though naturally starting with a different (arbitrary) zero position on the two micrometers. At *a* the subdued sunlight used was intensified and a marked break in the progress of the curve at once ensues. This is obviously a temperature or air-drift effect, acting contrariwise. As the mirrors are equidistant from the center, one beam is stronger than the other. The total or final displacement, moreover, is too large an amplitude, being over $x=0.15$ cm. But the difficulties and dangers are well given by figure 114.

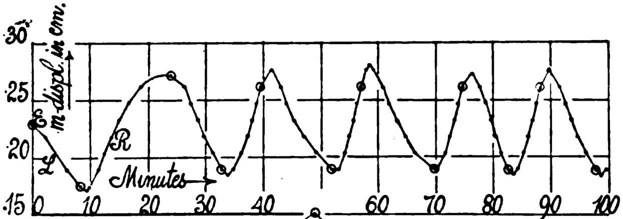
TABLE 6.—Slit-image and micrometer data. $x=0.142y$; $x'=0.89\Delta N$; $M=949$ grams; $m=0.62$ gram. Subdued sunlight.

t	10^2y	$10^4\Delta N$	10^3x	$10^4x'$	t	10^2y	$10^4\Delta N$	10^3x	$10^4x'$
sec.	*cm.*	*cm.*	*cm.*	*cm.*	*sec.*	*cm.*	*cm.*	*cm.*	*cm.*
0	14	2,085	20	1,856	28	55	1,430	78	1,273
1	6	2,186	8	1,946	29	58	1,372	82	1,221
2	+ 1	2,258	1	2,010	30	62	1,313	88	1,169
3	− 2	2,310	− 3	2,056	31	65	1,252	92	1,114
4	− 4	2,336	− 6	2,079	32	68	1,200	97	1,068
5	− 4	2,342	− 6	2,084	33	72	1,143	102	1,017
6	− 4	2,335	− 6	2,078	34	76	1,092	108	972
7	− 3	2,320	− 4	2,065	35	80	1,025	114	912
8	− 2	2,318	− 3	2,063	37*	84	957	119	854
9	− 2	"	− 3	"	39	83	966	118	860
10	− 2	"	− 3	"	40	85	941	121	837
11	0	2,292	0	2,040	41	88	881	125	784
12	+ 2	2,248	+ 3	2,001	42	92	826	131	735
13	5	2,198	7	1,956	43	95	795	135	708
14	9	2,155	13	1,920	44	98	732	139	651
15	11	2,110	16	1,878	45	100	692	142	616
16	15	2,056	21	1,830	46	102	665	145	592
17	18	2,002	26	1,782	47	103	648	146	577
18	21	1,944	30	1,730	48	105	621	149	553
19	25	1,896	35	1,687	49	106	584	150	520
20	28	1,845	40	1,642	50	108	551	153	490
21	31	1,791	44	1,594	51	110	520	156	463
22	35	1,735	50	1,544	52	112	480	158	427
23	38	1,680	54	1,495	53	115	455	163	405
24	41	1,630	58	1,451	54	115	441	163	392
25	44	1,586	62	1,412	57	115	440	163	391
26	47	1,540	67	1,371	60	116	....	165	....
27	50	1,492	71	1,328					

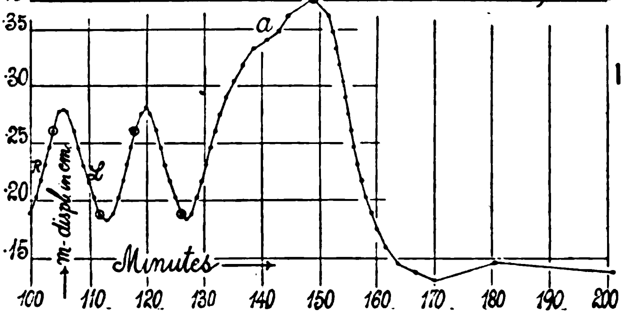
* Light subdued.

Hence in the following experiments part of a Welsbach mantle was used as a source of light; and as the deflections *x* are relatively quite large, it was not thought necessary to make fringe readings throughout for the purposes here in question. The observations, moreover, will be given graphically for convenience, and the tables, which were computed in full, removed.

90. Reversal at symmetrical positions.—In the graph (figs. 115, 116, constructed with the *m* displacement in centimeters as ordinates, in relation to time in minutes as abscissas), the large attracting mass *M* was reversed when the deflection *x* had reached a certain mean departure (at $x=0.38$ and 0.52 cm.) from the equilibrium position with which the series begin. Inertia carries the needle beyond the turning-points indicated by circles, and the segments are at first sinusoidal in form. They finally became more nearly straight in their branches or V-shaped. At first, furthermore, the *M* effect is stronger when this mass is on the right (*R*) than when on the left (*L*) of *m*; but eventually (after 100 minutes) the two effects are practically identical. Even if the needle were not quite symmetric, this effect is probably extraneous, since it



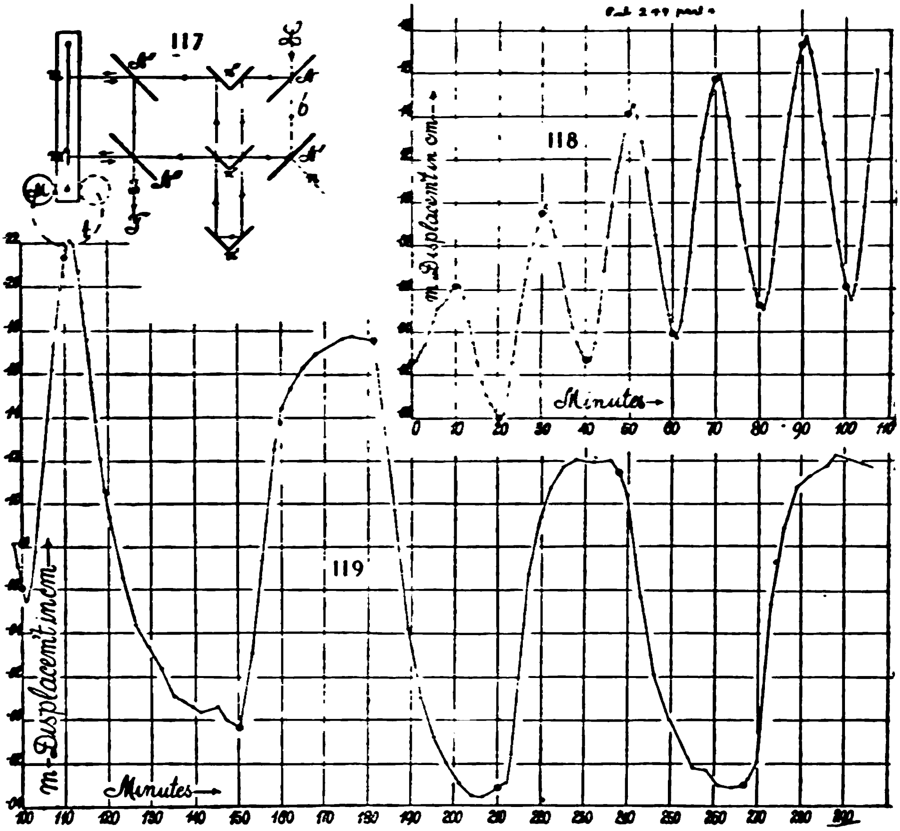
115



116

vanishes. After 125 minutes the needle was allowed to swing for a time exceeding its period. In the first branch some foreign influence shows itself at *a*. The second branch is very uniform and precipitate, but the double amplitude obtained is too large.

91. Reversal after equal time intervals.—In contrast with this the graph figures 118 and 119 show the motion when the reversal of *M* takes place every 10 minutes for 110 minutes and finally, after an interval exceeding the period of the needle. The curve shows the behavior of the needle during about 5 hours, and starts from the equilibrium position. The needle had been rehung. The turning-points of *M* are indicated by small circles and the excess of motion thereafter is due to inertia. During the first 100 minutes the motion proceeds with remarkable regularity. Both elongations are gradually approaching a maximum, from which, if the logarithmic decrement λ and the rotation coefficient *b* were known, the gravitational force could be computed. There is, nevertheless, a persistent drift of the needle to position much beyond its equilibrium position. This might be due to lack of symmetry in the location of *m* relatively to the two positions of *M*, but is probably the result of extraneous causes, and after 100 minutes shows a marked and sudden increase. During the remaining full periods this effect vanishes again and the double amplitudes run through successive values of 0.22, 0.18, 0.21, 0.15, 0.14, and 0.15 cm., the latter being nearly symmetrical to the equilibrium position. At the end of a period there is always some irregular oscillation of fluttering, but obviously



the motion of the needle is practically aperiodic. If we call the double amplitude x', the torsion coefficient b, the gravitational force F, we may write

$$x_a = \frac{F}{b}\frac{e^{\lambda}+1}{e^{\lambda}-1}$$

where $F/b = 0.115$ (double amplitude) nearly, from the static experiments of the next section. Hence (using the last values)

$$x_a = 0.15 = 0.115(\epsilon^{\lambda}+1)/(\epsilon^{\lambda}-1),$$

so that the damping coefficient would be about $\epsilon^{\lambda} = 7.6$. The observed coefficient would, however, seem to be larger than this.

92. Static elongation.—To complete the evidence as to the character of the behavior of the needle, measurements of the elongations were made at long intervals apart, with the attracting mass M alternately on one side and the other of the needle. The results are given in table 7. The apparatus was kept in the dark; nevertheless the individual results are curiously irregular, showing the interference of a foreign effect. It is difficult to suggest a cause, but laboratory tremor affecting the suspension and the warping of the straw shaft of the needle are here possibly associated with a temperature effect. The mean data of each day are consistent and from them the result $2x_a = 0.115$ cm. may be inferred. Thus the deflection due to $M = 949$g., at 4.2 cm. from the small weight $m = 0.61$g. is $x_a = 0.057$ cm., a little over a half millimeter when the distance from center of the needle is 12.6 cm. This result has already been used in § 91.

TABLE 7.—Maximum static elongations.

Date.	Time.	$10^2 y$	Mean $10^2 \Delta y$	$10^3 \times 2x_a$	Date.	Time.	$10^2 y$	Mean $10^2 \Delta y$	$10^3 \times 2x_a$
		cm.	*cm.*	*cm.*			*cm.*	*cm.*	*cm.*
Aug. 28	$5^h 15^m$	93	82	116	Aug. 30	$6^h 10^m$	115		
	6 25	175			Aug. 31	10 30	106	83	118
Aug. 29	10 50	170	80	114		11 55	176		
	12 25	88				12 55	86		
	2 5	170				2 5	174		
	4 3	86				3 35	90		
	5 50	162				4 40	188		
Aug. 30	10 20	168	78	111		5 45	102		
	11 47	104			Sept. 1	11 5	145	81	115
	2 30	175				12 30	73		
	4 55	85				1 20	165		
	5 43	193				5 00	75		

With these data we may inquire as to the time-limits of approximately uniformly varied motion at the outset of the experiment, with the needle ($m = 0.5$ g) in vacuo. We may write for the first departure from equilibrium

$$(1) \qquad \gamma Mm/R^2 - \tau x/h^2 = 2m\alpha$$

since x/h is the angular deflection if $2h$ is the length of the needle, α the acceleration, and τ the torsion modulus. At the elongation $\alpha = 0$ and therefore, statically, using the preceding data and an approximate γ

$$6.7 \times 10^{-8} \frac{10^3 \times 0.50}{(4.2)^2} = \frac{\tau}{h^2} 0.057$$

so that $\tau/h^2 = 3.3 \times 10^{-5}$. If during a small interval t the motion is appreciably uniformly varied

$$\gamma M/2R^2 = \alpha \quad \text{and} \quad x = \gamma M t^2/4R^2$$

Thus, if $t = 10^2$ sec.,

$$\alpha = 1.9 \times 10^{-6} \text{ cm./sec.}^2$$
$$x = 9.5 \times 10^{-3}$$
$$F = 2m\alpha = 1.9 \times 10^{-6}; \; \tau x/h^2 = 31.3 \times 10^{-8}$$

or the error is

$$31.3 \times 10^{-8}/1.9 \times 10^{-6} = 16.5 \times 10^{-2}, \text{ or about 17 per cent.}$$

Equation (1) may be stated, using the values of α and x,

$$\gamma M(m - \tau t^2/4h^2)/R^2 = 2m\alpha$$

which shows that to reduce the very large theoretical error t must be much within 100 seconds. Furthermore, m must be made as large as possible compatible with the given tenacity of the fiber and the importance of a long needle is manifest. M and R do not affect the theoretical result, but they do determine the micrometer reading, for

$$\Delta N \cos i = b\theta \text{ and } \theta = x/h \quad \text{or} \quad \Delta N = \frac{b}{h \cos i}\frac{\gamma M}{4R^2}t^2$$

If $t = 100$ sec., $b = 10$ cm.

$$\Delta N = 0.0107 \text{ cm.}$$

A whole fringe would thus be about 0.4 per cent, while fractions of a fringe could certainly be estimated in case an ocular micrometer or the like is used. There is, however, no need of totally neglecting $\tau t^2/4h^2$. It may be treated as a correction to eliminate the outstanding 17 per cent, if t is as large as 100 seconds. The problem is not easy, but it seems worth a serious trial.

93. Recent work.—In the further pursuit of this subject I endeavored to exhaust the apparatus, believing that even if the viscosity of air does not appreciably change, the annoyance due to convection currents could be eliminated. The experiment was disastrous, however; for in spite of the thick plate of glass, the vessel suddenly burst at high exhaustions, quite destroying the fine instrumental contents, among which I particularly regretted the quartz fiber. Another case of much greater strength and adapted for high vacua was then constructed and tested; but I have not, up to the present date, been able to find a suitable quartz fiber. In those examined, even if the motion of the needle was little more than creeping and with the old annoying tendency to rest persistently on the sides, the effect of gravitational attraction proved to be quite inadequate. The work, however, will be pursued during the present summer.

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